THE PENTAGON

A Mathematics Magazine for Students

Volume 65 Number 1 Fall 2005

Contents

Typeset in Scientific WorkPlace Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be provided in both electronic and typewritten form. The electronic copy can be sent as an e-mail attachment (preferred) or on disk. Either a TeX file or Word document is acceptable. The typewritten copy should be double-spaced with wide margins on white paper. Standard notational conventions should be respected. Any special symbols not typed should be carefully inserted by hand in black ink. Graphs, tables, or other materials taken form copyrighted works MUST be accompanied by an appropriate release form the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

Individual domestic subscriptions: $5.00 for two issues (one year) or $10.00 for four issues (two years); individual foreign subscriptions: $7.00 (US) for two issues (one year). Library rate: $10.00 (US) for two issues (one year) or $20.00 (US) for four issues (two years). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced. Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

Editor:

Charles N. Curtis
Department of Mathematics
Missouri Southern State University
3950 E Newman Road
Joplin, MO 64801-1595
curtis-c@mssu.edu

Business Manager:

Richard A. Laird
Department of Mathematics
Missouri Southern State University
3950 E Newman Road
Joplin, MO 64801-1595
laird-r@mssu.edu

Associate Editors:

The Problem Corner: Kenneth M. Wilke
Department of Mathematics
Washburn University of Topeka
Topeka, Kansas 66621
ken.wilke@washburn.edu

The Problem Corner: Pat Costello
Department of Mathematics and Statistics
Eastern Kentucky University
521 Lancaster Avenue
Richmond, KY 40475-3102
e-mail: pat.costello@eku.edu

Kappa Mu Epsilon News: Connie Schrock
Department of Mathematics
Emporia State University
Emporia, Kansas 66801
schrockc@emporia.edu

NEW KME Website!

The new national KME website can be found at

http://www.kappamuepsilon.org/

Among the items on the site:

- Contact information for national officers
- Initiation report form
- How to start a KME chapter
- Information on KME conventions

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. Currently, this is the National Secretary, Rhonda McKee. Her contact information is located in the list of National Officers on page 81 and under National Officers on the web site.

Subscription Renewals and Change of Address

Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 65 Number 1, if the code 65-1 appears on your label then this is your last issue!

Early renewals save us the cost of mailing out renewal notices. To renew, please send your check - just $10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subscriptions) - together with your name and address and a copy of your old address label to:

Rich Laird, *The Pentagon* Business manager
Department of Mathematics
Missouri Southern State University
3950 E. Newman Road
Joplin, MO 64804 USA

Please renew promptly to avoid gaps in your journal collection.

If you have a change of address, please don't forget *The Pentagon*! Send your change of address to the business manager at the above address. Failure to do so may cost KME additional postage fees!

Back Issues

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for $5.00 per copy. Please send inquiries to:

Rich Laird, *The Pentagon* Business manager
Department of Mathematics
Missouri Southern State University
3950 E. Newman Road
Joplin, MO 64804 USA

Canoeing Down the River of Recursion

Jo Marie Rozzelle, *student*

KS Delta

Washburn University
Topeka, KS 66621

Presented at the 2005 National Convention and awarded "top four" status by the Awards Committee.

1. Introduction

The problem discussed in this paper comes from an area of study called dynamic programming, commonly introduced in the field of operations research. A professor in the Washburn University Computer Science Department encountered the problem in a graduate course while working on his doctorate in computer science. The faculty advisor of this project anticipated that the problem could be solved using undergraduate discrete mathematics, such as recursion, and graph theory techniques. In other words, it was suspected that it would not be necessary to use complex algorithmic processes developed in the typical graduate level operations research course to complete the problem.

The problem can be found on pages 281-282 of [1] and is given below:

There are n trading posts along a river. At any of the posts you can rent a canoe to be returned at any other post downstream. (It is next to impossible to paddle against the current.) For each possible departure point i and each possible arrival point j the cost of a rental from i to j is known. However, it can happen that the cost of renting from i to j is higher than the total cost of a series of shorter rentals. In this case you can return the first canoe at some post k between i and j and continue your journey in a second canoe. There is no extra charge for changing canoes in this way.

The ultimate goal of this project is to find the minimum cost from any trading post i to any trading post j ($i < j$). However, before the minimum cost is found, it is useful and interesting to determine how many possible ways there are to go from trading post i to trading post node j. These are the two main goals of this paper.

2. Terminology and Symbolism

Although many concepts used in this project are similar to those which appear in an introductory graph theory course, some of the terminology used will not be conventional. In the figure below, consider a river with 4 trading posts, labeled 1, 2, 3, and 4 respectively.

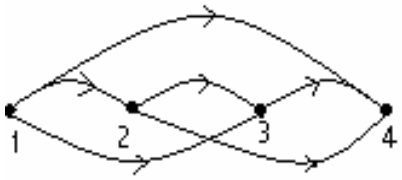


Figure 1

Figure 1 exemplifies a directed graph with 4 vertices. Each vertex of the directed graph will be called a node. The nodes will be labeled $1, 2 \ldots n$ in the order of the trading posts going downstream. This sequence of nodes will be called a river. From any node i to node j ($i < j$) there will be a directed arc symbolized (i, j), which is called a leg. The length of a leg (i, j) will be the positive integer $j - i$. A trip from node i to node j ($i < j$) is a sequence of legs. The first component of the first leg of the sequence must be i and the second component of the final leg must be j (observe that if a trip is made up of only one leg then the first component of the leg must be i and the second component of that same leg must be j). Also, for every leg in a sequence, the first component of that leg must equal the second component of the leg that immediately precedes it.

In Figure 1, notice there are exactly four distinct trips from node 1 to node 4, as listed below.

$$(1,2),(2,3),(3,4)$$
$$(1,3),(3,4)$$
$$(1,2),(2,4)$$
$$(1,4)$$

Each leg (i, j) (for $i < j$) on the river will have an associated positive

cost which will be symbolized $R(i, j)$. The total costs for any of the four trips above will be the sum of the cost of each leg. For Figure 1, the possible trip costs are as follows:

$$R(1,2) + R(2,3) + R(3,4)$$
$$R(1,3) + R(3,4)$$
$$R(1,2) + R(2,4)$$
$$R(1,4)$$

The minimum cost in Figure 1 will be the minimum of the four costs listed above. We will symbolize the minimum trip cost from node i to node j ($i < j$) as $C(i, j)$. A major goal of the paper is to develop a recursive formula which can be used to find $C(i, j)$ for any nodes i to j ($i < j$) on the river. There is no leg from i to i on the river, but the assignment: $C(i, i) = 0$ will simplify notation in the development which follows.

To ensure understanding of the process of finding the minimum cost, refer to Figure 2.

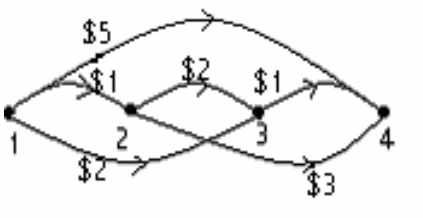


Figure 2

The directed graph in Figure 2 illustrates possible costs involved in getting from node 1 to node 4. There are obviously nodes 1,2,3,4 on the river and six different legs, $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$. To find the cheapest trip, one must determine the cost of all possible combinations of legs that form a trip from node 1 to node 4:

$$C(1,4) = \min \begin{cases} R(1,2) + R(2,3) + R(3,4) = 1 + 2 + 1 = 4 \\ R(1,3) + R(3,4) = 2 + 1 = 3 \\ R(1,2) + R(2,4) = 1 + 3 = 4 \\ R(1,4) = 5 \end{cases}.$$

Therefore, in this case $C(1,4) = 3$. Although this process is fairly painless when a small number of nodes are used, it becomes increasingly cumbersome as the number of nodes increase.

3. Counting the Number of Trips

As previously mentioned, a major objective of the paper is to develop and utilize a recursive formula for the minimum cost $C(1, n)$ for a trip down a river from node 1 to node n. Before this is done a related, interesting counting problem is considered. Exactly how many trips from node 1 to node n are possible? Examining some specific examples will assist in determining this number. In Figure 3 below the rivers with 2, 3, 4, and 5 nodes with the associated lengths of each leg on a given trip are listed.

Number of Nodes	**Trip**	**Leg Length**
2 nodes 1 trip (Note $1 = 2^0$)	$(1,2)$	1
3 nodes 2 trips (Note $2 = 2^1$)	$(1,2),(2,3)$ $(1,3)$	1,1 2
4 nodes 4 trips (Note $4 = 2^2$)	$(1,2),(2,3),(3,4)$ $(1,3),(3,4)$ $(1,2),(2,4)$ $(1,4)$	1,1,1 2,1 1,2 3
...	...	...
6 nodes 16 trips (Note $16 = 2^4$)	$(1,2),(2,3),(3,4),(4,5),(5,6)$ $(1,3),(3,4),(4,5),(5,6)$ $(1,2),(2,4),(4,5),(5,6)$ $(1,4),(4,5),(5,6)$ $(1,2),(2,3),(3,5),(5,6)$ $(1,3),(3,5),(5,6)$ $(1,2),(2,5),(5,6)$ $(1,5),(5,6)$ $(1,2),(2,3),(3,4),(4,6)$ $(1,3),(3,4),(4,6)$ $(1,2),(2,4),(4,6)$ $(1,4),(4,6)$ $(1,2),(2,3),(3,6)$ $(1,3),(3,6)$ $(1,2),(2,6)$ $(1,6)$	1,1,1,1,1 2,1,1,1 1,2,1,1 3,1,1 1,1,2,1 2,2,1 1,3,1 4,1 1,1,1,2 2,1,2 1,2,2 3,2 1,1,3 2,3 1,4 5

Figure 3

From these specific examples a conjecture can be made. It appears that on a river with n nodes the number of distinct trips from node 1 to node n is $2n-2$. Ultimately, this conjecture will be proven using mathematical induction, but first this process will be examined using basic counting techniques.

First note, in the example of a trip with 6 nodes, the leg length column contains only permutations of all possible positive integers less than or equal to 5 that sum to exactly 5.

Generally, it appears that the number of trips from node 1 to node n correspond exactly to the number of permutations of all positive integers less than or equal to n-1 that sum to n-1. But why should there be 2n-2 of these permutations? In the counting investigation that follows, an argument is made that there are exactly

$$\binom{n-2}{0}+\binom{n-2}{1}+\binom{n-2}{2}+\cdots+\binom{n-2}{n-2}$$

of these permutations. Using the binomial expansion, this value equals 2n-2. To avoid unnecessary complicated symbolism, rather than considering the general n nodes, the river with 6 nodes in Figure 3 is investigated. This will involve considering all permutations of the positive integers less than or equal to 5 that sum to 5.

k	Leg lengths using k numbers	Number of permutations using k numbers
1	5	$\binom{4}{0}=1$
2	4,1 1,4 3,2 2,3	$\binom{4}{1}=4$
3	3,1,1 1,3,1 1,1,3 2,2,1 2,1,2 1,2,2	$\binom{4}{2}=6$
4	1,1,1,2 1,1,2,1 1,2,1,1 2,1,1,1	$\binom{4}{3}=4$
5	1,1,1,1,1	$\binom{4}{4}=1$

Accordingly, the total number of permutations of the positive numbers less than or equal to five that sum to five is

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}.$$

(Using the binomial expansion this equals 24). This result is obviously consistent with the claim given before the example. Naturally, one may wonder why the combinations $\binom{4}{i}$ occur. To answer this question, the third possibility, $\binom{4}{2}$, is chosen arbitrarily to be examined. The following argument explains why the number of permutations of three positive numbers, all less than or equal to 5, which add to 5 must equal $\binom{4}{2}$.

To begin, consider the three integers as 3 boxes which must contain a total of five "X" symbols. For example, the leg lengths 3,1,1 could be symbolized XXX I X I X, and the leg lengths 2,1,2 could be symbolized XX I X I XX. The boxes are ordered and separated by vertical lines. To count the number of possible arrangements of five X symbols in the three ordered boxes, it is first necessary to place 1 X in each box (because there must be three positive numbers, each box must contain at least one number). With 3 X symbols in the fixed position, there remain 2 X symbols which have not yet been assigned a box. They can be assigned in the following ways:

X	X	
	X	X
X		X
XX		
	XX	
		XX

Counting the ways the 2 X symbols can be placed in the 3 ordered boxes is $\binom{4}{2}$. This is simply the number of distinguishable permutations of the symbols XX I I (i.e. 4!/(2!2!) = 6). A similar argument could be used on the other permutation possibilities which sum to 5.

The counting argument briefly outlined above could be generalized to n nodes. Although such an argument would better motivate an explanation of the final 2^{n-2} result, it is easier and more practical to simply prove the conjecture using strong mathematical induction.

Theorem 1 *On a river with nodes* $1, 2, ..., n$*, the number of trips from node 1 to node* n *is* 2^{n-2}.

The statement of the theorem will be symbolized $P(n)$ for $n \geq 2$ a positive integer.

Proof.

1. Base Case:

   When there are 2 nodes there is only one possible trip (1,2) from node 1 to node 2. Since $2^{2-2} = 2^0 = 1$, $P(2)$ holds.

2. Inductive Step:

Assume $P(i)$ is true for all $2 \leq i \leq t$, for some integer t. Consider an arbitrary river with $t+1$ nodes, $1, 2, 3, \ldots, t+1$. If the $t+1$ node is removed along with each leg leading into that node, then the result is a river with t nodes, $1, 2, 3, \ldots, t$. Now the induction hypothesis applies to this river with t nodes. There is/are:

1	trip	from node 1 to node	2
2	trips	from node 1 to node	3
2^2	trips	from node 1 to node	4
2^3	trips	from node 1 to node	5
		...	
2^{t-2}	trips	from node 1 to node	t

These values can now be applied to the original river with $t+1$ nodes. Each of the trips described above from node 1 to node i, $2 \leq i \leq t$ can be extended to node $t+1$ (without adding any additional trips) by adding the arc $(i, t+1)$ to any trip from node 1 to node i. Therefore, there are exactly $1+2^1+2^2+2^3+\cdots+2^{t-2}$ trips that go from node 1 to node $t+1$ through the nodes $2, 3 \ldots\ t$. The geometric series above sums to $2^{t-1}-1$ (using the well known formula $1+a^1+a^2+a^3+\cdots+a^{n-1} = (a^n-1)\,/\,(a-1)$. There is only one trip that has not been accounted for. This is the direct trip from node 1 to node $t+1$. Now, in all, there are $2^{t-1}-1+1 = 2^{t-1}$ trips from node 1 to node $t+1$. Assuming that $P(i)$ is true for all i, $2 \leq i \leq t$, it has been shown that $P(t+1)$ is also true. Therefore by strong mathematical induction, $P(n)$ is true for all $n \geq 2$. ■

It is clear that the results of this theorem can be generalized to Theorem 2 below, with changes in symbolism only.

Theorem 2 *On a river with n nodes $1, 2 \ldots n$ the number of trips from any node i to node j $(1 \leq i < j \leq n)$ is 2^{j-i-1}.*

4. Minimum Cost Recursion Development

The investigation which concluded with Theorem 2 outlined a process which carefully detailed and counted all possible paths from node 1 to node n on a river. This result will be useful in developing the recursion formula for the minimum trip cost C(1, n), which, as stated earlier, is the major goal of this project. Beginning with the minimum cost of a trip with two nodes, C(1,2), there is obviously only one possible trip to consider, namely on the leg (1,2). Specifically:

$$C\left(1,2\right)=R\left(1,2\right)=\min\left\{\left(C\left(1,1\right)+R\left(1,2\right)\right\}.\qquad(1)$$

The last expression follows using to the previous assignment that C(i, i)=0 for any node i.. When another node is introduced, the problem becomes more exciting. As the formula for finding the number of trips suggests, there are 2 distinct trips when there are three nodes (23-2). The minimum cost function C(1,3) is given by:

$$C\left(1,3\right)=\min\left\{\begin{array}{c}R(1,2)+R(2,3)\\R(1,3)\end{array}\right.\qquad(2)$$

From (1), $R(1,2)$ in the first expression of (2) can be replaced by $C(1,2)$. Also, $C(1,1)=0$ so $C(1,1)$ can be added to the second expression. Rewriting (2) with these modification yields:

$$C\left(1,3\right)=\min\left\{\begin{array}{l}C\left(1,2\right)+R\left(2,3\right)\\C\left(1,1\right)+R\left(1,3\right)\end{array}\right.=\min_{k=1,2}\left\{C\left(1,k\right)+R\left(k,3\right)\right\}.\ (3)$$

The procedure above can be adapted to accommodate more nodes. Examine adding an additional node (node 4) to the previous example.

$$C\left(1,4\right)=\min\left\{\begin{array}{c}R\left(1,2\right)+R\left(2,3\right)+R\left(3,4\right)\\R\left(1,3\right)+R\left(3,4\right)\\R\left(1,2\right)+R\left(2,4\right)\\R\left(1,4\right)\end{array}\right.\qquad(4)$$

Again, when there are four nodes our formula states there are 2^{4-2} possible trips. Formula (4) lists the costs for all possible trips from node 1 to node 4. The first two trips come from formula (2) with the final leg added. The next trip comes from formula (1) adding the leg necessary to reach the final node added. The final trip is the leg from the beginning node to the ending node. Using (3), the first two lines in the above expression can be replaced by $C(1,3)+R(3,4)$. Using (1), the third line can be replaced by $C(1,2)+R(2,4)$. The fourth line $R(1,4)$ is the cost of the

direct trip on leg $(1,4)$ plus $C(1,1)=0$. Thus, (4) can be rewritten:

$$C\left(1,4\right)=\left\{\begin{array}{l} C\left(1,3\right)+R\left(3,4\right) \\ C\left(1,2\right)+R\left(2,4\right) \\ C\left(1,1\right)+R\left(1,4\right) \end{array}\right. =\min_{k=1,2,3}\left\{C\left(1,k\right)+R\left(k,4\right)\right\} \quad (5)$$

Although the general method has been established, adding another node further solidifies this recursive process:

$$C\left(1,5\right)=\min\left\{\begin{array}{c} R\left(1,2\right)+R\left(2,3\right)+R\left(3,4\right)+R\left(4,5\right) \\ R\left(1,3\right)+R\left(3,4\right)+R\left(4,5\right) \\ R\left(1,2\right)+R\left(2,4\right)+R\left(4,5\right) \\ R\left(1,4\right)+R\left(4,5\right) \\ R\left(1,2\right)+R\left(2,3\right)+R\left(3,5\right) \\ R\left(1,3\right)+R\left(3,5\right) \\ R\left(1,2\right)+R\left(2,5\right) \\ R\left(1,5\right) \end{array}\right. \quad (6)$$

With $2^{5-2}=8$ trips when five nodes are considered, the first four trips represent the four trips in formula (4), with the addition of the leg to reach the final node. The next two trips represent the trips in formula (2) with the addition of the leg to reach the final node. The next trip is derived from formula (1) with the addition of the leg needed to reach the final node. The final trip represents the leg from the beginning node to the final node. Using (4), the first four lines of the expression can be replaced by $C(1,4)+R(4,5)$. Using (2), lines five and six can be replaced by $C(1,3)+R(3,5)$. Using (1), line 7 is replaced by $C(1,2)+R(2,5)$. The zero cost $C(1,1)$ can then be added to the last line. Thus, (6) can be rewritten:

$$C\left(1,5\right)=\min\left\{\begin{array}{l} C\left(1,4\right)+R\left(4,5\right) \\ C\left(1,3\right)+R\left(3,5\right) \\ C\left(1,2\right)+R\left(2,5\right) \\ C\left(1,1\right)+R\left(1,5\right) \end{array}\right. =\min_{k=1,2,3,4}\left\{C\left(1,k\right)+R\left(k,5\right)\right\}$$

From the pattern above, one can conjecture:

$$C\left(1,n\right)=\min_{k=1,2,\ldots,n-1}\left\{C\left(1,k\right)+R\left(k,n\right)\right\}.$$

The above minimum cost formula will be proven below using strong mathematical induction:

Theorem 3 *For a river with n nodes, $1,2\ldots n$, the minimum cost of a trip from node 1 to node n is given by the recursive formula:*

$$C(1,n)=\min_{k=1,2,\ldots,n-1}\{C(1,k)+R(k,n)\}$$

for $n\geq 2$, where $R(i,j)$ is the cost associated with leg (i,j) of the trip.

The statement of the theorem will be symbolized $P(n)$ for n an integer, $n \geq 2$.

Proof.

1. Base Case:

   Since $C(1,1) = 0$ and the only possible trip from node 1 to node 2 is leg $(1,2)$ with the associated cost $R(1,2)$, then $C(1,2)$ must equal $R(1,2)$. So $C(1,2) = \min_{k=1}\{C(1,1) + R(1,2)\}$. $P(2)$ is thus established.

2. Inductive step:

   Assume $P(i)$ for all i, $2 \leq i \leq t$, where $t \geq 2$. The ultimate goal is to prove that $C(1,t+1) = \min_{k=1,2,\ldots,t}\{C(1,k) + R(k,t+1)\}$ is also true.

   First, consider the direct trip, $(1,t+1)$, from the beginning node 1 to the ending node $t+1$. Since $C(1,1) = 0$, the cost of the direct trip is:

$$C(1,1) + R(1,t+1) \qquad (7)$$

Now the above cost (7) provides one possible candidate for minimum cost for the trip from node 1 to node $t+1$. Any other candidate for the minimum cost of the trip must pass through at least one of the nodes $2, 3 \ldots t$. Working backwards, the last leg of any such trip will be leg $(i,t+1)$ with associated cost $R(i,t+1)$ for $2 \leq i \leq t$. However, the induction hypothesis specifies the minimum cost of any trip from node 1 to node i, namely $C(1,i)$. Adding the minimum cost from node 1 to node i to the last leg cost from node i to node $t+1$ gives:

$$\begin{array}{l} C(1,2) + R(2,t+1) \\ C(1,3) + R(3,t+1) \\ \ldots \\ C(1,t) + R(t,t+1) \end{array} \qquad (8)$$

Included in (8) must be the minimum cost for any trip starting at node 1 onto node i, (possibly passing through other nodes along the way), and finally going on from node i to node $t+1$ on the leg $(i,t+1)$. The only other possible candidate for minimum cost of a trip is found in formula (7). It involves the direct trip via leg $(1,t+1)$. Combining (7) and (8), the minimum cost for a trip from node 1 to node $t+1$ must be exactly

$$C(1,t+1) = \min_{k=1,2,\ldots,t}\{C(1,k) + R(k,t+1)\}$$

.Thus, by strong mathematical induction, $P(n)$ is true for all $n \geq 2$. ■

In the above theorem, all trips from node 1 to node n for any integer $n \geq 2$ were considered. If the proof began with node i, $i \geq 1$, and ended with any other node j, $i < j$, $n \geq j$, a similar proof could be constructed with changes in symbolism only. The more general theorem which provides the minimum cost from any node i to any later node j on the river is given below.

Theorem 4 *For a river with n nodes, $\{1, 2 \ldots n\}$, the minimum cost of a trip from node i to node j, $1 \leq i < j \leq n$, is given by the recursive formula: $C(i,j) = \min_{k=i,i+1,\ldots,j-1} \{C(i,k) + R(k,j\}$ for $j \geq 2$, where $R(i,j)$ is the cost associated with leg (i,j) of the trip.*

5. Computer Program Implementation

Appendix A contains a visual basic program which utilizes this recursive formula in Theorem 4. A matrix $R = R(i,j)$ may be loaded with initial fixed costs for each associated leg (i,j), $1 \leq i < j \leq n$. The program transfers the matrix R into the desired minimum cost matrix $C = C(i,j)$ using Theorem 4. Figure 4 shows an example of an initial input cost matrix R for a river with 6 nodes, and Figure 5 shows the program output for the resulting minimum cost matrix C.

Figure 4

Figure 5

6. Conclusion

Often in mathematics, several different questions need to be answered before the final outcome can be obtained. This project is no exception, as the exact number of possible trips was established before the final formula for the minimum cost was determined. Certainly there are many real world applications of this problem. By changing the assumptions, one could determine the minimum cost of a plane trip from destination A to destination B. Also, one could examine the rates for insurance truck drivers on interstates, as they vary according to the danger level of that highway. An interesting application would be to explore the minimum insurance costs by varying the interstates traveled.

Appendix A: Computer Program

```
Dim i As Integer
Dim j As Integer
Dim k As Integer
Dim n As Integer
Dim intSum1 As Integer
Dim intSum2 As Integer
Dim intMin As Integer
n = grdMatrix.Rows - 1
'process the array
For i = 0 To (n - 2)
    For j = (i + 2) To n
        intMin = 0
        intSum1 = 0
        intSum2 = 0
        'initial value (so there is something to compare the next to)
        k = i
        intSum1 = Val(grdMatrix.TextMatrix(i, k))
                    + Val(grdMatrix.TextMatrix(k, j))
        For k = (i + 1) To (j - 1)
            intSum2 = Val(grdMatrix.TextMatrix(i, k))
                        + Val(grdMatrix.TextMatrix(k, j))
            If intSum1 > intSum2 Then
                intMin = intSum2
            Else
                intMin = intSum1
            End If
            intSum1 = intMin
        Next k
        grdMatrix.TextMatrix(i, j) = intMin
    Next j
Next i
```

Acknowledgements: This project was successfully completed with the assistance of my advisor, Dr. Al Riveland, Washburn University Mathematics and Statistics Department. I would like to express my appreciation for the guidance that he provided me. I also want to thank a fellows students, Fred Hollingshead for his time and support spent on this project, and Matt Hamilton who designed the visual basic program which illustrated the implementation of the minimum cost recursion formula. Finally, I would like to express my appreciation to the Washburn University CSI summer program for the grant I was allotted.

References

[1] G. Brassard and Paul Bratley, *Fundamentals of Algorithmics*, Prentice Hall, 1996.

[2] J. Dossey, A.D. Otto, L.E. Spence, and C. Vanden Eynden, *Discrete Mathematics*, 4th ed., Addison Wesley, 2002.

A Rational Integral and a Polynomial Decomposition

Dimitrios Kodokostas

Technological Education Institute of Larissa
Department of Computer Science Larissa, 41110 Greece
dkodokostas@alumni.nd.edu
dkodokostas@teilar.gr

1. An Integration Problem

A student practicing integration techniques for the first time will sooner or later meet the class of integrals of the form

$$I = \int \frac{Ax^2 + Bx + C}{(ax^2 + bx + c)^2} dx, A \neq 0 \neq a.$$

A tantalizing fact about these integrals (though not unique to them!) is that for some choices of the coefficients A, B, C, a, b, c the integral I is a rational function of x (i.e. the quotient of two polynomials of x) and for other choices it is not. For example if $(A, B, C, a, b, c) = (1, 0, 0, 1, 0, 0)$ then

$$\int \frac{x^2}{x^4} dx = \int \frac{1}{x^2} dx = \frac{-1}{x} + d, d \in \mathbb{R},$$

i.e. a rational function of x, while if $(A, B, C, a, b, c) = (1, -1, 0, 1, -1, 0)$ then

$$\begin{aligned} \int \frac{x^2 - x}{(x^2 - x)^2} dx &= \int \frac{1}{x^2 - x} dx \\ &= \int (\frac{1}{x - 1} - \frac{1}{x}) dx \\ &= \int \frac{1}{x - 1} dx - \int \frac{1}{x} dx \\ &= \ln(x - 1) - \ln(x) + d, d \in \mathbb{R}, \end{aligned}$$

which does not equal any rational function of x.

A natural question, then, is if one can be sure about the nature of the result just by checking the coefficients A, B, C, a, b, c. The integral I can always be computed via the method of partial fractions, counting on the help of the logarithmic and inverse trigonometric functions. A nice account of the partial fractions method can be found in [1]. Nevertheless the calculations are often tedious and moreover it is not always obvious if the final result is or is not a rational function of x (or for simplicity just "rational").

Our aim here is to give a complete set of conditions for the coefficients that characterize rationality of I. We are moreover going to provide a concise calculation of I.

Instead of stating our findings and then proving them rigorously, we first attempt a small investigation leading to them. We hope that the interplay between analysis and algebra will be more clear, and that each step will make more sense to the interested reader.

Recall that any indefinite integral actually represents a class of functions differing from each other by a constant, and it is in this sense that any equality between two such integrals below should be interpreted.

A good way to start is to investigate what happens when the binomial ax^2+bx+c appearing in the denominator possesses a double (necessarily) real root ρ. In this case the coefficients satisfy $b^2-4ac=0$, and ax^2+bx+c is written as $a(x-\rho)^2$; so then

$$\begin{aligned} I &= \int \frac{Ax^2+Bx+C}{(ax^2+bx+c)^2}dx \\ &= \int \frac{Ax^2+Bx+C}{a^2(x-\rho)^4}dx \\ &= \int \frac{A(t+\rho)^2+B(t+\rho)+C}{a^2t^4}dt \ \text{ (and using } x-\rho=t) \\ &= \frac{A}{a^2}\int \frac{1}{t^2}dt + \frac{2A\rho+B}{a^2}\int \frac{1}{t^3}dt + \frac{A\rho^2+B\rho+C}{a^2}\int \frac{1}{t^4}dt \\ &= \frac{A}{a^2}(\frac{-1}{t}) + \frac{2A\rho+B}{a^2}(\frac{-1}{t^2}) + \frac{A\rho^2+B\rho+C}{a^2}(\frac{-1}{3t^3}) + d, d \in \mathbb{R} \end{aligned}$$

which is a rational function of x, a small victory towards our goal!

In the case $b^2-4ac \neq 0$ though, it is not at all clear how to proceed. Experience suggests that we try computing I by transforming the integrand into a sum of simpler pieces. A promising way for achieving this is by decomposing the numerator as:

$$Ax^2 + Bx + C = k(ax^2 + bx + c) + (lx + m)(ax^2 + bx + c)' \quad (1)$$

where $k, l, m \in \mathbb{R}$, and the prime denotes derivative.

Let's assume for a moment the existence of such a decomposition. In due course it will be valuable to have at hand the relationships between the coefficients involved in it:

Relation (1) is equivalent to the following three relations:

$$ka + 2la = A \quad (2)$$
$$kb + lb + 2ma = B \quad (3)$$
$$kc + mb = C \quad (4)$$

For the integral itself, relation (1) allows us to write:

$$\begin{aligned} I &= \int \frac{Ax^2 + Bx + C}{(ax^2 + bx + c)^2} dx \\ &= \int \frac{k(ax^2 + bx + c)}{(ax^2 + bx + c)^2} dx + \int \frac{(lx + m)(ax^2 + bx + c)'}{(ax^2 + bx + c)^2} dx \\ &= \int \frac{k}{ax^2 + bx + c} dx - \int (lx + m)((ax^2 + bx + c)^{-1})' dx \\ &= \int \frac{k}{ax^2 + bx + c} dx - \frac{lx + m}{ax^2 + bx + c} + \int \frac{(lx + m)'}{ax^2 + bx + c} dx \\ &= \int \frac{k}{ax^2 + bx + c} dx - \frac{lx + m}{ax^2 + bx + c} + \int \frac{l}{ax^2 + bx + c} dx \end{aligned}$$

thus

$$I = \int \frac{k + l}{ax^2 + bx + c} dx - (lx + m)(ax^2 + bx + c)^{-1} \quad (5)$$

Clearly, I is rational if and only if the integral

$$I_0 = \int \frac{k + l}{ax^2 + bx + c} dx$$

is rational.

We distinguish two cases:

1. $b^2 - 4ac > 0$.

   Then $ax^2 + bx + c$ has two real, distinct roots ρ_1, ρ_2, and it can be written as

   $$ax^2 + bx + c = a(x - \rho_1)(x - \rho_2).$$

Then

$$\begin{aligned} I_0 &= (k+l)\int \frac{1}{a(x-\rho_1)(x-\rho_2)}dx \\ &= \frac{k+l}{a(\rho_1-\rho_2)}\int(\frac{1}{x-\rho_1}-\frac{1}{x-\rho_2})dx \\ &= \frac{k+l}{a(\rho_1-\rho_2)}(\ln(x-\rho_1)-\ln(x-\rho_2)+d), d\in\mathbb{R}. \end{aligned}$$

This is a rational function if and only if $k+l=0$.

2. $b^2-4ac<0$.

Then

$$\begin{aligned} ax^2+bx+c &= a[x^2+\frac{bx}{a}+\frac{c}{a}] \\ &= a[(x+\frac{b}{2a})^2-\frac{b^2}{4a^2}+\frac{c}{a}] \\ &= a[(x+\frac{b}{2a})^2+\frac{-(b^2-4ac)}{4a^2}]. \end{aligned}$$

Since $\frac{-(b^2-4ac)}{4a^2}>0$, we can write $\frac{-(b^2-4ac)}{4a^2}=f^2$. We have

$$\begin{aligned} I_0 &= (k+l)\int \frac{1}{a\left[\left(x+\frac{b}{2a}\right)^2+f^2\right]}dx \\ &= \frac{k+l}{a}\int \frac{1}{\left(x+\frac{b}{2a}\right)^2+f^2}dx, \text{ and using } x+\frac{b}{2a}=t \\ &= \frac{k+l}{a}\int \frac{1}{t^2+f^2}dt \\ &= \frac{k+l}{a\,|f|}\left(\arctan\frac{t}{|f|}+d\right) \\ &= \frac{k+l}{a\sqrt{\frac{-\left(b^2-4ac\right)}{4a^2}}}\left(\arctan\frac{t}{\sqrt{\frac{-\left(b^2-4ac\right)}{4a^2}}}+d\right), d\in\mathbb{R}. \end{aligned}$$

This is a rational function if and only if $k+l=0$.
Summarizing, for $b^2-4ac\neq 0$, the integral I is rational if and only if

$$k+l=0. \tag{6}$$

But notice that $k+l=0 \Longleftrightarrow Bb=2(Ac+Ca)$ and we are done!

The proof of the last equivalence is trivial: Whenever $k+l=0$ holds, (2) and (6) give $k=\frac{-A}{a}, l=\frac{A}{a}$, and then (3) implies $m=\frac{B}{2a}$. But then (4) is written as $\frac{-A}{a}c+\frac{B}{2a}b=C$ i.e. $Bb=2(Ac+Ca)$.

Conversely, let's assume that $Bb=2(Ac+Ca)$ holds. Recall that relations (2),(3),(4) also hold. So substituting the values of A, B, C into Bb=$2(Ac+Ca)$ this last relation becomes

$$\begin{aligned}
& (kb+lb+2ma)b=2((ka+2la)c+(kc+mb)a) \\
\iff\ & b^2(k+l)+2mab=2kac+4lac^2+2kac+2mab \\
\iff\ & b^2(k+l)=4kac+4lac \\
\iff\ & b^2(k+l)=4(k+l)ac \\
\iff\ & (b^2-4ac)(k+l)=0 \\
\iff\ & k+l=0 \text{ since } b^2-4ac\neq 0.
\end{aligned}$$

Thus, granted the existence of decomposition (1) (whenever $b^2-4ac\neq 0$) which will be proved shortly in Section 2, we can summarize the previous results in the following proposition:

Proposition 5 *Consider the integral*

$$I=\int \frac{Ax^2+Bx+C}{(ax^2+bx+c)^2}dx,$$

where $A\neq 0, B, C, a\neq 0, b, c$ are real numbers. Then

1. [I is a rational function of x] $\iff$[$b^2-4ac=0$ or $Bb=2(Ac+Ca)$].
2. If $b^2-4ac=0$, then
$$I=\frac{-A}{a^2(x-\rho)}-\frac{2a\rho+B}{a^2(x-\rho)^2}-\frac{a\rho^2+B\rho+C}{3a^2(x-\rho)^3}+d,$$
where $d\in\mathbb{R}$ and ρ is the root of ax^2+bx+c.
3. If $b^2-4ac>0$, then
$$I=\frac{k+l}{a(\rho_1-\rho_2)}\ln\left(\frac{x-\rho_1}{x-\rho_2}\right)-\frac{lx+m}{ax^2+bx+c}+d,$$
$d\in\mathbb{R}$.
4. If $b^2-4ac<0$, then

$$I=\frac{k+l}{a\sqrt{\frac{-(b^2-4ac)}{4a}}}\arctan\left(\frac{x+\frac{b}{2a}}{\sqrt{\frac{-(b^2-4ac)}{4a}}}\right)-\frac{lx+m}{ax^2+bx+c}+d,$$

$d\in\mathbb{R}$.

In the last two cases, k, l, m *are the real numbers satisfying* $ka+2la = A$, $kb + lb + 2ma = B$, $kc + mb = C$.

The proposition can be viewed as a very partial answer to the question of when the integral of a rational function is a rational function. The proposition also gives us a glimpse of the difficulties involved in investigating this question. One should perhaps expect this question to be difficult since even trivially simple rational functions like $\frac{1}{x}$ posses irrational indefinite integrals: $\int \frac{1}{x}dx = \ln x + c$, $c \in \mathbb{R}$.

2. A Polynomial Decomposition Result

As mentioned, the validity of the proposition depends on the fact that given binomials $Ax^2 + Bx + C$, $ax^2 + bx + c$ with $A, a, b^2 - 4ac \neq 0$, there exist decompositions of the form $Ax^2+Bx+C = k(ax^2+bx+c)+(lx + m)(ax^2 + bx + c)'$ for some $k, l, m \in \mathbb{R}$. This can be checked directly by solving the system of equations (2), (3), (4). Nevertheless, it is interesting to note that it also comes easily as a consequence of the following more general result.

Lemma 6 *Let* $p(x), q(x)$ *be real polynomials of degree* $n \geqq 1$*, and* $r(x)$ *a polynomial of degree* 1 *such that its root is not a root of* $p(x)$*. Then* $q(x)$ *decomposes (uniquely) as* $q(x) = kp(x)+m(x)r(x)$ *for some* $k \in \mathbb{R}$*, with* $m(x)$ *a real polynomial of degree* $\leq n - 1$.

Indeed, if the Lemma holds, let's set

$$\begin{aligned} p(x) &= ax^2 + bx + c, \\ q(x) &= Ax^2 + Bx + C, \\ r(x) &= p'(x) = (ax^2 + bx + c)', \end{aligned}$$

and let's note that the root $\frac{-b}{2a}$ of $r(x)$ is not a root of $p(x)$ if and only if $p(\frac{-b}{2a}) \neq 0$, i.e. whenever $a(\frac{-b}{2a})^2 + b(\frac{-b}{2a}) + c \neq 0$ or equivalently whenever $b^2 - 4ac \neq 0$ which is one of our assumptions for the coefficients a,b,c,A,B,C. Then the Lemma applies to guarantee the existence of $k, l, m \in \mathbb{R}$ such that

$$q(x) = kp(x) + (lx + m)r(x),$$

i.e.

$$Ax^2 + Bx + C = k(ax^2 + bx + c) + (lx + m)(ax^2 + bx + c)'$$

as desired.

By the way, notice also that the above argument shows that a binomial $ax^2 + bx + c$ has no double root if and only if it does not have a common root with its derivative $r(x) = p'(x) = 2ax + b$, a result probably familiar to many readers.

We now prove the Lemma: Let

$$\begin{aligned} p(x) &= p_n x^n + p_{n-1}x^{n-1} + \cdots + p_1 x + p_0, \\ q(x) &= q_n x^n + q_{n-1}x^{n-1} + \cdots + q_1 x + q_0, \\ r(x) &= r_1 x + r_0. \end{aligned}$$

We intend to prove the existence of a real number k and a real polynomial $m(x) = m_{n-1}x^{n-1} + m_{n-2}x^{n-2} + \cdots + m_1 x + m_0$ of degree $\leq n-1$ such that $q(x) = kp(x) + m(x)r(x)$. Since we do not demand the degree of $m(x)$ to be equal to $n-1$, but just less or equal to $n-1$, we make no assumption about m_{n-1} or any other m_i to be non-zero.

So we search for real numbers $k, m_0, m_1, \cdots, m_{n-2}, m_{n-1}$ such that

$$\begin{aligned} & q_n x^n + q_{n-1}x^{n-1} + \cdots + q_1 x + q_0 \\ = \; & k(p_n x^n + p_{n-1}x^{n-1} + \cdots + p_1 x + p_0) \\ & +(m_{n-1}x^{n-1} + m_{n-2}x^{n-2} + \cdots + m_1 x + m_0)(r_1 x + r_0) \end{aligned}$$

This is equivalent to the following system of equations:

$$\left(\begin{array}{l} q_0 = kp_0 + m_0 r_0 \\ q_1 = kp_1 + m_0 r_1 + m_1 r_0 \\ q_2 = kp_2 + m_1 r_1 + m_2 r_0 \\ \vdots \\ q_{n-2} = kp_{n-2} + m_{n-3} r_1 + m_{n-2} r_0 \\ q_{n-1} = kp_{n-1} + m_{n-2} r_1 + m_{n-1} r_0 \\ q_n = kp_n + m_{n-1} r_1 \end{array} \right). \tag{7}$$

This is a linear system with $n+1$ equations for the $n+1$ unknown numbers $k, m_0, m_1, ..., m_{n-2}, m_{n-1}$ (in this order) with determinant equal to

$$\begin{vmatrix} p_0 & r_0 & 0 & 0 & \cdots & & & \\ p_1 & r_1 & r_0 & 0 & \cdots & & & \\ p_2 & 0 & r_1 & r_0 & & & & \\ & & & & & 0 & & \\ \vdots & \vdots & & & \ddots & & & \\ & & 0 & & & & & \\ p_{n-2} & & & & & r_1 & r_0 & \\ p_{n-1} & & & & & & r_1 & r_0 \\ p_n & & & & & & & r_1 \end{vmatrix}.$$

This determinant is the so called *Sylvester resultant* for the two polynomials $p(x), r(x)$. The resultant is an extremely useful tool in deciding if two polynomials have a common root. This can be done using Sylvester's Theorem which states that two complex polynomials have a common complex root if and only if their resultant is equal to 0. The interested reader can consult for example [2], [3] or [4].

Now note that our real polynomials p, r would share a root if and only if they would share the unique real root of r (recall that r is linear). Note also that p, r can be considered as complex polynomials as well, and their possible real common root would also be a complex common root. So in this case Sylvester's Theorem would assure us that p, r have a real common root if and only if their resultant equals to 0.

Since by assumption p and r do not share a root, we conclude that their Sylvester resultant does not equal 0, i.e. the determinant of (7) is non-zero, thus this system possesses a solution which moreover is unique. QED.

Note that the Lemma cannot guarantee that the degree of $m(x)$ is $n-1$. If for example $p(x)$ is a multiple of $q(x)$, then $p(x) = kq(x)$ for some $k \in \mathbb{R}$, thus $p(x) = kq(x) + (0x^{n-1} + 0x^{n-2} + ... + 0x + 0)r(x)$, so the unique $m(x)$ has to be the zero polynomial which of course does not have degree $n - 1$.

References

[1] E. Moise, *Calculus*, Part I, Addison-Wesley Publishing Company, 1966.

[2] N. Conkwright, *Introduction to the Theory of Equations*, Ginn and Company, 1957.

[3] M. Pohst and H. Zassenhaus, *Algorithmic Algebraic Number Theory*, Cambridge University Press, 1989 & 1997.

[4] S. Wagon, *Mathematica in Action*, W. H. Freeman, 1991 (or Springer-Verlag, 1999 & 2000).

Applications in Abstract Algebra: Encryption/Coding and Calendar Calculations

Daniel Majcherek, *student*

CA Epsilon

California Baptist University
Riverside, CA 92504

Presented at the 2005 National Convention

1. Introduction

Abstract Algebra has a wide variety of applications, ranging from military operations to amusing algorithms. Perhaps one of the most well-known applications is coding. In this paper, we will mainly focus on EIGamal encryption, but we will also discuss the Diffie-Hellman Key Exchange, as well as a fun algorithm that can be used to find the day of the week of any day in history. However, it would be useful to mention two theorems that have been applied to EIGamal encryption: Fermat's Little Theorem and Euler's Theorem.

2. Fermat and Euler

Fermat's Little Theorem states that *if p is a prime number, then $a^p \equiv a \pmod{p}$.* In words, this theorem states that if p is a prime number, then a to the p power is congruent to a modulo p (in other words, a to the p power minus a is divisible by p) (See [6].) Any integer n, if it happens to satisfy Fermat's Little Theorem, whether or not it is prime, is called a Fermat pseudoprime to base a, and if that said integer is a Fermat pseudoprime to many different bases, then that number is most likely a prime number (See [5].)

Euler's Theorem is the general form of Fermat's Little Theorem. Let's let $f(n)$ denote Euler's totient function. This function $f(n)$ is simply the number of positive integers relatively prime to the integer n, given that $n > 1$. Euler's Theorem states that *if n is an integer greater than one, and $\gcd(a, n) = 1$, then $a^{f(n)} \equiv 1 \pmod{n}$.* In words, this theorem states that if integer n is larger than one and a and n are relatively prime, then a to the power of the value given by Euler's totient function is congruent to 1 modulo n.

Example using Fermat's Little Theorem: Let $a = 3$ and $p = 7$. (Note that p is a prime number.) By Fermat's Little Theorem, $3^7 \equiv 3 \pmod{7}$, which is true, since $3^7 = 2187 = 2184 + 3 = 7(312) + 3$, implying that $3^7 \equiv 3 \pmod{7}$, because we are left with a remainder of 3.

A specific case of Fermat's Little Theorem can be applied to the expansion of rational numbers, and this reveals an interesting property. We will use the following corollary of Fermat's Little Theorem:

Corollary: *Let $k > 0$ be the order of 10 modulo p (in other words, suppose p divides $99...9$ with k 9's in a row, and k smallest possible). Then $\frac{1}{p}$ is periodic with period k, and there is no smaller period.* (See [2].)

The general form of this corollary is $10^x \equiv 1 \pmod{p}$, although in this form, x may not be the smallest possible, buit it may be. Note that p has to be a prime number, where $p \neq 2$ and $p \neq 5$, because $\frac{1}{p}$ would not repeat for $p = 2$ or $p = 5$. Recall that the term "order" refers to the number of elements in a cyclic group.

Example: In the above corollary to Fermat's Little Theorem, let k be the period of $\frac{1}{37}$. To calculate k, we must calculate the order of 10 modulo 37 under multiplication. We begin with 1, the multiplicative identity. We have $10^1 = 10$, $10^2 = 26$, and $10^3 = 1$ (recall we are working modulo 37),

hence the order of 10 modulo 37 is 3, thus, $\frac{1}{37} = .02702702...$ is period 3 in the expansion. (See [2]) Note the group generated by 10 modulo 37 is $\{1, 10, 26\}$. In this case, $k = 3$ is the smallest possible period.

Also, this points out an interesting property of repeating decimals. Take $\frac{1}{7} = .142857\ldots$, where those number repeat. Add together the first half and the second half: $142 + 857 = 999$. This is a result of the above corollary. The repeating decimals always add up to 9's, unless we are dealing with a repeating decimal like $\frac{1}{3}$, in which case the repeating portion is a single number. Whenever the number of repeating digits in a repeating decimal is even and greater than one, add the first half and the second half to obtain the period k, where k is the number of 9's obtained (note that a number, such as $\frac{1}{37} = .02702702...$ would not follow this property because the number of repeating decimals is odd and not even).

Example using Euler's Theorem: Let $n = 6$. Then $f(n) = 2$, because there are two numbers relatively prime to 6, namely 1 and 5. Now we will pick a number, a, such that a is relatively prime to 6. Let $a = 1003$. Note that $1003^2 \equiv 1 \pmod 6$, hence $1006009 \equiv 1 \pmod 6$. In other words, 1006009 divided by 6 will have a remainder of 1.

3. ElGamal Encryption

Now we are ready to move on to EIGamal encryption. The is a method for public key encryption. Here are the steps to encrypt a message. Say two people are sending data to each other. We will call them #1 and #2. #1 first picks a very large prime number p and a generator a of the factor group under multiplication $(\mathbb{Z}/p\mathbb{Z})_X$ such that $2 \leq a \leq p-2$. Then, #1 must pick a secret key k (completely at random) such that $2 \leq k \leq p-2$, and in the process compute $a^k \pmod p$. The public key of #1 is then defined by $\left(p, a, a^k \pmod p\right)$, and the private key is defined as k.

Once the public key is found, then, #2 can send a message to #1, so long as #2 knows #1's public key. #2 must represent the message as some integer $m \leq p-1$, but his will only work if p is large enough (alphabetical notation may be used for m, for example). Now, #2 picks a random integer, q, such that $1 \leq q \leq p-2$. Once #2 calculates $d \equiv a^q \pmod p$ and $e \equiv m\left(a^k\right)^q \pmod p$ such that $1 \leq d, e \leq p-1$, then #2 can send what is called a ciphertext c to #1 such that $c = (d, e)$. Then #1 can decode the message by using the private key and the obtained ciphertext along with the formula $m \equiv d^{(p-1-k)}e \pmod p$. (See [4].)

Example: Suppose I want to send you the message "hi". You and I would follow the following steps. We will use alphabetical notation (i.e. $a = 01$, $b = 02$, $c = 03$, etc.). Let $p = 811$ with generator a of factor group $(\mathbb{Z}/p\mathbb{Z})_X$, $a = 2$. Let your secret key be $k = 113$. Hence $a^k \pmod p = 2^{113} \pmod{811} = 591$. You now know your public key, which is $(811, 2, 591)$, and your private key is $k = 113$.

Next, I can send you my message. I select my random integer to be $q = 3$. Using the letter notation, "hi" becomes $0809 = m$. Assume I have your public and private keys. Then, I can calculate d and e. Using the above formulas, $d \equiv a^q \pmod p = 2^3 \pmod{811} = 8$ and $e \equiv 809 \left(2^{113}\right)^3 \pmod{811} = 762$. Thus, I send you the ciphertext $c = (8, 762)$.

Now you must decode the message using $m \equiv (d^{p-1-k} e \pmod p) = 8^{811-1-113} \cdot 762 \pmod{811} = 809$, which is the original message (since we have an odd number of digits, it can be assumed that the first digit is a zero). This translates to "hi" in alphabetical notation.

4. Diffie-Hellman Key Exchange

The next application we will investigate is the Diffie-Hellman Key Exchange. This process allows two people to share a secret, even when people may be listening to them. This comes in very handy during military operations. The process was discovered by Whitfield Diffie and Martin Hellman (See [3].) Here are the steps to sharing a secret using this process.

Again, assume you and I will share a secret. First, we pick a large prime number p and a generator a of the factor group $(\mathbb{Z}/p\mathbb{Z})_X$ under multiplication (this is the same first step as in the EIGamal encryption process) such that $2 \leq a \leq p - 2$. Now, if I pick a random secret x such that $2 \leq x \leq p - 2$ and send you the information $a^x \pmod p$, and if you pick a secret y such that $2 \leq y \leq p - 2$ and send me the information $a^y \pmod p$, then we can calculate the shared secret key. Once we each receive the other's information, then I can calculate $(a^y)^x \pmod p = K$, and you calculate $(a^x)^y \pmod p = K$. The shared secret key is the calculated K. (See [3].)

Example: Let $p = 541$ and $a = 2$ (once again, a generates the entire factor group). Let my secret be $x = 5$ and your secret be $y = 7$. I send you $a^x \pmod p = 2^5 \pmod{541} = 32$, and you send me $a^y \pmod p = 2^7 \pmod{541} = 248$. The shared secret key is $(a^x)^y \pmod p = \left(2^5\right)^7 \pmod{541} = K = 97$.

5. Calendar Calculations

The final application of abstract algebra that we will discuss involves a rather entertaining algorithm. While it may not be related to Fermat's Little Theorem or Euler's Theorem, the mathematics behind this algorithm is similar. With this particular algorithm, one can calculate the day of the week of any date in history according to the calendar we are currently using. Here are the steps:

Assume that Sunday = 1, Monday = 2, etc., up to Saturday = 7.

- Step 1: Divide the last 2 digits of the year by 4 and discard the fraction.
- Step 2: Add the date of the month to that number.
- Step 3: Add the month's key value to the number. Key values are as follows (months represented with first letters, in chronological order):

$$\begin{aligned} J &= 1, F = 4, M = 4, A = 0, M = 2, J = 5, \\ J &= 0, A = 3, S = 6, O = 1, N = 4, D = 6 \end{aligned}$$

- Step 4: Subtract 1 if the month is a January or February of a leap year.
- Step 5: Add 0 if the year is in 1900's, 6 if in the 2000's, 4 if in the 1700's, and 2 if in the 1800's. If the year is not in one of the categories, add multiples of 400 until it meets one of the criteria.
- Step 6: Add the last 2 digits of the year; call the final result x.
- Step 7: For such that y satisfies $x \equiv y \pmod 7$, y is the number corresponding to the day of the week.

Example: What day of the week was December 9, 1421 (12-09-1421)? Simply follow the steps in order.

- Step 1: $\dfrac{21}{4} = 5$ if we discard the fraction.
- Step 2: $5 + 9 = 14$ (9 is the date of the month).
- Step 3: The key value for December is 6, so $14 + 6 = 20$.
- Step 4: We can skip step 4 since this is not a leap year.
- Step 5: Adding multiples of 400 gives us a year in the 1800's, so we must add $20 + 2 = 22$.
- Step 6: Adding the last two digits of the year gives us $22 + 21 = 43$.
- Step 7: Now, we have $x \equiv y \pmod 7$, therefore, $43 \equiv y \pmod 7$, such that $1 \leq y \leq 7$. Thus, $y = 1$, which means that this day in history was a Sunday.

6. Conclusion

This concludes our examination of applications in the fascinating world of abstract algebra. We have barely scratched the surface, though. There are so many more applications we have not even mentioned, even some in puzzle games such as the Rubik's Cube. Abstract algebra can be incredibly tedious and challenging, but some is also accessible, as you saw in Fermat's Little Theorem and Euler's Theorem, and applications such as EIGamal encryption and the Diffie-Hellman Key Exchange are not beyond our comprehension. Algorithms such as the one we examined and used to find the day of the week of any date can be understood by pre-middle school students if they were to learn congruence modulo notation! However, teaching young ones that 2+2 can equal different values depending on what modulo we are working in may not be such a good idea.

References

[1] Application: Calendar Calculations, http://web.usna.navy.mil, p. 1.

[2] Application: Decimal Expansions of Rational Numbers, http://web.usna.navy.mil, pp. 1-3.

[3] Application: Diffie-Hellman Key Exchange, http://web.usna.navy.mil, p. 1.

[4] Application: EIGamal Encryption, http://web.usna.navy.mil, pp. 1-3.

[5] Euler's Theorem, http://web.usna.navy.mil, p. 1.

[6] Fermat's Little Theorem, http://web.usna.navy.mil, p.1.

Optimal Strategies for the Dice Game Schultz

Kevin Reed, *student*

MO Theta

Evangel University
Springfield, MO, 65802

Presented at the 2005 National Convention

1. Introduction

Schultz is a game of chance involving dice. There are many strategies associated with this game, but the trick is identifying the most probable winning strategy. No prior research could be found in regard to Schultz, but some probability formulas have been taken from [1]. Before diving into probability, here is a subset of rules we will be investigating to simplify the problem.

2. Scoring

- A player begins with seven dice. The player rolls all dice the first time.
- If a "1" turns up, it could be worth 10 points.
- If a "5" turns up, it could be worth 5 points.
- If three of a kind turns up on one roll, those three dice together are worth ten times the number on the die, and 100 points if they are three ones. For example, a roll of 4,4,4 is worth 40 points. The player removes at least one score, all three dice if scoring three of a kind.

- If no score is available, the player “Schultz”-es and loses the points accumulated for that turn. A “turn” is defined by the successive rolls of dice until stopping, Schultzing, or rolling out.
- "Rolling out" means scoring with all dice some time during the turn and having no dice remaining. When a player succeeds in rolling out, those points are “locked in” or in other words cannot be lost. It can be thought of as storing those points and beginning a new turn.
- The player may choose to continue to roll or stop after any score.
- In the case that a player rolls a Schultz or chooses to stop, the next player begins with seven dice.
- If a player stops, the points are added to that current player’s score and the turn ends.
- If a player rolls out, the points are added to that player’s score. The player is then allowed to begin a new turn.
- The first player to pass a set amount of points (usually 500 or 1000) wins.

3. Schultz-ing

As mentioned previously, a player Schultz-es when rolling dice without receiving points. The probabilities begin fairly straight forward, following intuition.

First we will examine the chances for “Schultz”-ing when rolling n dice.

- $n = 1$:
$$P\left(\text{Schultz} \mid n = 1\right) = \frac{4}{6} \text{ or } .666666.$$
This is fairly self explanatory. A one or five scores when using only one die. So a six-sided die has four ways not to score.
- $n = 2$:
$$P\left(\text{Schultz} \mid n = 2\right) = \frac{4}{9} \text{ or } .444444.$$
The probability of either die not getting a 1 or 5 is $\frac{4}{6}$. Multiply these probabilities together and we get $\frac{16}{36}$ or $\frac{4}{9}$.

- $n = 3$: Three dice complicate the probability of not scoring. Dice that would not normally score will score when found in triplet.

$$P\left(\text{Schultz} \mid n = 3\right) = \frac{5}{18} \text{ or } .277777.$$

The probability of not receiving a 1 or 5 is $\left(\frac{4}{6}\right)^3$. However if all three dice are the same value, they score ten times that number. Thus the probability of Schultzing is $\left(\frac{4}{6}\right)^3 - \frac{4}{216}$ (ways of getting three of a kind but not 1 or 5) or $\left(\frac{4}{6}\right)^3$ −P(3 of a kind).

- $n = 4$: Four dice complicate the matter even more, because we must now consider where the three of a kind comes from.

$$P\left(\text{Schultz} \mid n = 4\right) = \frac{17}{108} \text{ or } .157407.$$

The probability of not receiving a 1 or 5 is $\left(\frac{4}{6}\right)^3$. The probability of receiving a score from three of a kind is

$$\binom{4}{3}\binom{4}{1}\left(\frac{1}{6}\right)^3\left(\frac{3}{6}\right) + \binom{4}{4}\binom{4}{1}\left(\frac{1}{6}\right)^4$$

Using this, we can find the probability of either three of a kind or four of a kind, so the probability of Schultzing is $\left(\frac{4}{6}\right)^4 - P\left(3, 4 \text{ of a kind}\right)$.

- $n = 5$:

$$P\left(\text{Schultz} \mid n = 5\right) = \frac{25}{324} \text{ or } .077160.$$

The probability of not receiving a 1 or 5 is $\left(\frac{4}{6}\right)^5$. In the usual fashion, we subtract the ways we could get three of a kind:

$$\binom{5}{3}\binom{4}{1}\left(\frac{1}{6}\right)^3\left(\frac{3}{6}\right)^2 + \binom{5}{4}\binom{4}{1}\left(\frac{1}{6}\right)^4\left(\frac{3}{6}\right) + \binom{5}{4}\binom{4}{1}\left(\frac{1}{6}\right)^5,$$

and the probability of Schultzing is $\left(\frac{4}{6}\right)^5 - P\left(3, 4, 5 \text{ of a kind}\right)$.

- $n = 6$: There are two possible ways of finding the probability of not scoring. When rolling six dice, it is beginning to look as though it would be easier to find the probability of scoring, and subtract from one.

$$P\,(\text{Schultz} \mid n = 6) = \frac{5}{162} \text{ or } .030864.$$

With six dice, things become very tricky. We found another route to the answer to verify our results. The probability of not receiving a 1 or 5 is $\left(\frac{4}{6}\right)^6$. We subtract from this

$$\binom{6}{3}\binom{4}{1}\left(\frac{1}{6}\right)^3\left(\frac{3}{6}\right)^3 + \binom{6}{3}\binom{4}{2}\left(\frac{1}{6}\right)^6$$

which subtracts the possibility of having three of a kind, while adding back in the possibility that the second three be three of a kind also. Also we subtract

$$\binom{6}{4}\binom{4}{1}\left(\frac{1}{6}\right)^4\left(\frac{3}{6}\right)^2 - \binom{6}{5}\binom{4}{1}\left(\frac{1}{6}\right)^5\left(\frac{3}{6}\right) - \binom{6}{6}\binom{4}{1}\left(\frac{1}{6}\right)^6$$

or $\left(\frac{4}{6}\right)^6 - P\,(3, 4, 5 \text{ of a kind}) - P\,(6 \text{ of a kind}) + P\,(\text{two 3 of a kind})$.

An alternative strategy is to add the case that three of the four non-scoring dice turn up twice, but not three times, and the case that two of the four non-scoring dice turn up twice, and the other two turn up once. This uses a different sort of notation:

$$\binom{x}{a, b, c, d}$$

This is interpreted as $\frac{x!}{a!b!c!d!}$. The alternative strategy yields

$$\text{Case I} \quad \binom{4}{3}\binom{6}{2, 2, 2}\left(\frac{1}{6}\right)^6$$

$$\text{Case II} \quad \binom{4}{2}\binom{6}{2, 2, 1, 1}\left(\frac{1}{6}\right)^6$$

We add these to get $\frac{5}{162}$.

- $n = 7$:

$$P\,(\text{Schultz} \mid n = 7) = \frac{35}{3888} \text{ or } .009002.$$

We use our new strategy to find the probability of not scoring with seven dice. There is only one case for seven dice, which is that three of the four non-scoring numbers show up twice, while the other non-scoring number shows up once. We get

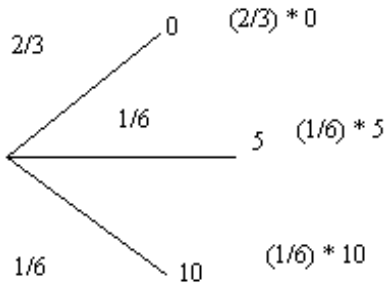
$$\binom{4}{1}\binom{7}{2, 2, 2, 1}\left(\frac{1}{6}\right)^7 = \frac{35}{3888}.$$

- $n = 8$:
$$P\,(\text{Schultz} \mid n = 8) = \frac{35}{5832} \text{ or } .006001.$$
Though this situation never arises in the game, we will look at it briefly as an example. We have
$$\binom{4}{0}\binom{8}{2,2,2,2}\left(\frac{1}{6}\right)^8 = \frac{35}{5832}.$$
- $n = 9$: Any combination of 9 dice will yield at least a 1, a 5, or three of a kind.

4. Strategy

Strategy sounds like a simple thing to determine, until it is approached from a mathematical standpoint. One particular strategy would stop after a certain point value has been gained for the turn. We will approach this strategy as with determining probabilities for "Schultz"-ing, with simple analysis. We determine this strategy based on expected values:

$$EV = Pr(\text{this turn's score}) + Pr(\text{rollout}) \cdot EV$$
Through some algebra we find
$$EV = \frac{E\,(\text{this turn's score})}{1 - \Pr\,(\text{rollout})}.$$
- 1 die: In 1-die Schultz, you can roll or not. So of course you would always roll in this version. There is a $\frac{2}{3}$ chance the player will Schultz, and a $\frac{1}{3}$ chance of rolling out (in this case scoring).
$$EV = \frac{1}{6}(5) + \frac{1}{6}(10) + \frac{2}{3}(0) \text{ this turn only}$$
$$EV = \left(\frac{\frac{15}{6}}{1 - \frac{1}{3}}\right) = \frac{15}{4}$$



- 2 dice: There are three point strategies for two-dice Schultz. This produces three separate trees.
  - 15 or 20 point stop

$$\begin{aligned} EV &= \frac{4}{9}(0)+\frac{8}{54}(0)+\frac{2}{54}(10)+\frac{2}{54}(15)+\frac{2}{54}(0)+\frac{2}{54}(15) \\ &\quad +\frac{2}{54}(20)+\frac{2}{36}(15)+\frac{1}{36}(10)+\frac{1}{36}(20) \text{ this turn only} \end{aligned}$$

$$EV = \frac{3.8888}{1-.2593} = 5.250.$$

  - 10 point stop

$$\begin{aligned} EV &= \frac{4}{9}(0)+\frac{8}{54}(0)+\frac{2}{54}(10)+\frac{2}{54}(15)+\frac{2}{9}(10)+\frac{2}{36}(15) \\ &\quad +\frac{1}{36}(10)+\frac{1}{36}(20) \text{ this turn only} \end{aligned}$$

$$EV = \frac{4.815}{1-.1852} = 5.909$$

  - 5 point stop

$$EV = \frac{4}{9}(0)+\frac{2}{9}(5)+\frac{2}{9}(10)+\frac{2}{36}(15)+\frac{1}{36}(10)+\frac{1}{36}(20) \text{ this turn only}$$

$$EV = \frac{5}{1-.1111} = .5625$$

    So now we look at our expected values and find that stopping after 10 points will earn a player the most points over time.

- 3 dice: At this point, we can see that drawing a tree has gotten somewhat out of hand. However, a spreadsheet can handle the 38 rows and 26 columns. The spreadsheets can be found in Appendix A.

In the spreadsheets, Sc stands for score, Event describes the output of a roll (? indicates some non-scoring die), Left indicates the number of dice remaining (after taking all scoring dice), Prob stands for the probability of rolling that combination of dice, RO stands for roll out (1 for Yes, 0 for No), and Sch stands for Schultz (1 for Yes, 0 for No). There is a maximum of three rounds (three dice). In the totals (shaded) columns, Tprob indicates the total likelihood of that three-round scenario playing out. Roprob shows the Tprob of scenarios concluding with a roll out (otherwise shows 0). EV stands for the Expected Value for the scenario and strategy, while Tots shows the total score for that scenario and strategy. On the far right of the spreadsheet, the grand totals are displayed. Schultz shows the number of ways to Schultz, Rollout shows the number of ways to roll out. PrRollout shows the probability of rolling out, and ExpVal indicates the expected value of using the given strategy (including rollouts and therefore extra turns). Tprob is simply a tool I used to be sure all probabilities were included (hence 1.0000 or 100%).

The same formulas are applied in the spreadsheet as had been before and we get the results:

Points until stop	E (single turn)	EV (this strategy)
25+	6.7671	$\frac{6.7671}{1-.2778} = 9.3698$
20	7.9039	$\frac{7.9039}{1-.2428} = 10.4383$
15	8.9841	$\frac{8.9841}{1-.1399} = 10.4457$
10	9.2979	$\frac{9.2979}{1-.0802} = 10.1092$
5	8.6806	$\frac{8.6806}{1-.0556} = 9.1913$

5. Conclusion

With a game of three die Schultz, stopping at 15 points has a better chance of being the winning strategy according to the expected value strategy than stopping at 20 points.

For every die added, the size of the table has increased almost 3 times. Following this general pattern, seven dice would have a table of roughly 3^7 (or 2187) rows. The first roll has 6^7 (279,936) possibilities, including repeats. This doesn't include the following rolls. Clearly, a table is not going to be a final solution to this problem either.

Future research will include writing an algorithm to be processed by a computer, or possibly by hand. By creating a program that would use recursive statements, the problem could be simplified for at least 4 and/or 5 dice. A recursive statement is a function that calls itself. An example would be an algorithm to trace the correct path through a maze. The algorithm would test all possible paths by moving to an open area, while saving the last position. If the algorithm hit a dead end, it would back out one space and try another path. If another path did not exist, it would continue to back out until an open path was available.

In the case of Schultz, a recursive approach could simplify the analysis by simply counting the 300,000+ rolls with the computer. A possible algorithm is:

$$EV = (\text{curProb } (\text{curScore} + EV(\text{dieLeft}, \text{ptsLeft})),$$

where curProb is the probability of getting that far along the "tree", curScore is the current score, dieLeft is the number of dice left for the current turn, and ptsLeft is the number of points left until the desired score is reached.

We may also set other parameters such as stopping if a certain number of dice are left and an alternate score has been reached, or if there are more than a certain number of dice when the desired score is reached.

Acknowledgements: I would like to thank my faculty advisor, mentor, and friend, Dr. Don Tosh. Without his help and guidance this paper would not have been. I would also like to acknowledge Missouri Theta's chapter of KME, for support and friendship.

References

[1] D. Tosh, Math 310: *Probability and Statistical Analysis*, Evangel Press, Ch. A3, A4, A6, 1994.

Appendix A

Schultz One Die

Round 1

Sc	Event	Left	Prob	RO	Sch	TProb	ROProb	EV	SchProb	TotS
0	?	0	0.6667	0	1	0.6667	0.0000	0.0000	0.6667	0
5	5	0	0.1667	1	0	0.1667	0.1667	0.8333	0.0000	5
10	10	0	0.1667	1	0	0.1667	0.1667	1.6667	0.0000	10

Sch	RO	TProb	EVnoCont	PrRO	PrSch	EV
1	2	1.0000	2.5000	0.3333	0.6667	3.7500

Schultz Two Dice Target 15,20

Round 1

Sc	Event	Left	Prob	RO	Sch
0	?,?	2	0.4444	0	1
5	5,?	1	0.2222	0	0
5	5,?	1	0.2222	0	0
5	5,?	1	0.2222	0	0
10	10,?	1	0.2222	0	0
10	10,?	1	0.2222	0	0
10	10,?	1	0.2222	0	0
10	5,5	0	0.0278	1	0
15	10,5	0	0.0556	1	0
20	10,10	0	0.0278	1	0

Sch	RO	TProb	EV	PrRO	PrSch
3	7	1.0000	3.8891	0.2593	0.7407

Round 2

Start	Sc	Event	Left	Prob	RO	Sch
2	0	schultz	2	1.0000	0	1
1	0	?	1	0.6667	0	1
1	5	5	0	0.1667	1	0
1	10	10	0	0.1667	1	0
1	0	?	1	0.6667	0	1
1	5	5	0	0.1667	1	0
1	10	10	0	0.1667	1	0
0	0	rollout	0	1.0000	1	0
0	0	rollout	0	1.0000	1	0
0	0	rollout	0	1.0000	1	0

TProb	ROProb	EV	SchProb	TotS
0.4444	0.0000	0.0000	0.4444	0
			0.0000	
0.1481	0.0000	0.0000	0.1481	0
0.0370	0.0370	0.3704	0.0000	10
0.0370	0.0370	0.5556	0.0000	15
			0.0000	
0.1481	0.0000	0.0000	0.1481	0
0.0370	0.0370	0.5556	0.0000	15
0.0370	0.0370	0.7407	0.0000	20
			0.0000	
0.0278	0.0278	0.2780	0.0000	10
			0.0000	
0.0556	0.0556	0.8333	0.0000	15
			0.0000	
0.0278	0.0278	0.5555	0.0000	20

The Problem Corner

Edited by Pat Costello and Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 2006. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall, 2006 issue of The Pentagon. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)622-3051)

PROBLEMS 590-596

Problem 590. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Evaluate the following sum: $\displaystyle\sum_{n=1}^{\infty}\left(4^n\cos^2\frac{\pi}{2^{n+2}}\right)^{-1}$

Problem 591. *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, Massachusetts.*

Express

$$\cos A\cos B\sin(A-B)+\cos B\cos C\sin(B-C)$$
$$+\cos C\cos A\sin(C-A)$$

as the product of three sines.

Problem 592. *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, Massachusetts.*

The points $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ are the vertices of a square S. Find an equation in x and y whose graph in the xy-plane is S.

Problem 593. *Proposed by Kenneth M. Wilke, Washburn University, Topeka, Kansas.*

Let

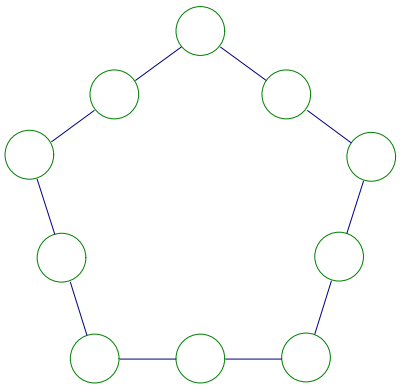
$$N = p^2 + q^2 + r^2 + s^2,$$

where p, q, r, and s are positive integers such that $pq = rs$. Prove or disprove that N is prime.

Problem 594. *Proposed by the editor.*

The sequence $x_0, x_1, x_2, \ldots$ is defined by the conditions $x_0 = 0, x_1 = 1$, and $x_{n+1} = \dfrac{nx_n + x_{n-1}}{n+1}$ for $n \geq 1$. Determine $\lim\limits_{n\to\infty} x_n$.

Problem 595. *Proposed by the editor.*



Place the first 10 odd primes $3, 5, 7, 11, 13, 17, 19, 23, 29, 31$ into the circles on the pentagon so that the sum of the entries on each side is the same.

Problem 596. *Proposed by the editor.*

A phone number has the *lazy-finger* property when the next number that you dial (on a touch tone phone) is either the same number or a number up or down one button or a number to the left or right one button. For example, 555-2365 has the *lazy-finger* property. How many 7-digit phone numbers that start with a 5 have the *lazy-finger* property?

Please help your editor by submitting problem proposals.

SOLUTIONS 572, 578, 580-584

Problem 572. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let z_1, z_2, and z_3 be nonzero complex numbers such that $z_1^3+z_2^3+z_3^3 = 0$. Show that $\left\{z_1^9+z_2^9+z_3^9\right\} / \left\{z_1^3z_2^3z_3^3\right\}$ is an integer, and determine its value.

Solution *by Russell Euler and Jawad Sadek jointly, Northwest Missouri State University, Maryville, Missouri.*

The value of the desired expression is 3. We replace z_1, z_2, and z_3 with x, y, and z, respectively. Using the formula for the sum of two cubes, we get

$$\begin{aligned} x^9+y^9 &= \left(x^3+y^3\right)\left(x^6-x^3y^3+y^6\right) \\ y^9+z^9 &= \left(y^3+z^3\right)\left(y^6-y^3z^3+z^6\right) \\ x^9+z^9 &= \left(x^3+z^3\right)\left(x^6-x^3z^3+z^6\right). \end{aligned}$$

By adding the sides, we get

$$\begin{aligned} 2\left(x^9+y^9+z^9\right) &= \left(x^3+y^3\right)\left(x^6-x^3y^3+y^6\right) \\ &\quad +\left(y^3+z^3\right)\left(y^6-y^3z^3+z^6\right) \\ &\quad +\left(x^3+z^3\right)\left(x^6-x^3z^3+z^6\right). \end{aligned}$$

Using the given equality $x^3+y^3+z^3=0$, we get

$$\begin{aligned} 2\left(x^9+y^9+z^9\right) &= -z^3\left(x^6-x^3y^3+y^6\right)-x^3\left(y^6-y^3z^3+z^6\right) \\ &\quad -y^3\left(x^6-x^3z^3+z^6\right) \\ &= -x^6\left(z^3+y^3\right)-y^6\left(z^3+x^3\right)-z^6\left(x^3+y^3\right) \\ &\quad +3x^3y^3z^3 \\ &= x^9+y^9+z^9+3x^3y^3z^3. \end{aligned}$$

This implies that

$$x^9+y^9+z^9 = 3x^3y^3z^3.$$

It follows that

$$\frac{x^9+y^9+z^9}{3x^3y^3z^3} = 3.$$

Solution *by Eric Dixon, student, Kettering University, Kettering, Michigan. (Revised slightly by the editor).*

Note that

$$\frac{z_1^9+z_2^9+z_3^9}{z_1^3z_2^3z_3^3}=\frac{z_1^6}{z_2^3z_3^3}+\frac{z_2^6}{z_1^3z_3^3}+\frac{z_3^6}{z_1^3z_2^3}.$$

Consider the quantity $\frac{z_1^6}{z_2^3z_3^3}$. Since $z_1^3=-\left(z_2^3+z_3^3\right)$, we have

$$\begin{aligned}\frac{z_1^6}{z_2^3z_3^3} &= \frac{\left\{-\left(z_2^3+z_3^3\right)\right\}^2}{z_2^3z_3^3}\\ &= 2+\frac{z_2^6+z_3^6}{z_2^3z_3^3}\\ &= 2+\frac{z_2^3}{z_3^3}+\frac{z_3^3}{z_2^3}.\end{aligned}$$

Similarly,

$$\frac{z_2^6}{z_1^3z_3^3}=2+\frac{z_1^3}{z_3^3}+\frac{z_3^3}{z_1^3}$$

and

$$\frac{z_3^6}{z_1^3z_2^3}=2+\frac{z_1^3}{z_2^3}+\frac{z_2^3}{z_1^3}.$$

Adding these last three equations and using $z_1^3+z_2^3+z_3^3=0$ yields

$$\begin{aligned}\frac{z_1^9+z_2^9+z_3^9}{z_1^3z_2^3z_3^3} &= 6+\frac{z_2^3+z_3^3}{z_1^3}+\frac{z_1^3+z_3^3}{z_2^3}+\frac{z_1^3+z_2^3}{z_3^3}\\ &= 6-\frac{z_1^3}{z_1^3}-\frac{z_2^3}{z_2^3}-\frac{z_3^3}{z_3^3}\\ &= 3.\end{aligned}$$

Also solved by Clayton W. Dodge, University of Maine, Orono, Maine and the proposer.

Editor's Comment: Two solutions were chosen to illustrate different approaches to this problem. The proposer's (and Dodge's) solution used the identity

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-ac-bc\right),$$

with $a=z_1^3$, $b=z_2^3$, and $c=z_3^3$.

Problem 578. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain*

Let ABC be a triangle such that $\sin A$, $\sin B$, and $\sin C$ are in arithmetic progression. Prove that $\tan\left(\frac{A}{2}\right)\tan\left(\frac{C}{2}\right) = \frac{1}{3}$.

Solution *by Joe Flowers, Texas Lutheran University, Seguin, Texas.*

Since $\sin A$, $\sin B$, and $\sin C$ are in arithmetic progression, we must have $\sin B - \sin A = \sin C - \sin B$ or $2\sin B = \sin A + \sin C$. By the Law of Sines, this can be written as $2\sin B = \frac{a}{b}\sin B + \frac{c}{b}\sin B$, so that $2 = \frac{a+c}{2b}$ or $2b = a + c$. Thus the semiperimeter of the triangle is

$$s = \frac{a+b+c}{2} = \frac{3b}{2}.$$

Using the known formulas

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \text{ and } \tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{s(s-c)}},$$

we have

$$\tan\frac{A}{2}\tan\frac{C}{2} = \frac{s-b}{s} = \frac{1}{3}.$$

Also solved by Vincent Ferlini, Keene State College, Keene, New Hampshire, Andrew Rockett, C.W. Post Campus of Long Island University, Brookville, New York, and the proposer.

Problem 580. *Proposed by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.*

Let x, y, z be nonnegative real numbers. Show that if $x^2 + y^4 + z^6 \leq 3$, then $x + 2y + 3z \leq 6$.

Solution *by the proposers.*

By Bernoulli's Inequality, we have that if $n \geq 1$ and $x \geq 0$, then $x^n - 1 \geq n(x-1)$. Applied to this problem, we have $x^2 - 1 \geq 2(x-1)$, $y^4 - 1 \geq 4(y-1)$, and $z^6 - 1 \geq 6(z-1)$. Adding these inequalities and rearranging the terms gives $x^2 + y^4 + z^6 \geq 2x + 4y + 6z - 9$. Since $x^2 + y^4 + z^6 \leq 3$, we have $2x + 4y + 6z - 9 \leq 3$, and the desired inequality now follows.

Problem 581. *Proposed by Thomas Chu, Austin, Texas.*

Let triangle ABC be a right triangle with legs a and b and the right angle at C. Let O denote the circumcenter of triangle ABC. Let P be a point located on AB so that when CP is drawn, CP is perpendicular to AB. Show that the ratio of OP to c is given by $\dfrac{\left|a^2-b^2\right|}{2\left(a^2+b^2\right)}$.

Solution *by Clayton W. Dodge, University of Maine, Orono, Maine.*

Point O is, of course, the midpoint of the hypotenuse AB, whose length is c. Draw OP, and consider the similar right triangles. By similar right triangles, we have $OP = AP - OA = \dfrac{b^2}{c} - \dfrac{c}{2} = \dfrac{2b^2-c^2}{2c} = \dfrac{\left|b^2-a^2\right|}{2c}$ and finally $\dfrac{OP}{AB} = \dfrac{\left|b^2-a^2\right|}{2c^2}$, which is equivalent to the desired result.

Also solved by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri and the proposer.

Problem 582. *Proposed by Pat Costello, Eastern Kentucky University, Richmond Kentucky.*

Suppose that one fills in the ten blanks in the number

$$9\_\ \_8\_7\_6\_5\_4\_3\_2\_1\_0$$

with all of the ten digits $0, 1, 2, ..., 9$. What is the probability that the resulting 20-digit number is divisible by 528?

Solution *by Elizabeth Stoner, student, Northwest Missouri State University, Maryville, Missouri (revised by the editor).*

Let N represent the 20-digit number 9_ _8_7_6_5_4_3_2_1_0 after the blanks have been filled in with all ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 in some order. If N is divisible by 528, it must be divisible by the factors of 528. The canonical representation of 528 is $24 \cdot 3 \cdot 11$.

Now N is always divisible by 3 since the sum of the digits of N is divisible by 3. Note that 8 divides N when the last three digits of N are divisible by 8; thus the last blank (to the right) must be filed with either 2 or 6. Similarly, 16 divides N when the last four digits are divisible by 16; thus N must end in 1120, 3120, 5120, 7120, 9120, 0160, 2160, 4160 or 8160. Also, 11 divides N whenever the number formed by alternately adding and subtracting the successive digits of N is divisible by 11. Thus the first (left-most) blank must be filled in with 9 and all other digits will

add to their respective opposites to make a sum of zero.

Finally, the total sample space of possible N has 10! elements, while the number of N which are divisible by 528 is $4 \cdot 2 \cdot 7!$. (Note that the possibility that N ends in 9120 must be discarded since the left-most blank must be filled with the digit 9 to insure divisibility by 11. Ed.) Thus the probability that 528 divides N is $\dfrac{8 \cdot 7!}{10!} = \dfrac{1}{90}$.

Also solved by Clayton W. Dodge, University of Maine, Orono, Maine, Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri, and the proposer. One incorrect solution was received.

Problem 583. *Proposed by James R. Bush, Waynesburg College, Waynesburg, Pennsylvania.*

For an integer $n \geq 2$, consider the remainder term $A_n x + B_n$ which occurs when x^n is divided by the trinomial $x^2 - x - 1$. Find $\lim\limits_{n\to\infty} \dfrac{A_n}{B_n}$.

Solution *by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Applying Euclid's algorithm, we have

$$x^n = \left(x^{n-2} + a_{n-3}x^{n-3} + \cdots + a_1 x^1 + a_0\right)\left(x^2 - x - 1\right) + A_n x + B_n.$$

Since the zeros of $x^2 - x - 1$ are $\alpha = \dfrac{1+\sqrt{5}}{2}$ and $\beta = \dfrac{1-\sqrt{5}}{2}$, then

$$\alpha^n = \alpha A_n + B_n$$

and

$$\beta^n = \beta A_n + B_n.$$

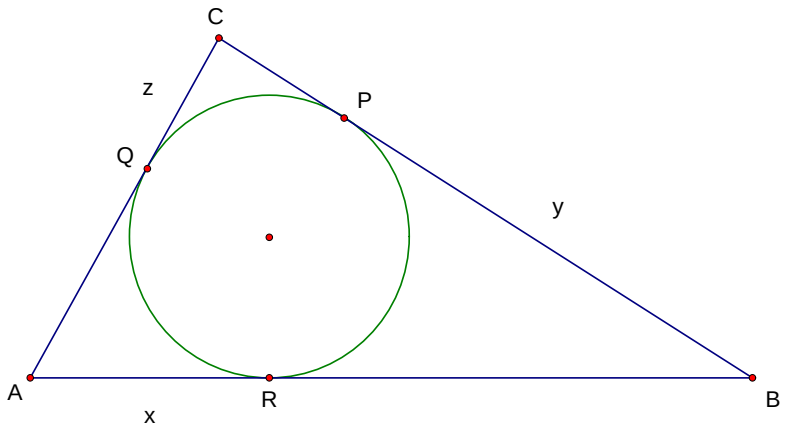
Solving this system of linear equations, we get $A_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta} = F_n$ and $B_n = \dfrac{\alpha\beta^n - \beta\alpha^n}{\alpha - \beta} = \dfrac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} = F_{n-1}$, where F_n is the nth Fibonacci number. (Note that $\alpha\beta = -1$). Therefore,

$$\lim_{n\to\infty} \frac{A_n}{B_n} = \lim_{n\to\infty} \frac{\alpha^n - \beta^n}{\alpha^{n-1} - \beta^{n-1}} = \alpha = \frac{1+\sqrt{5}}{2}.$$

Also solved by Clayton W. Dodge, University of Maine, Orono, Maine, Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri, and the proposer.

Problem 584. *Proposed by Thomas Chu, Austin, Texas.*

In triangle ABC, the incircle is tangent to AB, BC, and CA at P, Q, and R, respectively. Let $x = AR$, $y = BP$, and $z = CQ$. Let r be the radius of the inscribed circle. Prove that $\dfrac{x+y+z}{xyz} = \dfrac{1}{r^2}$.



Solution *by Clayton W. Dodge, University of Maine, Orono, Maine.*

The area K of triangle ABC is given by $K = rs$, where $s = x+y+z$ is the semiperimeter of the triangle. Also

$$K^2 = s(s-a)(s-b)(s-c) = sxyz,$$

so that $sxyz = r^2s^2$. Thus

$$\frac{1}{r^2} = \frac{s^2}{sxyz} = \frac{s}{xyz} = \frac{x+y+z}{xyz}.$$

Also solved by Scott H. Brown, Auburn University, Montgomery, Alabama, Eric Dixon, student, Kettering University, Kettering, Michigan, Ovidiu Furdui, Western Michigan University, Kalamzaoo, Michigan, and the proposer.

A late solution was received from Allen Hoffmeyer, student, Georgia College and State University, Milledgeville, Georgia.

Kappa Mu Epsilon News

Send news of chapter activities and other noteworthy KME events to
Connie Schrock, KME Historian
Department of Mathematics, Computer Science, and Economics
Emporia State University
1200 Commercial Street
Campus Box 4027
Emporia, KS 66801
or to
schrockc@emporia.edu

Installation Report

Illinois Iota
Lewis University, Romeoville, Illinois

The Illinois Iota chapter of Kappa Mu Epsilon was installed at Lewis University in Romeoville, Illinois, on Saturday, February 26, 2005. The installation ceremony was held at 6:00 p.m. in the Faculty Dining Hall, and began with a greeting from Brother James Gaffney, the president of Lewis University. Don Tosh, President-Elect of KME, was the installing officer. Margaret Juraco, already a member of KME and a prime supporter of starting a new chapter, served as the conductor.

Illinois Iota has 13 charter members. Student charter members are Ellen Deinzer, Tiffany Eischen, Natalie Kremer, Elaina Ktistou, Jack Lelko, Rachel Oesterreicher, Joel Pommier, and Steve Weierman. Faculty charter members are: Steven Berger, Thomas Dupre, Margaret Juraco, Paul Kaiser, and Ray Klump.

The officers installed during the ceremony were: Jennifer Jackson, President; Rachel Oesterreicher, Vice President; Jack Lelko, Secretary; Ellen Deinzer, Treasurer; and Margaret Juraco, Corresponding Secretary/ Faculty Sponsor.

Missouri Nu
Columbia College

The Missouri Nu chapter was installed at Columbia College in Columbia, Missouri, on Friday, April 29, 2005. The installation was held in Dorsey Chapel at 4:00 pm following a reception that was held for initiates and attendees. Don Tosh, National President, was the installing officer. Ann Bledsoe served as the conductor. She and Larry West, the chair of the Mathematics and Computer Science Department, were already members of KME and instrumental in forming the new chapter. Charter students

members initiated were: Ashley Bonine, Jenna Holdmeyer, Jason Kerr, Shahthureen Khan, Sanel Krgo, Jamie Netherton, Michael Perkins, Charles Phillips, Daniel Roemer, Kelly Sharp, Than Som, and Nobuyuki Tsuchiya. Faculty charter members initiated were Arlin Epperson, David Heise, Natasha Latushkina, Yihsiang Liow, Suzanne Tourville, and Paul Wiedemeier. The following officers were also installed during the ceremony: James Netherton, President; Michael Perkins, Vice-President; Jenna Holdmeyer, Secretary; Shahthureen Khan, Treasurer; Ann Bledsoe closed the ceremony with a story of how she met her husband through KME.

Pennsylvania Sigma
Lycoming College

The installation of the Pennsylvania Sigma chapter took place on April 1, 2005 in the Jane Schultz Room of the Wertz Student Center at Lycoming College, Williamsport, Pennsylvania following a dinner for those present. 24 students and 5 faculty members formed the charter group, while approximately 30 family members, students, faculty and administrators were also in attendance. Taking part in the ceremony were officers of the math club, known as the Association of Mathematically Interested Students, at Lycoming College. These officers included Christopher Reed, President, Kathryn Chastek, Vice President, Philip Mann, Secretary, and Rebecca Jervey, Treasurer. The installation was performed by Dr. Robert Bailey, KME National President, who also addressed the group on the history of Kappa Mu Epsilon. Chapter officers will be appointed at a later date for the next school year. Dr. Santhusht de Silva will serve as Corresponding Secretary, and the Faculty Sponsor will be Dr. Eileen Peluso.

Virginia Delta
Marymount University

The installation of the Virginia Delta chapter took place on March 26, 2004 in the Reinsch Auditorium at Marymount University, Arlington, Virginia. 13 students and 5 faculty members formed the charter group, while approximately 150 family members, students, faculty and administrators were also in attendance. Taking part in the ceremony were officers of the Math Club at Marymount University, including Tanya Kazakova, President, Anna Wolfram, Vice President, Lindsey Redding, Secretary, and Julie Rogers, Treasurer. The installation was performed by Dr. Robert Bailey, KME National President. Dr. Elsa Schaefer serves as Corresponding Secretary, and the Faculty Sponsor is Dr. Stephan Cawthorne. A reception at the home of Dr. Elsa Schaefer for initiates and guests followed the ceremony.

West Virginia Beta
Wheeling Jesuit University

The installation of the West Virginia Beta chapter of Kappa Mu Epsilon was held on March 11, 2005, in the new Acher Science Center on the campus of Wheeling Jesuit University in Wheeling, West Virginia. Dr. Pat Costello, Past President of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Ted Erickson, Chair of the Department of Mathematics, arranged the ceremony. Six students and three faculty constituted the founding group of the new chapter at Wheeling Jesuit. Those initiated were: Christopher Tartamella, Theresa Glandon, Megan Beakman, Locksley Todd, Kirsten Williams, Julie Sobutka, Onkar Pandit, Kimberly Roth, and Theodore Erickson. Also in attendance was Julianne Maher, Senior Vice President for Academic and Student Affairs at Wheeling Jesuit University.

Dr. Costello began the evening ceremony with an introduction to the organization and a brief history of KME. Besides presenting the new chapter with a charter and crest, each student received their membership certificate, keypins, and a 15-puzzle at the installation. Since two students were away at university athletic events, it was decided that officers would be elected at a later meeting of the new chapter.

Faculty member Ted Erickson accepted the responsibilities of corresponding secretary and faculty sponsor. After the installation, the students, faculty, and Dr. Maher walked to the Troy Theater in Swint Hall where the annual university-wide Honors Convocation was held. Dr. Costello and Reverend Joseph Hacala, University President, got to shake hands of all the charter members of the West Virginia Beta chapter of KME.

Chapter News

AL Alpha – Athens State University
Chapter President– Holly Gasque, 15 Current Members, 15 New Members. Other spring 2005 officers: Michelle Gist, Vice–President; Mimi Cook, Secretary; Elizabeth Pylant, Treasurer; Dottie Gasbarro, Corresponding Secretary.

KME members of Alabama Alpha worked a service/food booth at the Athens Ole Time Fiddlers Convention in October to raise funds for local projects and meetings/socials. Members also hosted a speaker and reception for Women's History Month this spring.

AL Gamma – Montevallo University

7 New Members. Corresponding Secretary-Don Alexander.

New Initiates – Sally Clark, Katherine Lanier, Stefanie Ledbetter, Mary Maxwell, Ivan Smith, Amanda Fox, Tanya Wilson.

AL Zeta – Birmingham Southern College

Chapter President– Matt Woods., 10 Current Members, 3 New Members. Other spring 2005 officers: Meredith Kirkpatrick, Vice–President; Wiley Truss, Secretary; Wiley Truss, Treasurer; Mary Jane Turner, Corresponding Secretary.

The formal spring semester initiation of the Alabama Zeta chapter of Kappa Mu Epsilon was held April 27, 2005 in the Olin Mathematics and Computer Science Building on the campus of Birmingham-Southern College. Three new members were initiated. BSC alumna and former KME member Molly Gibson presented a talk entitled "Mathematics in Management Sciences and Electric Utility Pricing".

CA Delta – California State Polytechnic University

Corresponding Secretary-Patricia Hale.

New Initiates – Ernest Shen, Brandon Duffey, Phillip Gonzalez, Sarah Freed, Loc Tuong Le, Eunice Yang, Aily Xie, Hoa Q. Lam, Brian Chen, Jason Pham, Godwin Shen, Tracy Nguyen, Lisa A. Schulman.

CA Epsilon – California Baptist University

4 New Members. Corresponding Secretary-Catherine Kong.

New Initiates – Shelly Britton, Sarah Cherry, Melisa Palomo, Michelle Papp.

CO Delta – Mesa State College

Chapter President – Kandice Abramson, 180 Current Members, 7 New Members. Other spring 2005 officers: Katherine Stadelman, Vice-President; Cami Nielsen, Secretary; Robert Miller, Treasurer; Erik Packard, Corresponding Secretary.

CT Beta – Eastern Connecticut State University

9 New Members. Spring 2005 officers: Mizan R. Khan, Treasurer; Christian L. Yankov, Corresponding Secretary.

The CT Beta initiation was held April 28, 2005.

New intiates – Nichole Caisse, Sheryl Garcia, Katie Hardy, Meaghan Kehoegreen, Erin Guinn, Hearther Souza, Michelle Sposato, Jeffrey Weber, Gordon Willey.

GA Alpha – University of West Georgia

Chapter President– Ginger Jones. 13 Current Members, 17 New Members. Other spring 2005 officers: Matt Jones, Vice–President; Max Perkins, Secretary; Dimitri Plaks, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

The Georgia Alpha Chapter of KME held its 31st annual Initiation Ceremony on April 24, 2005. After the 17 new members were initiated,

Chapter Officers as listed above were elected for 2005-2006 and a reception was held in honor of the 2005 Initiates. At the reception, the winners of this year's mathematics scholarships and awards were announced (all of whom are KME members). The Burson Calculus Award went to Matt Jones, The three Crider Award winners were Heather Morse, Sara Nofziger Reid, and David Yarbrough. The Whatley Scholarship went to Matt Jones. The Cooley Scholarship went to Ginger Jones. The Pittman Scholarship went to Ron Chase. The Martin Scholarship went to Corey Pittman, and the Boyd Award in Mathematics went to Sara Nofziger Reid.

GA Beta – Georgia College & State University

17 New Members. Corresponding Secretary-Dr. Michael Marion.

New Initiates – Christopher J. Angles, Lisa M. Cash, Jessica A. Edwards, Brandi N. Hart, Laurel S. Ivester, Amy F. Kelley, Thomas L. Lewis, Yi Liu, Pamela D. Merritt, Daniel C. Michael, Althea M. Mumelo, Amanda S. Murrey, Bao G. Nguyen, Leya M. Peters, Lila F. Roberts, Endalenesh Tsegaye, Martin J. Wattenbarger.

GA Delta – Berry College

11 New Members. Corresponding Secretary-Ron Taylor.

New Initiates – Kevin Jameson Baugh, Carrie Nicole Carden, Carly Michelle Donahue, Courtney Deann Griffin, Jessica Grace Grundy, Adam Jackson Kiefer, Frank Robert Petruzielo, Jr., Jeffrey David Pullen, Byron James Schueneman, Elise Jacqueline Trisler, Meng An Tsai.

IA Alpha – University of Northern Iowa

Chapter President– Cindee Calton-45 members, 4 New Members. Other spring 2005 officers: Doug Hoffman, Vice–President; Lynne Dieckman, Secretary; Donald Daws, Treasurer; Mark D. Ecker, Corr. Sec.

Student member Jake Ferguson presented his paper "Probability in Sports" at our first spring KME meeting on January 31, 2005 at Professor Mark Ecker's home. Our second meeting was held on February 28, 2005 at Professor Cathy Miller's house where student member Lynne Dieckman presented her paper, "Eye Color Inheritance Pattern in the Common Fruit Fly". At our third meeting on March 28, 2005 at Professor Syed Kirmani's residence, student member Moon Lee presented her paper on "The Cantor Set". Student member Doug Hoffman addressed the spring initiation banquet with "The Probability of Poker Hands". Our banquet was held at The Ridges Association Poolhouse on April 25, 2005 where four new members were initiated.

IA Delta – Wartburg College

Chapter President– Justin Peters. 34 Current Members, 18 New Members. Other spring 2005 officers: Brian Borchers, Vice–President; Jill Seeba, Secretary; Joee Williams, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

We sponsored two teams to participate in the Mathematical Modeling Competition. We encouraged a number of students to attend the Iowa Sectional Meeting of the Mathematical Assosciation of America in Pella. We planned the annual banquet and initiation ceremony, which was held on April 10; eighteen members were initiated. We sent three teams of students to participate in the Iowa Collegiate Math Competition in Ames, Iowa. Our annual end of the year picnic was held on May 23.

New Initiates– Kevin Balster, Erin Benoy, Brian Borchers, Allison Davis, Cassandra J. Frush, Nathanial M. Juchems, Andy Kruse, Kyle T. Morse, Jason Nielsen, Jodie Pearson, Justin Peters, Patrick S Rhomberg, Brad Schaefer, Jill Seeba, David Thomas Voves, Jason Welstead, Josephine Williams, and Benjamin Bosquet

IN Beta – Butler University

Chapter President– Eric Nelson. 20 Current Members, 0 New Members. Other spring 2005 officers: Brittany Brown, Vice–President; Scott Guttman, Secretary; Dirk Conner, Treasurer; Amos Carpenter, Corresponding Secretary.

In addition to our monthly meetings, we had two invited speakers. "The Science of Magic and the Magic of Science" by Bob Friedhoffer and "The Graduate School in Mathematics" by Dr. Scott Parsell.

IN Gamma – Anderson University

Corresponding Secretary-Stanley Stephens.

New Initiates – Melissa S. Bailey, Brian D. Baker, Bradley J. Mitchell.

KS Alpha – Pittsburg State University

Chapter President– Cassie Stuckey. 35 Current Members, 14 New Members. Other spring 2005 officers: Kristie Julian, Vice–President; Timothy McVey, Secretary; John Cauthon, Treasurer; Tim Flood, Corresponding Secretary.

KS Beta – Emporia State University

Chapter President– Chris Dobbs. Other spring 2005 officers: Shannon Wooton, Vice–President; Meaghan Jackson, Secretary; Jason Miller, Treasurer; Connie Schrock, Corresponding Secretary.

This year members of KS Beta received several awards. Jason Miller received the Taylor Award for a book scholarship. Shannon Wooton was named Shepherd Scholar and received Susan B. Anthony Award. Mindy Baker and Leah Childers were presented outstanding KME member awards at KME picnic. KS Beta was voted most outstanding organization at ESU.

KS Beta has also conducted and helps out with several events. They conducted an activity at Riverside Elementary School and William Allen White Elementary School to increase 4th graders interest in math. ESU hosts Expanding Your Horizons each spring for middle level girls and

KME members help. There was also a Middle School Family Math night that many of the students planned and worked at.

Chris Dobbs, Dr. Hollenbeck, Jacob Magnusson, Jason Miller, and Dr. Schrock attended the national convention, where Chris Dobbs presented "Lightning and the Application of Mathematics."

KS Delta –Washburn University

Chapter President– Fred Hollingshead. 30 Current Members, 13 New Members. Other spring 2005 officers: Jo Marie Rozzelle, Vice–President; Jan Misak, Secretary; Jan Misak, Treasurer; Kevin Charlwood, Corresponding Secretary.

The Kansas Delta chapter of KME met for three luncheon meetings with the Washburn Math Club during the semester. Our four students preparing to present their research work at the KME national convention at Schreiner University in Texas gave practice talks at some of these meetings. The chapter's annual KME initiation banquet was held on February 22, 2005 with 13 new initiates, including one faculty member as an honorary member. Faculty members Kevin Charlwood, Donna LaLonde, Al Riveland and Ron Wasserstein took students Ken Beck, Carolyn Cole, Matt Hamilton, Fred Hollingshead, Zeb Kramer, Kristin Ranum and Jo Marie Rozzelle to the National KME Convention at TX Mu in Kerrville, TX in April of 2005. Carolyn, Fred, Zeb and Jo Marie all presented at the meeting, with Carolyn, Fred and Jo Marie all winning "top four" placements in the judging of both written and oral presentations of their papers.

New Initiates – Kenneth Dale Beck, Paul D. Bedore, Megan Elizabeth Caudle, Melissa Ann Fast, Nicole M. George, Stephanie M. Herbster, Emily A. Huelskamp, Aaron Keller, Roscoe J. Mutz, Fai Y. Ng, Kristin L. Ranum, Michael V. Roe, Hwa Chi Liang.

KS Gamma – Benedictine College

Corresponding Secretary-Linda Herndom.

New Initiates—Charles Auman, Kelly Bach, David Mulvany.

KS Zeta – Southwestern College

Corresponding Secretary-Bhaskara Rao.

New Initiates – Sarah C. Benton, Ben Carden, Alex Turner Chen, Moses M. Mungania.

KY Beta – University of the Cumberlands

Chapter President-Joshua White, 38 Current Members, 15 New Members. Other spring 2005 officers: Stephanie Isaacs, Vice–President; Gretchen Phelps, Treasurer; Jonathan Ramey, Corresponding Secretary.

On February 25, 2005, the Kentucky Beta chapter held an initiation and a joint banquet with Sigma Pi Sigma, physics honor society at the Cumberland Inn. Kappa Mu Epsilon inducted fourteen new student members and one new faculty member at the banquet, presided over by

outgoing president, Joshua White. As an additional feature, senior awards were given by the department at the banquet.

Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 14. He spoke on "TWISTING AND TYING: A Mathematician's Humor or 'Isn't Math Fun!?!' " On April 15, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. On April 22, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held the annual spring picnic at Briar Creek Park.

LA Delta – University of Louisiana at Monroe

Spring 2005 officer: Andrew J. Hetzel.

New Initiates – April Boyett, Stephanie Evans, Brittany Jarrette, Michelle Lamanette, Heath Long, Francisco Picasso, Elizabeth Sihvonen, Laura Uyeda, Erin Vidrine, Jonathan Cox, David Hare.

MA Alpha – Assumption College

Chapter President– Kathryn Fleming. 9 Current Members, 11 New Members. Other spring 2005 officers: Christine Elkinson, Vice–President; Rebecca Freyenhagen, Secretary; Charles Brusard, Corresponding Secretary.

Eleven new members were initiated at a chapter meeting on April 28, 2005. Elise Gross, one of the new members, spoke on her research project "Exploring a Bijective Proof of the Hook Formula for Standard Young Tableaux." Chapter officers for 2005-2006 will be chosen in the fall.

MD Beta – McDaniel College

Chapter President – Jessica Bradford, 29 Current Members, 7 New Members. Other spring 2005 officers: Shannon Pusey, Vice-President; Kristie Springston, Secretary; Christopher Todd, Treasurer; Dr. Linda Eshleman, Corresponding Secretary.

We held an induction of 7 new members with a talk on the History of the Chapter by Dr. James Lightner a founding member of our Chapter and a former National President; Dr. Linda Eshleman was also honored as she retires after 36 years at the College. Other activities included a career night/dinner for mathematics majors with 3 alumni returning to talk and election of new officers for the 2005-2006 year. The chapter designed and sold T-shirts and sponsored a fantastic mathematics majors picnic to end the semester. 11 seniors are graduating, with 7 of them being inducted into Phi Beta Kappa.

New Initiates – Sarah Bloss, Benjamin Breakall, Matthew Crawford, Vincent Fabbri, Jeffrey Meyer, April Muir, Sean Uthe, Bradley Yoder.

MD Delta – Frostburg State University

Chapter President – Matt Miller, 29 Current Members, 8 New Members. Other spring 2005 officers: Ted Langan, Vice-President; Kimberly Embrey, Secretary; Creg Barnhart, Treasurer; Dr. Mark Hughes, Corresponding Secretary.

The Maryland Delta Chapter had an organizational meeting in February where we planned for fundraisers to be held in March. As with last year, members held a Pi Day back sale and a candy Easter egg sale. These events were rather successful with about $95 raised altogether. Several members worked hard to prepare the cookies and brownies for Pi Day.. Officers for the upcoming school year were elected during our April meeting. Other April activities included helping the FSU Mathematics Symposium run smoothly and the presentation of a lecture on the Riemann Zeta Function by student Cody Thompson. This lecture was the culmination of Cody's independent study on this topic supervised by Dr. Mark Hughes.

New Initiates – Sarah Bloss, Benjamin Breakall, Matthew Crawford, Vincent Fabbri, Jeffrey Meyer, April Muir, Sean Uthe, Bradley Yoder.

MI Alpha– Albion College

Corresponding Secretary–Mark Bollman

New Initiates–William Green, Dana Sauter, Peter Grostic, Jennifer Kamer, Megan McGown, Jessica Garrison, Mark Bollman(faculty), David Reimann(faculty), Karla McCavit(faculty), and Darren Mason(faculty)

MI Beta – Central Michigan University

Chapter President – Phillip Grebur, 15 Current Members, 10 New Members. Other spring 2005 officers: Craig Miller, Vice-President; Nicole Edgerton, Secretary; Eric Weisenauer, Treasurer; Arnie Hammel, Corresponding Secretary.

Five student members and one faculty attended the national convention at Schreiner University in April. The members and faculty of Schreiner are to be congratulated for putting on an outstanding convention. Several Michigan Beta members participated in the Fall Mathematics Challenge in November, 04. This is an undergraduate team test of 10 problems in three hours given at the colleges in Michigan. The mathematics department and KME members at CMU hosted the MUMC (Michigan Undergraduate Mathematics Conference) this year in October, 04. KME members had a holiday get together at the Hammel's in December. The spring picnic for the Actuarial Club, the Star Club, KME and department faculty was held at department chair Tom Miles' residence.

MI Delta – Hillsdale College

Chapter President – Michael Nikkila, 24 Current Members, 10 New Members. Other spring 2005 officers: Joel Clark, Vice-President; Erin Bartee, Treasurer; Dr. John H. Reinoehl, Corresponding Secretary.

MI Delta held free tutoring for students of mathematics classes. MI Delta also sponsored speeches on game theory by Hillsdale College faculty members.

MO Alpha – Southwest Missouri State University

Chapter President– Michael Sallee. 43 Current Members, 8 New Members. Other spring 2005 officers: April Williams, Vice–President; John Hammond, Secretary; Samantha Cash, Treasurer; John Kubicek, Corresponding Secretary.

During the Spring 05 semester the Missouri Alpha Chapter held four monthly meetings at which presentations were made by three faculty members and one student. Two students, April Williams and Brandon Alexander, along with the faculty sponsor, John Kubicek, attended the National Convention at Schreiner University. April served on the awards committee at the Convention.

New Initiates – Charles Bostic, Matthew Collins, Joshua Elliott, Colleen Garrett, Peter Hansen, David Lindell, Megan Reineke, Uriah Williams.

MO Beta – Central Missouri State University

Corresponding Secretary-Rhonda McKee.

New Initiates – Linlin Dong, Robert Jolls, Lincoln Maina, Brian Pattiz, Jonathan Peterson.

MO Gamma – William Jewell College

Chapter President– Michelle Richards. 15 Current Members, 2 New Memebers. Other spring 2005 officers: James K. Wyllie, Vice–President; Gregory R. Ziegler, Secretary; Dr. Mayumi Sakata, Treasurer; Dr. Mayumi Sakata, Corresponding Secretary.

New Initiates – James K. Wyllie, Gregory Robert Ziegler.

MO Iota – Missouri Southern

Corresponding Secretary-Chip Curtis.

New Initiates– Nicole Barnes, Amy Barnicle, William Beene, Yvette Bowden, Micah Griffin, Rahila Khan, Jamin Perry, Kevin Talbert, and Sarah Wallace.

MO Kappa – Drury University

Corresponding Secretary-Charles Allen.

New Initiates – Nathan Bloomfield, Sergey Borodich, Tim Itschner, Nikki Kennedy, Jennifer Mann, Moones Mellonli, Sharon Peppard, Scott Sigman, Brandon Snow, Brandon Williams.

MO Lambda – Missouri Western State College

Chapter President– Robert Smith . 45 Current Members, 7 New Members. Other spring 2005 officers: Whitney Lowrey, Vice–President; Heather

Goforth, Secretary; Daniel Cassity, Treasurer; Don Vestal, Corr. Sec.

New Initiates – Keith Michaek Anderson, Trevor Butner, Daniel Cassity, Heather Dawn Goforth, Janessa Michelle Hovey, Whitney M. Lowrey, Taylor W. Talbott.

MO Nu – Columbia College

Chapter President-Jamie Netherton. Other spring 2005 officers- Michael Perkins, Vice President; Jenna Holdmeyer, Secretary; Shahthureen Khan, Treasurer; Ann Bledsoe, Corresponding Secretary.

New Initiates-Ashley Bonine, Jenna Holdmeyer, Jason Kerr, Shahthureen Khan, Sanel Krgo, Jamie Netherton, Michael Perkins, Charles Phillips, Daniel Roemer, Kelly Sharp, Than Som, and Nobuyuki Tsuchiya. Faculty charter members initiated were Arlin Epperson, David Heise, Natasha Latushkina, Yihsiang Liow, Suzanne Tourville, and Paul Wiedemeier.

MO Theta – Evangel University

Chapter President– Andrew Reed, 14 Current Members, 0 New Members. Other spring 2005 officers: Tiffany Brown, Vice–President; Don Tosh, Corresponding Secretary.

Monthly meetings were held during the spring semester. Six members were initiated in the first meeting on February 1. Don Tosh and ten members attended the national convention in Kerrville, TX, at which Don Tosh was installed as National President of KME. The final meeting of the semester was at Don Tosh's home where the students presented him with a picture of their trip.

New Initiates – Andrea Feir, Andrew J. Hanson, Dianne Henry, Rachel Manley, Tiffany Roth, Joshua Thomassen.

MS Alpha – Mississippi University for Women

Chapter President – Robert Banik, 15 Current Members, 1 New Member. Other spring 2005 officers: Sarah Sheffield, Vice-President; Joshua Huwe, Secretary; Stefanie Cummings, Treasurer; Dr. Shaochen Yang, Corresponding Secretary.

On Jan. 24, 2005, elections were held. Initiation was on Feb. 15, 2005. There was a dinner and game night on Apr. 8, 2005. MS Alpha had 4 meetings in the spring of 2005.

New Initiate – Mary Hawkins.

NE Beta – University of Nebraska at Kearney

Current President – Kevin Vogel, 17 Current Members, 6 New Members. Other spring 2005 officer: Siva Kommuri, Vice-President; Lindsay Higgins, Secretary; Adam Haussler, Treasurer; Dr. Katherine Kime, Corresponding Secretary.

KME had a "Math Fund Day" at a local elementary school. KME members prepared mathematical activities for the children, including puzzles, card tricks, and measurement activity entitled "Are you a square

or a rectangle", and several others. This occurred after a period in which students had been taking tests, and was a good break from the routine for them. The school principal found the event successful.

New Initiates – John Auwerda, Neil Hammond, Dustin Kozal, Amber Norman, Abby Norman, Adam Sevenker, Andrea Skinner, Douglas Wadas.

NE Delta – Nebraska Wesleyan University

Chapter President– Jennifer Choutka. 13 Current Members, 5 New Members. Other spring 2005 officers: Kristen Houchin, Vice President; Zachary Brightweiser, Secretary; Zachary Brightweiser, Treasurer; Melissa Erdmann, Corresponding Secretary.

As part of our end of year picnic, we were pleased to induct five new members into the Nebraska Delta section of Kappa Mu Epsilon at the country home of Dr. Richard Vogt in Elmwood, Nebraska.

New Initiates – Zachary Brightweiser, Matthew Hampl, Kristen Houchin, Scott Schroetlin, Brenda Treffer.

NH Alpha – Keene State College

16 New Members. Corresponding Secretary-Vincent J. Ferlini.

New Initiates – Matthew Beauchesne, Matthew Brewer, Theresa Dube, Brittany Faulkner, Kathleen Gambino, Valerie George, Kate Kraft, Kelly Naro, Tania Osborn, Jessica Paiva, Melanie Paradis, Jennifer Richinick, Katy Robbins, Kimberly Simpson, Yulia Shurygina, Eileen Zavoluk.

NY Eta – Niagara University

Chapter President– Matt Nethercott. 20 Current Members, 21 New Members. Other spring 2005 officers: Adam Meyer, Vice–President; Megan Zdrojewski, Secretary; Megan Zdrojewski, Treasurer; Robert Bailey, Corresponding Secretary.

Our main event this semester was the initiation of new members. This took place April 27, 2005 and was attended by approximately 50 people. Dr. William Price presented an entertaining talk and demonstration featuring magic and mathematics.

New Initiates – Marcus Allen, Melissa A. Allen, Sandra Lucille Barnstead, Kelley Andrea Boyd, Dr. Marlo Brown, Erica Carey, Jerome Charles Haick, Jennifer L. Jones, Karrie A. Jones, Jennifer Maria Kemak, Katherine S. McAllister, Mary Patricia McDonald, Ashley R. McMaster, Shauna Marie Morrow, Stephanie Lee Nardozzi, Jessica M. Nichols, Whitney E. Roberts, Alicia Springer, Kari Swartz, Annie Katherine Wachob.

NY Lambda – Long Island University

Chapter President– Laura G. Silverman. 20 Current Members, 19 New Members. Other spring 2005 officers: Henry A. Ciapas, Vice–President; Andrea M. Lorusso, Secretary; Dr. Mahmoud Zeinalian, Faculty Sponsor; Dr. Andrew M. Rockett, Corresponding Secretary.

Eighteen students were initiated into the New York Lambda Chapter

by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of April 3rd, bringing the Chapter membership to 264. Laura G. Silverman's after-dinner presentation on Bernoulli's Principle was the "dress rehearsal" for her presentation at the KME national convention held at Schreiner University in Kerrville, Texas, April 14-16. Our evening concluded with the announcement of the 2004-2005 departmental awards: the Claire F. Adler Award to Dina Anaxagorou, the Lena Sharney Memorial Award to Robert E. Mashburn, the Joseph Panzeca Memorial Award to Stephanie C. Cardone and Thomas N. Gropf, and the Hubert B. Huntley Memorial Award to Jason C. Williams; the Dean Schmidt Graduate Scholarship Award to Xiaochun Li; and the presentation by Dr. James V. Peters of four MAA students memberships.

NY Mu – St. Thomas Aquinas College

Corresponding Secretary-J. Keane.

New Initiates – Melinda Behrens, Jeanette Marie Durfee, Anne Marie Harley, Jennifer Nilsson, Nicole Tantillo, Ian Sudman.

NY Nu – Hartwick College

Chapter President– Rasha Jawabri. 20 Current Members, 19 New Members. Other spring 2005 officers: Muhammad Qadi, Vice–President; Elizabeth McCabe, Secretary; Martin Svetoslavov, Treasurer; Ron Brzenk, Corresponding Secretary.

New Initiates—Jeanette Johnson, Mary M. Kellogg, Jason J. Tabor, Andrew S. Williams.

NY Omicron – St. Joseph's College

32 New Members. Corresponding Secretary-Barbara Thorpe.

New Initiates – Manelie Augustin, Kimberly Byrne, Stephanie Igneri, Crystal Jacobowitz, Gregory Luna, Jennifer Manzione, Aleksey Podushko, Itchelle Whyte, Sandra Abdelbarr, Gaetano Accardi, Lauren Andrews, Patricia Antonacci, Paul James Bodnar, Hayley C. Burns, Crystal Cantley, Andrew Casale, Megahan Disken, Stephanie Galvao, Kimberly Gordon, Taryn Ann Camille Gray, Jennifer Marie Mabanta, Jamie McGee, Wendy Nuss, Jessica Pace, Jennifer Lynn Pang, Jamie Lynn Press, Sally-Maria Selvaggi, Brian Strack, Christine Marie Vaccaro, Jill Marie Valente, Marieann Vitillo, Christine Marie Zeck.

OH Alpha– Bowling Green State University

Corresponding Secretary-Dr. Warren Wm. McGovern.

New Initiates– Andrea D. Bissell, Michelle K. Couch, Rachel L Crowe, Lauren A. Hablitzel, Ryan E. Henderson, Ryan A. Hershey, Lauren R. Hoffman, Matthew S. Hufnagel, Sarah E. Kainec, Alexis C. Kallas, Aricka S. Kerns, Andrew J. Kuntz, Martin P. Lay, Mark E. Limes, Michelle L. Marsh, Jessica A. Marvin, Derek E. Meyer, Corrie J. Mills, Lonie D. Moore, Kristin D. Mowery, Dustin P. Patterson, Erin L. Platzer, Joseph A. Pochedly, Lori A. Rethman, Sara K. Rutschilling, Susanna Strayer, and Heather M. Umstead.

OH Epsilon – Marietta College

Chapter President – Mary Kinsler, 20 Current Members, 13 New Members. Other 2005 spring officers: Kristen Martin, Vice-President; Dr. John C. Tynan, Corresponding Secretary

The initiation of thirteen new members was held on April 12, 2005.

New Initiates– Xiao Qiu Bao, Christopher J. Cheng, Shannon R. Glancy, Alan M. Grubb, William Glenn Hollandsworth, Joseph D. Mader, Kristen A. Martin, Harrison D. Potter, Charles R. Reynolds, Justin F. Rosenberg, Matthew D. Williamson, Meredith E. Rogers, and Tongying Yang.

OH Eta– Ohio Northern University

Corresponding Secretary-Donald Hunt.

New Initiates– David Chelmins, Kevin Deily, Amanda Dittmar, Matthew Hartman, Dustin House, Christopher Lemon, Michael Paulus, Jeremy Schroeder, and Jack Skinner.

OH Gamma– Baldwin-Wallace College

Corresponding Secretary–Dr. David Calvis.

New Initiates– Melissa Anne Barner, David C. Bensch, John Alexander Brandau, Karyn L. Cirino, Corey D. Clay, Steven G. Dieterle, Jacqueline M. Estep, Katherine C. Hastings, David M. Kast, Edward R. Lambert, Mary Elizabeth Mazurkiewicz, Andrew L. Miskimen, Megan E. Saad, Paul M. Shaniuk, Jennifer L. Slaga, Sarah E. Slavik, Gretchen E. Waugaman, Monty A. Wesoloski, Artion Pipa, and Michelle J. Robb.

OH Zeta – Muskingum College

Corresponding Secretary-Dr. Richard Daquila.

New Initiates—Shannon Chandler, Danielle Hutter, Mindy Linger, Brandon McFarland, Megan Mellinger, Cory Williams.

OK Alpha – Northeastern State University

Chapter President– Mary Kelly–Harper. 57 Current Members, 18 New Members. Other spring 2005 officers: Leticia Stone, Vice–President; Teri O'Neal, Secretary; Andy Hathcoat, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our spring activities began with the story "The Mathematical Romance of Poly___ and Ray_____," which was published in the Fall, 1946 issue of The Pentagon. The story omits 83 mathematical words that must be inserted to complete the story. Movies and popcorn were the main features of the February meeting. The movies were "Multiplication Algorithms" and "Space Filling Curves." During the March meeting, we provided materials for members to make personal copies of a game similar to "SET." Our speaker this semester was Dr. Gary McGrath from the Mathematics Dept. of Pittsburg State University. He spoke on "Euler and Undergraduate Mathematics." The end of the semester ice cream social honored KME member and math major, Alcides R. Silva Neto, who just finished a tour of duty in Iraq. He was presented with his KME

membership certificate (which he was unable to pick up in person, because of his call), as well as a KME shirt as a token of appreciation.

New Initiates – Ashley D. Cahwee, Shon T. Farris, Andrae L. Freeman, Josh N. Hamit, Sarah B. Hurley, Ana L. Ihle, Adonna M. Jennings, Amy S. Joslin, Amy C. Karnes, Jessica M. Keeler, Lindsay M. Keeling, Brock O. Marks, Candy L. Morgan, Elizabeth L. Perry, Courtney D. Richardson, Jeff A. Smith, Memory A. Stidman, Tabatha V. Woods.

OK Delta – Oral Roberts University

Corresponding Secretary-Vincent Dimiceli.

New Initiates– Willy Bustinza, Tim Butterworth, Gabriel Cap, Nathan Francis, Rachel Holeman, Jessica Kellogg, Nickolas LaSorte, Vijay Luvian Masillamoni-Karlsson, Aaron James McCready, Matthew Sterns, Siridej Peter Thavornrat, and Stephen Twyman

OK Gamma – Southwest Oklahoma State University

6 New Members. Corresponding Secretary-Bill Sticka.

New Initates – James Carver, Kristen Hanks, Ashley McMunn, Dustin Wulff, Santosh Bhatt, Moin Khan.

PA Alpha – Westminster College

Chapter President – Lauren Beichner. 29 Current Members, 10 New Members. Other spring 2005 officers: Sarah Spardy, Vice-President; Christie Grewe, Secretary; Amanda Ganster, Treasurer; Carolyn Cuff, Corresponding Secretary.

We helped to host the annual Spring Banquet for the Mathematics and Computer Science department where 10 new members (including one new faculty member) were inducted.

PA Beta – LaSalle University

Chapter President – Meridyth Mascio, 13 Current Members, 2 New Members. Other spring 2005 officers: Tom Plick, Vice-President; Ami Edwards, Secretary; Melissa Meyer, Treasurer; Dr. Anne E. Edlin, Corresponding Secretary.

We has several presentations this semester, including two student presentations and a guest speaker form Independence Blue Cross who talked about careers in actuarial science.

New Initiates – Joshua H. Shrader, Thomas C. Orzechowski, Meridyth A. Mascio, Thomas A. Plick, Ami M. Edwards, Charles E. Buehrle, Kelly Strobel, Ryan Hall, Samantha Ciccimaro.

PA Delta – Marywood University

Chapter President – Stacy Duink, 3 Current Members, 2 New Members. Other spring 2005 officers: Elizabeth Dittrich, Secretary; Sr. Robert Ann van Ahnen, Corresponding Secretary.

New initiates – Elizabeth Dittrick, Stacy Duink.

PA Iota– Shippensburg University

Corresponding Secretary –John Cooper

New Initiates– Kristina Yost, Shaun Henry, Chris Radley, and Dave Turner.

PA Kappa – Holy Family University

Chapter President – Emily Kerner, 2 Current Members, 3 New Members. Corresponding Secretary-Sister Marcella Louise Wallowicz CSFN.

Three new members were inducted into the PA Kappa Chapter on Friday, April 1, 2005.

PA Mu – Saint Francis University

Chapter President – Bridget Campbell, 13 Current Members, 0 New Members. Other spring 2005 officers: Lesley Wenzel, Vice-President; Shannon Decker, Secretary; Rebecca Dombrowski, Treasurer; Dr. Peter Skoner, Corresponding Secretary.

The mathematics education students who are members of the Pennsylvania Mu chapter attended the annual meeting of the Pennsylvania Council of Teachers of Mathematics held on October 28-28, 2004 in Erie. Two KME members, Bridget Campbell and Tsega Workalemahu, assisted faculty member Paul Deskevich in his presentation on the Powerball Lottery.

The Pennsylvania Mu chapter helped host the Eleventh Annual Science Day on Friday, November 19, 2004. A record total of 426 high school students from 33 area high schools attended. The day included a 16-team science bowl run by Saint Francis University students, presentations by faculty, alumni, and industry professionals, and a variety of games and challenges.

An induction ceremony was held on Tuesday, February 8, 2005, in the JFK student center lounge. Inductions featured the largest cohort ever, 25 students, to bring the total membership of Pennsylvania Mu to 237. The induction ceremony followed the dinner.

New Initiates – Daniel Ampadu, Scotty Boshinsky, Kimberly Brennan, Scott Caldwell, Andria D'Amato, Arlie Eichenlaub, Mary Fantuzzo, Andrew Farabaugh, Emily Gehrlein, Lauren Gompers, Christian Hartnett, Michael Hughes, Paul Johns, Sarah Keller, Amanda Long, Matt Middleton, Mitch Nagle, Robert Pipech, Tarah Schloss, Jennifer Vitko, Robert Weney, Daniel Wetklow, Jesse Williamson, Kristy Wolfel, Alexander Yeisley.

PA Pi– Slippery Rock University

Chapter President – Mark Kratz, 9 Current Members, 6 New Members. Other spring 2005 officers: T. J. Deems, Vice-President; Angel Streitman, Secretary; Peter Schallot, Treasurer; Elise M. Grabner, Corr. Sec.

PA Pi had an active year as participants at the Allegheny Mountain Section of the MAA held in Slippery Rock this spring. Our newest inductees presented their works at this meeting.

New Initiates– Mark A. Kratz, Kory F. Fish, Thomas Deems, Jr., Richard Busi, Peter Schallot, and Angel Streitman

PA Rho – Thiel College

5 New Members. Corresponding Secretary-John C. Nichols.

New Initiates – Ronald C. Anderson, Randall L. McPherson, Alexander R. Miller, Joseph F. Page, Michael S. Young.

PA Sigma– Lycoming College

Corresponding Secretary-Santu deSilva.

New Initiates– Byron E. Arenella, Jason M. Black, Karissa L. Brown, Lindsey M. Carr, Josemar A. Castillo, Kathryn A. Chastek, Santu deSilva, Jessica E. Gough, Jason Gresh, Nicole E. Gugliucci, David K. Haley, Robert Hofford, Joshua R. Hogan, Marni Holt, Rebecca B. Jervey, William J. Knapp, Philip T. Mann, Gregory J. Moses, Kevin Nester, Christina Nestlerode, Eileen M. Peluso, Larry Pritchett, Rachael J. Raughley, Christopher K. Reed, Jana L. Reitz, Michael P. Russell, Gene Sprechini, Randall Utsch, and Cui Yin

PA Theta– Susquehanna University

Corresponding Secretary-Kenneth Brakke.

New Initiates—Kristin Aurand, Katherine B. Beem, Rachel Bradley, Laura K. Buzzell, Dristen M. Deak, Adam Dreibelbis, Ashley Edwards, Jeremy Fry, Jason Herrold, Stephanie Ihnat, Robert Nowicki, Katherine Plocharczyk, Brian Schmidt, Suzanne Thomas, Quan Tran, Eric Warner, and Laura Zettlemoyer.

SC Epsilon – Francis Marion University

Chapter President— Ashley Godbold Turbeville. 0 Current Members,19 New Members. Other spring 2005 officers: Regina Quick, Vice—President; Erin Cox, Secretary; Taka Sakamoto, Treasurer; Damon Scott, Corresponding Secretary.

On 18 March, 2005, Kappa Mu Epsilon installed the South Carolina Epsilon Chapter on the campus of Francis Marion University. Dr. Kenneth Bernard of Virginia State University conducted the installation, which was followed by a dinner. Thirteen students and six members of the faculty were founding members.

KME sponsored a spring pizza and ice cream social for the mathematics department and all mathematics majors.

SC Gamma – Winthrop University

Chapter President— Philip Gear. 12 Current Members,5 New Members. Other spring 2005 officers: Shantelle Prioleau, Vice—President; Anesha Simms, Secretary; Ian Finlayson, Treasurer; Dr. Frank Pullano, Corresponding Secretary.

KME sponsored a spring pizza and ice cream social for the mathematics department and all mathematics majors.

New Initiates– Ian Finlayson, Philip Schuyler Gear, Shantelle Monique Prioleau, Anesha Lavon Simms, and Jessica Desiree Sinkler.

SD Alpha – Northern State University

Chapter President—Nichole Wentz. 14 Current Members, 7 New Members. Other spring 2005 officers: Wendy Nisiewicz, Vice-President; Justin Downes, Secretary; Amy Martens, Treasurer; Dr. Raj Markanda, Corresponding Secretary.

We celebrated the National Mathematics week by having a pizza party and watching the video: The Right Spin: The story of a dramatic rescue in space and the mathematics behind it.

New Initiates– Mike Melko, Marsha Delagardelle, Wendy Nisiewicz, Amy Martens, Justin Downes, Nichole Wentz, and Brady Shank

TN Delta – Carson-Newman College

Chapter President— Chris Anderson. 20 Current Members, 6 New Members. Other spring 2005 officers: Sara Patton, Vice—President; Philip Barger, Secretary; Holly Gragg, Treasurer; B. A. Starnes, Corresponding Secretary.

TN Gamma – Union University

Chapter President– Brian Taylor. 16 Current Members, 5 New Members. Other spring 2005 officers: Jennifer Ellis, Vice–President; Denise Baughman, Secretary; Willie George, Treasurer; Bryan Dawson, Corresponding Secretary.

Five new members were initiated at our annual banquet, held April 11 at the Old Country Store. Former chapter officer Breanne Oldham was the speaker. The highlight of the semester was a trip to Kerrville, TX for the biennial convention. Five students and one faculty member attended; Jennifer Ellis presented her paper "Primes and Primality Testing: A Technological/Historical Perspective," Willie George served on the Auditing Committee, and Dr. Dawson served on the Nominating Committee. The semester ended with a joint picnic with the ACM chapter where various awards were presented, including the Joe Tucker Scholarship (Tony Winkler) and the Outstanding Research Award (Jennifer Ellis). At graduation chapter president Brian Taylor received this year's Tigrett Medal, Union University's highest student award.

TX Alpha – Texas Tech University

Chapter President– Latasha R. Smith. 25 Current Members, 38 New Members. Other spring 2005 officers: Laura A Reddy, Vice–President; Candace Cyrek, Secretary; Nancy A Gerrish, Treasurer; Dr. Anatoly B. Korchagin, Corresponding Secretary.

New Initiates – Kimberly J. Villa, Rafael Beckmann, Pierre V. McLeod, Travis D. Wells, Latasha R. Smith, Candace L. Cyrek, Shing-Ling Chiem, Jennifer S. White, Shane E. Albers, Jennifer N. Brewer, James R. McQuery, Sara E. Pugh, Daniel J. Tollerene, Charles J. Kolb, Kent T. Weinheimer, Christopher E. Middleton, Robert N. Hutchison, Jay D.

Parnell, Laura A. Reddy, Ronnie S. Williams, Mark W. Cralle, Amanda L. Allen, Brian R. Hermis, Nancy A. Gerrish, Clifford L. Colvin, Thomas A. White, Kunwook Kang, Stephen T. Johnson, Mark A. Ickert, Jordan Been, Bryan D. Kinsler, Richard T. Lane, Christopher D. Hard, Hillari J. Neely, Pamela G. Wait, Michael K. Riordon, Nicholas W. Lewoczko, Arthur N. Nesty.

TX Eta – Hardin–Simmons University

Chapter President- Melissa McClanahan, 243 Current Members, 8 New Members. Other spring 2005 officers: Mica Henson, Vice-President; Stephanie Irwin, Secretary; Frances Renfroe, Corresponding Secretary.

The 29th annual induction ceremony for the Texas Eta Chapter was held March 22, 2005. There were eight new members: Cari Ford, Samantha Mangum, McKade Marshall, Timothy Osborn, Miranda Rousett, Lindsay Thompson, Joshua Vaughn, and Jennifer White. With the induction of these members, membership in the local chapter stands at 251.

Leading the induction ceremonies were President Lindsey Mankins, Vice President Melissa Schaeffer, and Secretary Mica Henson. Following the induction ceremony, membership shingles and pins were presented to the 2004 inductees. The Burnam Medal, a departmental award, was presented to Larry Smedley as the outstanding graduating senior. A plaque was also presented to Ms. Frances Renfroe who is retiring. Following these presentations and the taking of a club picture, KME adjourned, and the members, inductees, and chapter sponsors enjoyed pizza and cold drinks.

During the fall semester Hardin-Simmons University with the help of KME members and faculty hosted the Big Country Council of Teachers of Mathematics and Science Pre-UIL Math and Science Meet. Approximately two hundred and fifty middle school and high school students participated. In February, Hardin-Simmons University hosted Math Counts. Five middle school teams (twenty students) participated.

Newly elected chapter officers for the 2005-2006 year are: President Melissa McClanahan, Vice-President Mica Henson, and Secretary-Treasurer Stephanie Irwin. Dr. Ed Hewett, Dr. James Ochoa, and Dr. Andrew Potter are chapter sponsors. Frances Renfroe is the corresponding secretary of the chapter.

New Initiates- Cari Ford, Samantha Mangum, McKade Marshall, Timothy Osborn, Miranda Rousett, Lindsay Thompson, Joshua Vaughn, and Jennifer White

TX Gamma – Texas Woman's University

Chapter President – Lindsay Renfro, 19 Current Members, 14 New Members. Other spring 2005 officers: Melinda Smith, Vice-President; Jaime Rioias, Secretary; Marian Marvin, Treasurer; Dr. Mark Hammer,

Corresponding Secretary.

Activities: 14 new National KME members inducted, members organized and attended Math vs. Computer Science Bowling event, members volunteered at the annual Harlan C. Miller Memorial Lecture Series and Awards Dinner at TWU where outstanding Mathematics and Computer Science scholars at TWU are recognized, several members attended KME National Convention in Kerrville, TX and 2 members presented (Lindsay Renfro and Tony Llopis), members volunteered at local schools' Edible Car Contest, members attended End-of-Semester dinner. 4 Meetings in Spring 2005: New officers elected.

New Initiates – Tracie Abeyratne, Elizabeth Bowers, Martha Cisney, Heather Dorsey, Kathleen Elberson, Virginia Foster, Kathryn Fritz, Tony Llopis, Lacey Pemberton, Jaime Riojas, Quentin Schmieding, Melinda Smith, Anita Valle, Angie Voss.

TX Lambda– Trinity University

Corresponding Secretary-Diane Saphire.

New Initiates – Amanda Barth, John Buabbud, Sarah Carolan, Mary Christensen, Susan Dixon, Kevin Doyle, Stephanie Gates, Rebecca Haas, Samantha Hammer, Leslie-Anne Juarez, William Kerr, Hunter Mabrey, Brian McCarty, Garland Miller, Ashley Mynar, Evan O'Dea, Christopher Reynolds, Scott Schwartz, Juan Suarez, and Ashley Yanchak.

TX Mu – Schreiner University

Chapter President- Rebekha Collins, 6 Current Members, 4 New Members. Other spring 2005 officers: Charnelyn Fortune, Vice–President; Matthew Casey, Secretary; Matthew Casey, Treasurer; William M. Silva, Corresponding Secretary

Texas Mu hosted the 35th Biennial National Convention.

New Initiates – Matthew B. Casey, Nicholas C. Landes, Aaron C. Mayes, Lauren Williamson.

WV Alpha - Bethany College

Corresponding Secretary-Dr. Mary Ellen Komorowski.

New Initiates – Dana Marie Culbert, Kirsten L. Trbovich, Kelly Jean Chaney, Kenneth Louis Hopper, Hillary Allese Norman, Chad Jacob Shepherd, Deidra Rashelle Hall, Michael Patrick Mihalyo, Jr., Jacquelyn L. White, Raymond Michael Dick, and Bruce M. Clark

Report of the 35th Biennial Convention.

Kappa Mu Epsilon
April 14-16, 2005
Schreiner University
Kerrville,Texas

Thursday, April 14, 2005

Registration and hotel check in took place at the Y.O. Ranch Resort Hotel and Conference Center. At 7:00 p.m. a mixer was held in the Cailloux Campus Activity Center at Schreiner University. The mixer included ping pong, pool, refreshments and the showing of an episode of the television series "Numb3rs." The national council and regional directors met at 8:00 p.m.

Friday, April 15, 2005

First General Session and Business Meeting

The first general session of the convention began at 8:30 Friday morning. Participants were welcomed to the Schreiner University Campus by C. Timothy Summerlin, president of the University. Don Tosh, KME National President Elect gave a response and then the group was greeted by Rebekha Collins, President of the host chapter, Texas Mu. Rhonda McKee unveiled the new KME web page, www.kappamuepsilon.org.

The first business meeting began with roll call by KME National Secretary Rhonda McKee. Seventy-three students and 28 faculty members (total 101) from 18 chapters were present.

Information from the petitions for two new chapters was distributed to the chapters present. The petitions were from Villa Julie College in Stevenson, Maryland, and Western New Mexico University in Silver City, New Mexico. The vote to accept the petitions will take place at the final business meeting on Saturday.

Bryan Dawson, chair of the Nominating Committee, introduced the nominees for the offices to be filled. The nominees for president elect were Ron Wasserstein and Vince Dimicelli. Connie Schrock, the current historian, agreed to serve a second term and was therefore the only nominee for that position. Each candidate gave some information about him or herself. The vote on the nominees will take place at the final business meeting on Saturday.

First Paper Session

Five papers were presented in the Friday morning session.

- "An Inverse Problem for Harmonic Oscillators" by Chad Forhsee, MO Kappa
- "Optimal Strategies for the Dice Game 'Schultz"' by Kevin Reed, MO Theta
- "Canoeing Down the River of Recursion" by Jo Marie Rozzelle, KS Delta
- "Patterns in Fibonacci and Fibonacci-type Sequences" by Eileen Bracciodieta and Anna Santos, NY Omicron
- "A Glimpse Into Mathematics in Theology" by Richard Rechtein, MO Beta

Second Paper Session

After lunch, seven more papers were presented.

- "On Health Care: Making an Informed Decision" by Fred Hollingshead, KS Delta
- "Bernoulli's Principle", by Laura Silverman, NY Lambda
- "An Introduction to the Theory and Uses of Ab Initio Electron Structure Techniques" by Zeb Kramer, KS Delta
- "Primes and Primality Testing: A Technological/Historical Perspective", by Jennifer Ellis, TN Gamma
- "Predicting Undergraduate Re-Enrollment—A Bayesian Approach" by Lindsay Renfro, TX Gamma
- "RATS Transformations in Base 4" by Andrew Ray, MO Beta
- "The Topeka Public School Busing Problem: It's Simplex" by Carolyn Cole, KS Delta

Sectional Meetings

After a refreshment break, the faculty met in one room, while the students met in another. Both groups were charged with discussing ideas and issues related to KME.

Banquet

As is tradition, the George R. Mach Distinguished Service Award was presented after the banquet meal. The recipient was Arnold (Arnie) Hammel of the Michigan Beta Chapter at Central Michigan University. Arnie served as National President Elect from 1989-1993 and as National President from 1993-1997. He is the original creator of the KME web site and has also served the Michigan Beta chapter as corresponding secretary for many years.

The banquet speaker was Dr. John Snow of Sam Houston State University. His topic was "Farming and Algebra: A Genetic Algorithm for Drawing Ordered Sets."

Saturday, April 16, 2005

Third Paper Session

The third paper session took place on Saturday morning. Three papers were presented at this session.

- "Determination of TMRCA Using Dynamic Population Models" by Tony Llopis, TX Gamma
- "Applications in Abstract Algebra: Encryption/Coding and Calendar Calculations" by Daniel Majcherek, CA Epsilon
- "Lightning and the Application of Mathematics" by Chris Dobbs, KS Beta

Second General Session and Business Meeting

The second general session began with reports from all the national officers. Printed copies of these reports were distributed to each chapter in attendance.Reports from the sectional meetings of the previous day revealed that both the student group and the faculty group spent quite a bit of time discussing the KME web page. These reports were followed by reports from the auditing committee and resolutions committee. During the election of officers, Ron Wasserstein was elected President Elect and Connie Schrock was elected to serve a second term as Historian.

The Awards Committee was charged with choosing the top four papers at the convention. They made their report to Don Tosh who called all presenters to the front and congratulated them on a job well-done. The top four papers were given by (in no particular order) Carolyn Cole, Fred Hollingshead, Lindsay Renfro and Jo Marie Rozzelle. Each of these top

four presenters was given a check in the amount of $100. All other presenters received a check for $50.

After the new officers were installed, Treasurer John Kubicek distributed checks for travel allowances to each chapter in attendance and the meeting adjourned.

List of Committee Members

- Auditing Committee: Al Riveland, KS Delta, Vince Dimicelli, OK Delta, Willie George, TN Gamma
- Awards Committee: Cynthia Woodburn, KS Alpha, Arnie Hammel, MI Beta, Meg Huddleston, TX Mu, April Williams, MO Alpha, Jack Harper III, Mi Beta, Jason Miller, KS Beta
- Nominating Committee: Bryan Dawson, TN Gamma, Jo Ann Fellin, KS Gamma, Andy Rockett, NY Lambda, Fred Hollingshead, KS Delta
- Resolutions Committee: Catherine Kong, CA Epsilon, Elana Epstein, NY Omicron, Kristin Ranum, KS Delta, Missy Libbert, MO Beta
- Paper Selection Committee: Don Tosh, MO Theta, Catherine Kong, CA Epsilon, Chip Curtis, MO Iota, Ron Brzenk, NY Nu, Leah Childers, KS Beta, Mindy Baker, KS Beta

Report of the National President

The last two years as President seem to have flown by. My job has been made easier by the excellent work of the national officers. Within this past biennium, nine new chapters have been installed. I was privileged to install three of the chapters. They are California Baptist University (CA Epsilon), April 2003; Thiel College (PA Rho), February 2004; and Marymount University (VA Delta), March 2004. The remaining six are St. Joseph's College (NY Omicron), installed by Andrew Rockett, May 2004; Texas A&M University at Corpus Christi (TX Nu), installed by Don Tosh, May 2004; Lewis University (IL Iota), installed by Don Tosh, February 2005; Wheeling Jesuit University (WV Beta), installed by Pat Costello, March 2005; Francis Marion University (SC Epsilon), installed by Kenneth Bernard, March 2005; and Lycoming College (PA Sigma), which I installed on April 1, 2005. An additional chapter will be installed at Columbia College (MO Nu) by Don Tosh on April 29 this year. The current number of active chapters now stands at 137 and we have well over 70,000 members since the inception of KME – our numbers continue to grow. Please take the opportunity to congratulate these new chapters. I

encourage you to interact with them through the structure of our regional conventions.

Recently, two petitions have been received. They are from Western New Mexico University (Silver City, NM) and Villa Julie College (Stevenson, MD). By the time you read this, it is possible that the National Council will have approved of these petitions and that you will have had the opportunity to vote on them at this convention.

I have corresponded with several other colleges, universities, and individuals who are interested in starting new chapters. Inquiries have been received from Alabama, Arkansas, Arizona, California (2), Georgia (2), Iowa, Illinois, Massachusetts, Maryland, Missouri, North Carolina, New Mexico, New York, Pennsylvania (2), South Carolina (2), Tennessee, Texas (2), Vermont and West Virginia. The majority of these requests have come from connecting with our web site and using the information contained there. We are in the process now of revising the site. This involves changing the physical location of the site and will involve a change in the URL address. We are grateful for the efforts of Kirk Jones, KY Alpha, who has been maintaining the site for the past 2 years and Rhonda McKee, MO Beta, who is arranging for the update of the site.

The regional structure is important to the National Council and the report by Don Tosh gives details about the activities of the various regions. We are grateful for the efforts that our Regional Directors put forth to keep things running smoothly. I would like to thank those whose terms have ended and who will not be continuing as directors. They are Carol Harrison (New England), Bryan Dawson (Southeast) and Gerry East (South Central). Thanks also to Peter Skoner (Great Lakes) who is midway through his second term and Cynthia Woodburn (North Central) who has agreed to take on a second term.

Corresponding Secretaries and Faculty Sponsors of our active chapters are also deserving of our gratitude. These individuals provide a crucial link between the national, regional and local organizations and our chapters would not survive without these key persons in place. It is through their encouragement that students prepare and present papers at a convention, such as this one. Since this is really the main purpose of the convention, we would like to thank each of the presenters who spent many hours in researching, writing, editing and practicing the talks that we were privileged to hear. We also owe a debt of gratitude to all those faculty and students who agreed to serve on the various committees, some at almost the last minute! My sincere thanks go to the members of the Texas Mu chapter, who made all the local arrangements and handled all the nitty-gritty details and problems that seem to continuously arise at these affairs.

After several years of discussion, the National Council was able to revise and update the KME Constitution, and it was ratified in 2003 at the National Convention. I would like to thank Rhonda McKee for arranging for the printing and distribution of this important document. By now, your chapter should have received its copy.

During the past biennium, I was privileged to represent Kappa Mu Epsilon at a meeting of the Association of College Honor Societies in Seattle (February 2004). It was beneficial for me to speak to other individuals who represent honor societies of other disciplines. The various sessions were built around solving common problems and trying to improve things on not only the national level, but also the regional and local arenas as well. Incidentally, ACHS publishes a booklet which lists and gives details about all the honor societies belonging to the ACHS. If you would care to have a copy mailed to your chapter, please let me know.

I would like to recognize the outstanding work which was done by the individuals who recently retired from their positions, having produced our official journal, The Pentagon for the past several years. They are Steve Nimmo, editor, and Larry Scott, business manager. The new editor and business manager are Chip Curtis and Rich Laird, respectively, and we are grateful to them for volunteering for these challenging tasks. The first copy under their leadership has been received by most of you and the tradition of publication excellence appears to be continuing. I would also like to recognize the recently retired editor of the Problem Corner, Ken Wilkie, who has probably set a record for longevity in this position. As Ken is phasing out of the position, he has been assisted by Catherine Kong with many of the problems submitted to the column. Pat Costello has agreed to take over the task of Problem Corner Editor within in the next few months, so thanks are due to both Catherine and Pat for their help.

I would specifically like to acknowledge the help I have received from members of the National Council. John Kubicek, our National Treasurer, has held his position for 2 years now and has done a commendable job of providing periodic financial reports, keeping track of checks, filling out income tax forms, and many other details the job entails. Our National Secretary, Rhonda McKee, diligently maintains records of initiations in addition to keeping a current log of information about each chapter's corresponding secretary. She also sends out periodic supplies to each chapter for its initiations and has taken on the task of updating our web page. Connie Schrock, our National Historian, collects news from each chapter and serves as an editor for The Pentagon in this capacity. Our National President-Elect, Don Tosh, was responsible for the collection, selection, and scheduling of papers at this convention. As well, he scheduled and

provided publicity for the regional conferences last year. Don is also involved in distributing keypins and producing and mailing the certificates that each new initiate receives. At the end of this convention, Don will be taking over as National President, so I hope you will give him the same support and assistance that you have provided me.

As I come to the end of my term, I want to say how much I have enjoyed the experience of serving as President, even during the hectic times. I hope that you will keep working with the National Council, your region, and your local chapter to continuously improve Kappa Mu Epsilon.

Bob Bailey
National President

Report of the President Elect

The president elect is responsible for working with regional directors in the coordination of regional activities and regional conventions which are held in the spring of even numbered years. There were three regional conventions held in the spring of 2004. Two were in the Southeastern Region. One was hosted by VA Alpha at Virginia State University and the other was hosted by TN gamma at Union University. There was also one convention held in the North Central Region. It was hosted by KS Beta at Emporia State University. I was able to attend all three regional conventions. I would like to encourage chapters to consider hosting regional conventions for next year.

The president-elect is also responsible for accepting nominations for the George R. Mach Distinguished Service Award. This award was established by the National Council in 1987 and is named in honor of George Mach in recognition and appreciation for his 21 years of exemplary service as a member of the National Council. It is to be awarded each biennium to a person who has made major contributions to the Society. We were pleased to award this honor to Arnie Hammel this year, and I would like to encourage you or your chapter to send in nominations for people that you believe have served KME in a distinguished way.

The President-Elect also arranges the presentation of student papers, and we were pleased again this year to see the number, variety, and quality of the papers presented. We have continued to use the grading system that was first adopted two years ago, and it seems to expedite the judging of papers. In a departure from previous years, all presenters this year will receive a cash award and the four top papers will receive additional prizes.

Don Tosh
President-Elect

Report of the National Treasurer

35th Biennium (March 2, 2003 – March 20, 2005)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium

BIENNIUM ASSET REPORT

Total Assets (March 1, 2003)		$48,982.24
Current Assets		
Great Southern Bank		
Checking	16,253.71	
Savings 1	10,605.00	
Savings 2 CD	34,235.79	
Total Current Assets		$61,094.50

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	51,960.00	
Installation fees received	675.00	
Interest income	926.96	
Overpayment in	212.00	
Gifts & misc. income	105.62	
Total Biennium Receipts		$53,879.58
Expenditures		
Association of College Honor Soc	1,133.36	
Administrative expenses	3,022.33	
National Convention expenses	8,176.75	
Regional Convention expenses	2,182.58	
Council meetings travel	2,053.65	
Certificates, jewelry, & shipping	11,569.58	
Installation expenses	68.69	
Inventory expenses	71.53	
Overpayment returns	205.00	
Pentagon expenses	13,143.78	
Miscellaneous expenses	137.57	
Total biennium Expenses		$41,767.32
Biennium Cash Flow		+ $12,112.26

The 34th biennium Cash Flow was + $ 11,057.28. We continue to realize the savings from having the pins and membership certificates processed through Evangel University. We have maintained our goal of maintaining assets of at least $30,000. The financial condition of Kappa Mu Epsilon is sound.

I want to thank Al Riveland for the excellent records he kept as National Treasurer and the help he provided to me as I took over as National Treasurer.

John Kubicek
National Treasurer

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,598 new members in 114 chapters during the 35th Biennium that ended March 16, 2005. That brings the total membership of KME to 70,187.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. In the fall of each year, I send out supplies to each chapter. The supplies include information brochures, membership cards, a hard copy of the initiation report form and one or two copies of the brochure "A Matter of Honor." I also take minutes of all business meetings of the organization and all meetings of the national council.

Last November, the national council charged me with the responsibility of overseeing the design of a new web page for KME. We now own the domain name kappamuepsilon.org and professional web designers have created a new web page for us. The page is online but is a work in progress. I will have the responsibility of maintaining the page.

Rhonda McKee
National Secretary

Joint Report of the Pentagon Editor and Business Manager

The Pentagon, introduced in 1941, is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the central theme of *The Pentagon*. Continuing a tradition, papers given "Top 4 Status" by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. Many other student papers presented at the KME National and regional conventions are included as well. The key to having one's paper published in *The Pentagon* is submission of all of the necessary materials.

All new initiates receive a two-year subscription to *The Pentagon* and can continue their subscriptions for $5.00 per year. Having a current address is obviously vital in insuring that you receive your copy. Please check the address on your most recent issue and e-mail corrections to the Business Manager if necessary at laird-r@mssu.edu. Because of difficulties in establishing our non-profit status with the local post office, the Fall, 2005 issue was not mailed until about six weeks ago, but we anticipate that the Spring, 2006 issue should be sent out on schedule in late May.

Manuscripts received by *The Pentagon* other than those presented at our conventions are still refereed by faculty volunteers. In the past few months, forty-two referees have volunteered. The service of these individuals is invaluable.

Another regular feature of *The Pentagon* is the Problem Corner, to which you are encouraged to submit problems and solutions. Ken Wilke has ably edited the Problem Corner for the past 30 years, longer than all previous editors of the Problem Corner put together. The Fall, 2005 issue began a transition to a new editor, with Catherine Kong and Ken Wilke working jointly. Many thanks, Ken and Catherine.

Thanks also to Connie Shrock for collecting and editing the Chapter News, to the regional directors for contributing reports of the regional conventions and forwarding student papers from the conventions, and to the National Committee for its assistance with a myriad of details crucial to the success of *The Pentagon*.

Finally, a special thank you to Steve Nimmo and Larry Scott, the previous editor and business manager, respectively, for their extensive instruction and assistance as we took over those positions.

Charles Curtis, Editor
Richard Laird, Business Manager

Kappa Mu Epsilon National Officers

Don Tosh *President*
Department of Science and Technology
Evangel College
1111 N. Glenstone Avenue
Springfield, MO 65802
toshd@evangel.edu

Ron Wasserstein *President-Elect*
262 Morgan Hall
Washburn University
1700 SW College Avenue
Topeka, KS 66621
ron.wasserstein@washburn.edu

Rhonda McKee *Secretary*
Department of Mathematics
Central Missouri State University
Warrensburg, MO 64093-5045
mckee@cmsu1.cmsu.edu

John Kubicek *Treasurer*
Department of Mathematics
Southwest Missouri State University
Springfield, MO 65804
jdk114@smsu.edu

Connie Schrock *Historian*
Department of Mathematics
Emporia State University
Emporia, KS 66801-5087
schrockc@emporia.edu

Rhonda McKee *Webmaster*
Department of Mathematics
Central Missouri State University
Warrensburg, MO 64093-5045
mckee@cmsu1.cmsu.edu

KME National Website:
http://www.kappamuepsilon.org/

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
TX Nu	Texas A&M University - Corpus Christi, Corpus Christi	8 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005