The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before January 1, 2009. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring, 2009 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)622-3051)

NEW PROBLEMS 624-631

Problem 624. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, Illinois.*

Given a tetrahedron, prove that two triangles can be formed such that the lengths of the six triangle sides equal to the lengths of the six edges of the tetrahedron. Prove that the converse is not true.

Problem 625. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, Illinois.*

All of the integers from 1 through 999999 are written in a row. All of the zeros are erased. Each of the remaining **digits** is separately inverted and the sum S is computed. Let T be the sum of the reciprocals of the digits 1 through 9. Show that S/T is an integer and find it.

Problem 626. *Proposed by David Rose, Florida Southern College, Lakeland, FL.*

Two values are randomly selected from the uniform distribution on the interval (0,L). They create three subintervals of the interval [0,L]. What is the probability that the lengths of the three subintervals are the lengths of the sides of some triangle?

Problem 627. *Proposed by Ken Dutch, Eastern Kentucky University, Richmond, KY.*

Suppose that the artist Krypto wants to form several rows of blocks 10 feet wide. He only wants to use two types of blocks - one type is one foot wide and the other is two feet wide. He wants to form a row for every possible pattern of blocks (order matters). How many rows will have have to make? How many of each type of block will he have to use?

Problem 628. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let F_n be the nth Fibonacci number defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that

$$\left(\frac{1}{\sqrt{n}} \sum_{k=1}^{n} F_k \tanh F_k \right)^2 + \left(\frac{1}{\sqrt{n}} \sum_{k=1}^{n} F_k \, sech F_k \right)^2 \leq F_n F_{n+1}$$

Problem 629. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a, b, c be positive real numbers greater than or equal to 1. Prove that

$$\frac{a^{1/a}}{b^{1/b} + c^{1/c}} + \frac{b^{1/b}}{a^{1/a} + c^{1/c}} + \frac{c^{1/c}}{b^{1/b} + a^{1/a}} < 2$$

Problem 630. *Proposed by the editor.*

Suppose that $\log_x y + \log_y x$ is a positive integer. Prove that $(\log_x y)^n + (\log_y x)^n$ is an integer for all positive integers *n*.

Problem 631. *Proposed by the editor.*

The Columbus State University Problem of the Week for March 10, 2008 asked for the three smallest positive integers that could not be written as the difference of two positive prime numbers. These turn out to be primes. Prove that there are infinitely many positive primes that cannot be written as the difference of two positive prime integers. Also prove that there are infinitely many pairs of positive integers (n, n+2) that cannot be written as the difference of two positive primes.