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NEW KME Website!

The new national KME website can be found at

http://www.kappamuepsilon.org/

Among the items on the site:

- Contact information for national officers
- Initiation report form
- How to start a KME chapter
- Information on KME conventions

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster. Currently, this is the National Secretary, Rhonda McKee. Her contact information is located in the list of National Officers on page 81 and under National Officers on the web site.

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Greetings to KME on its 75th Anniversary

The year 2006 marks the 75th anniversary of the founding of Kappa Mu Epsilon. On April 18, 1931, Dr. Kathryn Wyant, Dr. L.P. Woods, and 22 other faculty and students became charter members of Oklahoma Alpha, Northeastern Oklahoma State Teachers College, Tahlequah. The national organization elected its first officers the same day. Additional chapters were soon added in Iowa, Kansas, Missouri, and Mississippi, bringing the total number of chapters to six by the end of 1932.

The object of the Society is fivefold:

- to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program;
- to help the undergraduate realize the important role that mathematics has played in the development of western civilization;
- to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought;
- to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and
- to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics.

The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

KME Facts

- First president of KME: Kathryn Wyant. (Her title was President Pythagoras. Other officers included Vice-President Euclid and Treasurer Newton.)
- Year of the first national KME convention: 1933 (held in Tahlequah)
- Number of states in which a KME national convention has been held: 18
- State that has been home to the most national conventions: Kansas (eight conventions)
- State with the greatest number of active KME chapters: Pennsylvania (18 active chapters)
- First issue of *The Pentagon*: Fall, 1941
- First article in that issue: *Mathematics and National Defense*, by W. L. Hart, Chairman of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America
- Name of the Mathematics Club at Northeastern Oklahoma State Teachers College prior to the establishment of KME: *The Pentagon*
- Individual with the longest service in a national KME position: Kenneth M. Wilke (32 years as Editor of the Problem Corner of *The Pentagon*, 1974 to the present)
- Problem Corner contributor whose proposals spanned the longest period: Charles W. Trigg (33 years, 1953 to 1986)

An application of algebra (from the Fall, 1941 issue):

"From an old French source comes the study of the equation,

one-half full bottle equals one-half empty bottle.

If each member of this equation is multiplied by 2, there results,

full bottle equals empty bottle."

Predicting Undergraduate Re-Enrollment: A Bayesian Approach

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and
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TX Gamma

Texas Woman's University
Denton, TX 76204

Presented at the 2005 National Convention and awarded "top four" status by the Awards Committee.

1. Introduction

Texas Woman's University, located in Denton, Texas, enrolls more than 10,000 students, with approximately 51% of students studying at the undergraduate level. Increasingly forced to rely on tuition as a main source of revenue, educational institutions such as TWU are looking to more accurately and effectively predict total enrollment for each academic year. Having a reliable prediction of total tuition-based income for any given year allows administrators to budget accordingly and avoid financial catastrophe ([3]). A significant part of the University's enrollment consists of undergraduate students who are present during a certain year and who re-enroll the next academic year. Predicting the continuing rates of TWU undergraduate students, in a Bayesian statistical context, will be the focus of this paper.

Many educational institutions—seeking to predict subgroups of student enrollment just as we are—employ logistic regression methods ([3] and [5]) and often rely on a general frequentist interpretation of any results. Our Bayesian model, however, directly uses the laws of probability to ensure that our results come with a probabilistic interpretation.

First, we perform exploratory analysis on historical enrollment data by breaking the yearly student populations into different subgroups, seeking consistent proportions of continuing students. Ultimately, we discover that a high level of consistency exists when the student populations are stratified by class level. Therefore, we model our population prediction of continuing students based on stratification of students into freshman, sophomore, junior, and senior levels of classification.

In Section 2 we show the results of the exploratory analysis on the stratified population of undergraduate students for fall semesters 2000-2003. Then, in Section 3, we present our Bayesian model that specifically incorporates the *a priori* historical information derived from our exploratory analysis. In Section 4, we use SAS to apply our model to the stratified population of Fall 2003 undergraduate students and obtain a prediction on the total number of re-enrolled undergraduate students for Fall 2004. We then compare these results to the actual total of Fall 2003 students who re-enroll in Fall 2004. In Section 5 we apply our Bayesian model to Fall 2004 undergraduate students and obtain a prediction on the total number of undergraduate students we can expect to re-enroll in Fall 2005.

2. Exploratory Analysis of Historical Data

At any point during a given academic year, a university contains a finite number of enrolled students. First, we are interested in predicting the rates at which students of a particular fall semester will re-enroll at the university the following fall semester. Once the rate of re-enrollment is determined, we use it to find the total re-enrollment. To help us with this prediction on the rate, we observe historical enrollment data from the 2000 through 2003 fall semesters. The data will be stratified into undergraduate classifications, so that enrollment patterns are clearly exhibited and ideal for modeling with a Bayesian algorithm.

In Figure 2.1 below, we present a chart of fall semester student enrollment. Here, each column under the heading "Fall 12th Day Total" represents a finite population of students, such that each student in a particular finite population stratum either re-enrolls the following fall semester ($y = 1$) or does not re-enroll ($y = 0$). Thus, to determine the number of re-enrolling students for each finite population, we sum their response variable. Once the re-enrollment frequency has been determined, we can obtain a re-enrollment rate. Notice a definitive pattern of re-enrollment rates by classification (i.e. freshmen, sophomores, juniors, and seniors), with rates changing less than 5% from year to year. In keeping with this pattern, we will break up the population into h disjoint strata of size N_h, where $h \in \{1, 2, 3, 4\}$.

According to Bolfarine and Zacks ([2]), “Stratification may result in significant gains in precision since it makes it possible to divide a heterogeneous population into homogeneous strata”. Clearly this is the case when comparing the re-enrollment rates by classification level from year to year in Figure 2.1.

In Section 3, we will outline and describe in detail a model that can be used to predict fall semester re-enrollment for each stratum.

Figure 2.1: SAS Exploratory Analysis of Re-Enrollment Rates

		FALL $12th$-DAY TOTAL				FALL RE-ENROLL STUDENTS				FALL RE-ENROLL RATE			
LEVEL	CLASS	2000	2001	2002	2003	2000	2001	2002	2003	2000	2001	2002	2003
UNDERGRAD	FRESHMAN	815	880	999	1107	513	569	665	727	62.9%	64.6%	66.5%	65.6%
	SOPHOMORE	711	694	755	924	502	523	573	692	70.6%	75.3%	75.8%	74.8%
	JUNIOR	1017	977	1127	1234	805	792	909	974	79.1%	81.0%	80.6%	78.9%
	SENIOR	1680	1617	1658	1755	644	701	765	833	38.3%	43.3%	46.1%	47.4%

3. Bayesian Model

In Section 2, we presented the results of our exploratory analysis of historical data and identified the patterns that will contribute to our model. Specifically, we will create a model capable of predicting re-enrollment totals for a given year and individual class level, where the undergraduate level is represented by stratum $\{1, 2, 3, 4\}$. Thus, we can break up the total population of undergraduate students into 4 approximately homogeneous sub-populations where the population size is $N = N_1 + N_2 + N_3 + N_4$.

For each classification stratum within a current fall semester, we want to predict the number of students that will re-enroll at the university for the following fall semester. To do so, we will consider a previous fall semester's information to help us project the re-enrollment rates of the current year's 12th-day-enrolled students. Without loss of generality, suppose we are in a given stratum h; then the vector of responses for the current year's students is

$$\mathbf{y}_h^c = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_{N_h^c} \end{bmatrix},$$

where $\mathbf{y}_h^c$ is a $N_h^c \times 1$ vector, and the superscript c references the current year's data. We assume that each y_i is Bernoulli for $i \in \{1, 2, ..., N_h^c\}$. Thus, the current year's re-enrollment total, $\mathbf{T}_h^c$, is

$$T_h^c = \mathbf{1}_h^{'} \mathbf{y}_h^c = \sum_{i=1}^{N_h^c} y_i,$$

where $\mathbf{1}_h$ is an $N_h^c \times 1$ vector of ones. To predict the total re-enrollment T_h^c for the current year, we will use last year's total re-enrollment

$$T_h^{c-1} = \mathbf{1}_h^{'} \mathbf{y}_h^{c-1} = \sum_{i=1}^{N_h^{c-1}} y_i,$$

where $\mathbf{y}_h^{c-1}$ is an $N_h^{c-1} \times 1$ vector of responses from last year's data, and $\mathbf{1}_h$ is an $N_h^{c-1} \times 1$ vector of ones.

The distribution $f\left(T^c \mid \theta_h\right)$ of T^c is binomial, denoted as $b\left(N_h^c, \theta_h\right)$, where θ_h is the unknown re-enrollment rate for stratum h. To understand this, note that $y_i \sim Br\left(\theta_h\right)$, $i \in \{1, 2, ..., N_h^c\}$. Assuming each trial is independent, we have

$$\begin{aligned} & (\theta_h)^{y_1}(1-\theta_h)^{1-y_1}\cdots(\theta_h)^{y_{N_h^c}}(1-\theta_h)^{1-y_{N_h^c}} \\ = \; & (\theta_h)^{\sum_{i=1}^{N_h^c} y_i y_1}(1-\theta_h)^{N_h^c-\sum_{i=1}^{N_h^c} y_i y_1} \\ = \; & (\theta_h^c)^{T^c}(1-\theta_h^c)^{N_h^c-T^c}. \end{aligned}$$

We note that T^c can be any value between 0 and N_h^c. Thus, the probability of T_h^c students re-enrolling out of N_h^c is

$$f(T^c \mid \theta_h) = \binom{N_h^c}{T^c}(\theta_h)^{T^c}(1-\theta_h)^{N_h^c-T^c}, \qquad (1)$$

which is a binomial distribution. In general, the data model $f(\cdot \mid \theta)$ with parameter θ is called a *likelihood* function. Similarly, the distribution of last year's total re-enrollment has the following binomial distribution:

$$f\left(T^{c-1} \mid \theta_h\right) = \binom{N_h^{c-1}}{T^{c-1}}(\theta_h)^{T^{c-1}}(1-\theta_h)^{N_h^{c-1}-T^{c-1}}, \qquad (2)$$

where θ_h still represents the re-enrollment rate for stratum h.

First we want to model our uncertainty on the rate of re-enrollment, θ_h. Obviously, θ_h is a value between 0 and 1. Thus, we will define our *prior* model on θ_h using a beta distribution,

$$\pi(\theta_h) = \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)}(\theta_h)^{\alpha_0-1}(1-\theta_h)^{\beta_0-1}, \; \theta_h \in [0,1], \qquad (3)$$

with mean

$$E(\theta_h) = \frac{\alpha_0}{\alpha_0+\beta_0}$$

and variance

$$var(\theta_h) = \frac{\alpha_0\beta_0}{(\alpha_0+\beta_0)^2(\alpha_0+\beta_0+1)}.$$

In this context, α_0 can be thought of as the frequency of re-enrolling students from a particular fall semester and β_0 represents the frequency of non-re-enrolling students. For example, if we believe, *a priori*, that the re-enrollment rate θ_h of a particular classification stratum is equal to 0.6, then we can define $\alpha_0 = 6$ and $\beta_0 = 4$, which will center the prior distribution on θ_h at 0.6. If we desired our prior knowledge to have a more significant effect on the final prediction, we would increase the values of α_0 and β_0 while maintaining a prior distribution mean of 0.6.

Next we will update our prior uncertainty on θ_h using T_h^{c-1} from the historical data. To do this, we need to obtain the *posterior distribution*, $\pi\left(\theta_h \mid T_h^{c-1}\right)$, on the re-enrollment rate given last year's data. Using

Bayes' rule the posterior distribution is

$$\begin{aligned}\pi\left(\theta_h \mid T_h^{c-1}\right) &= \frac{f\left(T_h^{c-1}, \theta_h\right)}{f\left(T_h^{c-1}\right)} \\ &= \frac{f\left(T_h^{c-1}, \theta_h\right)\pi\left(\theta_h\right)}{f\left(T_h^{c-1}\right)}, \qquad (4)\end{aligned}$$

where $f\left(T_h^{c-1} \mid \theta_h\right)$ is the likelihood function for last year's data, $\pi\left(\theta_h\right)$ is the prior distribution, and $f\left(T_h^{c-1}\right)$ is the marginal density of the data obtained by integrating over the product of the likelihood and prior distributions with respect to θ_h:

$$f\left(T_h^{c-1}\right) = \int_0^1 f\left(T^{c-1} \mid \theta_h\right)\pi\left(\theta_h\right)d\theta_h.$$

Substituting equations 1 and 3 into 4, the posterior distribution on θ_h given T_h^{c-1} is a beta distribution (Gelman, Carlin, Stern, and Rubin, 2004):

$$\pi\left(\theta_h \mid T_h^{c-1}\right) = \frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)}\left(\theta_h\right)^{\alpha-1}\left(1-\theta_h\right)^{\beta-1}, \theta_h \in [0,1]. \qquad (5)$$

The posterior distribution $\pi\left(\theta_h \mid T_h^{c-1}\right)$ has mean $E\left(\theta_h\right) = \dfrac{\alpha_1}{\alpha_1+\beta_1}$ and variance $var\left(\theta_h\right) = \dfrac{\alpha_1\beta_1}{\left(\alpha_1+\beta_1\right)^2\left(\alpha_1+\beta_1+1.\ \right)}$, where $\alpha_1 = \alpha_0 + T_h^{c-1}$ and $\beta_1 = \beta_0 + \left(N_h^{c-1} - T_h^{c-1}\right)$. The complete derivation for the posterior distribution is provided in Appendix A.

Finally, our ultimate objective is to predict total re-enrollment T_h^c given last year's re-enrollment pattern T_h^{c-1}. To do this, we will apply the following Bayesian algorithm to obtain the *posterior predictive distribution*, which is derived in full in Appendix B:

$$\begin{aligned}f\left(T_h^c \mid T_h^{c-1}\right) &= \frac{f\left(T_h^c, T_h^{c-1}\right)}{f\left(T_h^{c-1}\right)} \\ &= \frac{f\left(T_h^c \mid T_h^{c-1}\right)f\left(T_h^{c-1}\right)}{f\left(T_h^{c-1}\right)} \\ &= \frac{1}{f\left(T_h^{c-1}\right)}\int_0^1 f\left(T_h^c \mid T_h^{c-1}, \theta_h\right)\pi\left(T_h^{c-1} \mid \theta_h\right)\pi\left(\theta_h\right)d\theta_h \\ &= \int_0^1 f\left(T_h^c \mid T_h^{c-1}, \theta_h\right)\pi\left(\theta_h \mid T_h^{c-1}\right)d\theta_h.\end{aligned}$$

Assuming T_h^c and T_h^{c-1} independent, this gives:

$$f\left(T_h^c \mid T_h^{c-1}\right) = \int_0^1 f\left(T_h^c \mid \theta_h\right) \pi\left(\theta_h \mid T_h^{c-1}\right) d\theta_h. \qquad (6)$$

Notice that the integrand of equation 6 is the product of the likelihood on T_h^c and the posterior distribution. Substituting equations 1 and 4 into equation 6, the posterior predictive distribution becomes a *beta binomial*

$$\begin{aligned} f\left(T_h^c \mid T_h^{c-1}\right) &= \frac{\Gamma\left(N_h^c+1\right)}{\Gamma\left(T_h^c+1\right)\Gamma\left(N_h^c-T_h^c+1\right)} \\ &\quad \times \frac{\Gamma\left(\alpha_1+T_h^c\right)\Gamma\left(N_h^c+\beta_1-T_h^c\right)}{\Gamma\left(\alpha_1+\beta_1+N_h^c\right)} \times \frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)} \end{aligned} \qquad (7)$$

where $T_h^c = 0, 1, 2, \ldots, N_h^c$, with mean $E\left(T_h^c\right) = N_h^c \dfrac{\alpha_1}{\alpha_1+\beta_1}$ and variance

$$var\left(T_h^c\right) = N_h^c \frac{\alpha_1\beta_1\left(\alpha_1+\alpha_1+N_H^c\right)}{\left(\alpha_1+\beta_1\right)^2\left(\alpha_1+\beta_1+1\right)}$$

([4], pp. 476-477). The quantities α_1 and β_1 are defined the same as discussed in the specification of the posterior distribution.

Lastly, we will use SAS to apply this model to our TWU historical enrollment data. The results of our programming, along with the prediction we have generated, will be presented in Section 4.

4. Results

The output of the SAS program used to implement our prediction model from Section 3 is shown below:

Figure 4.1: SAS Prediction of Re-Enrollment Rates and Totals

											BAYESIAN POSTERIOR RE-ENROLL					
	FALL 12th DAY TOTAL				FALL RE-ENROLL			FALL RE-ENROLL RATE (%)			RATE (%)		PREDICT		2003 ACTUAL RE-ENROLL	
CLASS	2000	2001	2002	2003	2000	2001	2002	2000	2001	2002	MEAN	SD	MEAN	SD	TOTAL	RATE (%)
FRESHMAN	815	880	999	1107	513	569	665	62.9%	64.6%	66.5%	66.4%	01.4%	735	22	727	65.6%
SOPHOMORE	711	694	755	924	502	523	573	70.6%	75.3%	75.8%	75.8%	01.4%	701	19	692	74.8%
JUNIOR	1017	977	1127	1234	805	792	909	79.1%	81.0%	80.6%	80.6%	01.1%	996	20	974	78.9%
SENIOR	1680	1617	1658	1755	644	701	765	38.3%	43.4%	46.1%	45.8%	01.1%	805	29	833	47.4%
GRAND TOTAL	4223	4168	4539	5020	2464	2585	2912	58.3%	62.0%	64.1%	63.9%	00.6%	3212	48	3226	64.2%

THE TOTAL PREDICTION FOR FALL 2003 RE-ENROLLMENT IS 3237.

As you can see, our model has predicted the Fall 2004 re-enrollment of Fall 2003 students with a very high degree of accuracy. The rates under the column heading "Bayesian Posterior Re-Enroll" are the means of the posterior distribution, equation 5, on the rate for each class level. As noted in Section 3, the posterior distribution on the rate of re-enrollment is part of the integrand of equation 6, which is then used to obtain the posterior predictive distribution on the total number of students from each stratum that we can expect to re-enroll in Fall 2004. The individual predictions for the class levels—which are the expected values (means) of the respective posterior predictive distributions—are listed under the "Predict" heading in Figure 4.1.

We predicted that approximately 735 freshman students enrolled in Fall 2003 would re-enroll for the Fall 2004 semester. In fact, 727 were present on the 12th day in Fall 2004. We predicted that 701 sophomores would continue from Fall 2003 to Fall 2004, and the actual frequency was 692. You may review similar predictions for juniors and seniors in Figure 4.1. Also note that for each stratum, our predicted total is within 1.5 standard deviations of the actual total.

Finally, we can observe the outstanding accuracy of our total prediction. The predicted total number of Fall 2003 undergraduate students expected to re-enroll in Fall 2004 (found by summing the predicted totals from each stratum) was 3,237. Also included in Figure 4.1 is the "Grand Total," which is simply the result of applying our Bayesian algorithm to the totals from each year without stratification and yielded an error of 14 students. As you can see, the stratification of students by classification level provided a slightly increased level of accuracy—an impressive error of only 11 students.

Given that our results were obtained through a Bayesian algorithm, we could choose to provide intervals on our predictions that would have purely probabilistic interpretations, rather than weaker, varying "confidence" levels generated by classical models. Certainly this application demonstrates the power of using a Bayesian model that adheres strictly to the laws of probability. The predictions for each stratum are just the mean of the posterior distribution with a specified standard deviation "SD".

Our Bayesian prediction model and its results give enrollment management personnel the convenience of a probabilistic interpretation versus an interpretation of confidence. In Section 5, we will present our Bayesian prediction results for the re-enrollment of Fall 2004 students in Fall 2005.

5. Future Prediction

In conclusion, we will predict the total number of Fall 2004 undergraduate students that we can expect to re-enroll in Fall 2005. Using the same model and methodology that we employed for our Fall 2004 prediction, we find that we can expect 3,484 undergraduates to re-enroll.

Figure 5.1: SAS Output of Fall 2005 Re-Enrollment Prediction

	FALL 12th DAY TOTAL				FALL RE-ENROLL			FALL RE-ENROLL RATE (%)			BAYESIAN POSTERIOR RE-ENROLL			
											RATE (%)		PREDICT	
CLASS	2001	2002	2003	2004	2001	2002	2003	2001	2002	2003	MEAN	SD	MEAN	SD
FRESHMAN	880	999	1107	1102	569	665	727	64.6%	66.5%	65.6%	65.7%	01.3%	725	22
SOPHOMORE	694	755	924	1014	523	573	692	75.3%	75.8%	74.8%	74.9%	01.3%	760	20
JUNIOR	977	1127	1234	1358	792	909	974	81.0%	80.6%	78.9%	79.0%	01.1%	1074	21
SENIOR	1617	1658	1755	1955	701	765	833	43.3%	46.1%	47.4%	47.3%	01.1%	926	31
GRAND TOTAL	4168	4539	5020	5429	2585	2912	3226	62.0%	64.1%	64.2%	64.2%	00.6%	3488	50

THE TOTAL PREDICTION FOR FALL 2004 RE-ENROLLMENT IS 3484.

In Figure 5.1 above, the column titled "Bayesian Posterior Re-Enroll" gives the predicted mean and standard deviation re-enrollment that we can expect for Fall 2005. In each case, we use the mean of the posterior predictive distribution as our predicted re-enrollment total. Accordingly, we expect 725 freshmen to re-enroll, 760 sophomores to re-enroll, and so on. Without stratification, we would expect 3,488 of Fall 2004 undergraduates to re-enroll in Fall 2005, but as we have seen before, the stratified prediction of 3,484 should provide an increased level of accuracy.

We expect this prediction for Texas Woman's University's undergraduate re-enrollment populations to both demonstrate the same level of precision as our previous predictions and provide the convenience of probabilistic interpretations that comes with the Bayesian approach.

Appendix A

Here we shall derive the posterior distribution function from the prior and likelihood functions used in our predictions.

Recall that our prior distribution 3 is defined to be

$$\pi\left(\theta_h\right) = \frac{\Gamma\left(\alpha_0+\beta_0\right)}{\Gamma\left(\alpha_0\right)\Gamma\left(\beta_0\right)}\left(\theta_h\right)^{\alpha_0-1}\left(1-\theta_h\right)^{\beta_0-1},\ \theta_h\in[0,1],$$

and our likelihood function 2 is

$$f\left(T^{c-1}\mid\theta_h\right)=\binom{N_h^{c-1}}{T^{c-1}}\left(\theta_h\right)^{T^{c-1}}\left(1-\theta_h\right)^{N_h^{c-1}-T^{c-1}}.$$

From Equation 4 and using Bayes' Rule, we expect our posterior distribution to have the form

$$\pi\left(\theta_h\mid T_h^{c-1}\right)=\frac{f\left(T_h^{c-1},\theta_h\right)}{f\left(T_h^{c-1}\right)}=\frac{f\left(T_h^{c-1}\mid\theta_h\right)\pi\left(\theta_h\right)}{f\left(T_h^{c-1}\right)}, \qquad (8)$$

where the denominator $f\left(T_h^{c-1}\right)$ is equal to $\int_0^1 f\left(T_h^{c-1}\mid\theta_h\right)\pi\left(\theta_h\right)d\theta_h$ and evaluates to a simple proportionality constant. As a result, we have

$$\pi\left(\theta_h\mid T_h^{c-1}\right)\propto f\left(T_h^{c-1}\mid\theta_h\right)\pi\left(\theta_h\right). \qquad (9)$$

The product of the likelihood and prior functions on the right-hand side of the proportionality statement above can be written as

$$\binom{N_h^{c-1}}{T_h^{c-1}}\left(\theta_h\right)^{T_h^{c-1}}\left(1-\theta_h\right)^{N_h^{c-1}-T_h^{c-1}}\frac{\Gamma\left(\alpha_0+\beta_0\right)}{\Gamma\left(\alpha_0\right)\Gamma\left(\beta_0\right)}\left(\theta_h\right)^{\alpha_0-1}\left(1-\theta_h\right)^{\beta_0-1}.$$

Since $\binom{N_h^{c-1}}{T_h^{c-1}}$ and $\frac{\Gamma\left(\alpha_0+\beta_0\right)}{\Gamma\left(\alpha_0\right)\Gamma\left(\beta_0\right)}$ are constants, we will rewrite the product above as

$$K\left(\theta_h\right)^{T_h^{c-1}}\left(1-\theta_h\right)^{N_h^{c-1}-T_h^{c-1}}\left(\theta_h\right)^{\alpha_0-1}\left(1-\theta_h\right)^{\beta_0-1}, \qquad (10)$$

where $K=\binom{N_h^{c-1}}{T_h^{c-1}}\frac{\Gamma\left(\alpha_0+\beta_0\right)}{\Gamma\left(\alpha_0\right)\Gamma\left(\beta_0\right)}$. To derive the form of the posterior for $\pi\left(\theta_h \mid T_h^{c-1}\right)$, we collect all the elements of equation 10 that contain θ_h and then have

$$\begin{aligned} & (\theta)^{T_h^{c-1}+\alpha_0-1}(1-\theta)^{N_h^{c-1}-T_h^{c-1}+\beta_0-1} \\ = \; & (\theta)^{\left(\alpha_0+T_h^{c-1}\right)-1}(1-\theta)^{\left[\beta_0+\left(N_h^{c-1}-T_h^{c-1}\right)\right]-1} \\ = \; & (\theta)^{\alpha_1-1}(1-\theta)^{\beta_1-1}, \end{aligned} \qquad (11)$$

since $\alpha_1=\alpha_0+T_h^{c-1}$ and $\beta_1=\beta_0+\left(N_h^{c-1}-T_h^{c-1}\right)$.

We recognize equation 11 as being the kernel of a beta distribution and can therefore use a constant of the form

$$C=\frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)}$$

so that the area of the beta distribution represented by $C\int_0^1(\theta)^{\alpha_1-1}(1-\theta)^{\beta_1-1}$ evaluates to one, as it should.

Appendix B

Here we will derive the posterior predictive distribution 6, which by definition is found from integrating the product of the likelihood 1 and posterior 5 distributions with respect to θ_h.

$$\begin{aligned} f\left(T_h^c \mid T_h^{c-1}\right) &= \int_0^1 f\left(T_h^c \mid \theta_h\right)\pi\left(\theta_h \mid T_h^{c-1}\right) d\theta_h \\ &= \int_0^1 \binom{N_h^c}{T_h^c}\left(\theta_h\right)^{T_h^c}(1-\theta)^{N_h^c-T_h^c} \\ &\quad \cdot\frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)}\left(\theta_h\right)^{\alpha_1-1}(1-\theta)^{\beta_1-1} d\theta_h \\ &= \binom{N_h^c}{T_h^c}\frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)} \\ &\quad \cdot\int_0^1\left(\theta_h\right)^{T_h^c}(1-\theta)^{N_h^c-T_h^c}\left(\theta_h\right)^{\alpha_1-1}(1-\theta)^{\beta_1-1} d\theta_h. \end{aligned}$$

Then, letting $C=\binom{N_h^c}{T_h^c}\frac{\Gamma\left(\alpha_1+\beta_1\right)}{\Gamma\left(\alpha_1\right)\Gamma\left(\beta_1\right)}$ and combining like terms, we continue and have:

$$f\left(T_h^c \mid T_h^{c-1}\right)=C\int_0^1\left(\theta_h\right)^{\left(T_h^c+\alpha_1\right)-1}(1-\theta)^{\left(N_h^c-T_h^c+\beta_1\right)-1} d\theta_h. \qquad (12)$$

Next, letting $\alpha_2 = T_h^c + \alpha_1$ and $\beta_2 = N_h^c - T_h^c + \beta_1$, we see that

$$f\left(T_h^c \mid T_h^{c-1}\right) = C \int_0^1 (\theta_h)^{\alpha_2 - 1} (1-\theta)^{\beta_2 - 1} \, d\theta_h. \tag{13}$$

The integrand above is the kernel of a beta distribution with parameters α_2 and β_2. Since, by definition, any analytical probability density function integrates to one, it follows that a beta distribution with the form

$$f(X \mid \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (X)^{\alpha-1} (1-X)^{\beta-1}$$

will satisfy the condition $\int_0^1 f(X \mid \alpha, \beta)\, dX = 1$. This implies that the integrand in 13 integrates to a constant, as shown below:

$$\int_0^1 (\theta_h)^{\alpha_2-1} (1-\theta_h)^{\beta_2-1} \, d\theta_h = \frac{\Gamma(\alpha_2)\Gamma(\beta_2)}{\Gamma(\alpha_2+\beta_2)}.$$

Therefore,

$$\begin{aligned} f\left(T_h^c \mid T_h^{c-1}\right) &= C \frac{\Gamma(\alpha_2)\Gamma(\beta_2)}{\Gamma(\alpha_2+\beta_2)} \\ &= \binom{N_h^c}{T_h^c} \frac{\Gamma(\alpha_1+\beta_1)}{\Gamma(\alpha_1)\Gamma(\beta_1)} \frac{\Gamma(\alpha_2)\Gamma(\beta_2)}{\Gamma(\alpha_2+\beta_2)} \\ &= \frac{\Gamma(N_h^c+1)}{\Gamma\left(T_h^c+1\right)\Gamma\left(N_h^c - T_h^c + 1\right)} \frac{\Gamma(\alpha_1+\beta_1)}{\Gamma(\alpha_1)\Gamma(\beta_1)}, \end{aligned}$$

which is the beta binomial function we expected to derive for the posterior predictive distribution.

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An Inverse Problem for Harmonic Oscillators

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1. Introduction and Statement of Problem

We are considering underdamped harmonic oscillators that are modeled by homogeneous, ordinary differential equations. An example of such a harmonic oscillator is the horizontally-aligned spring-mass system below.

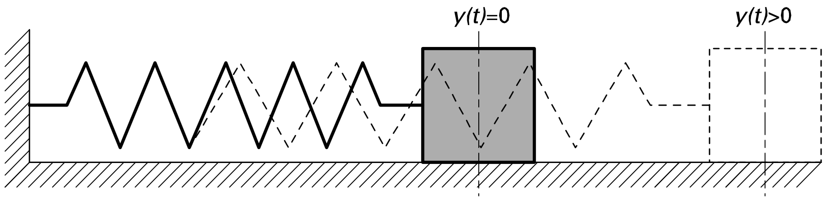

Figure 1

We let m be the mass of the object attached to the spring, and $c > 0$ is the damping coefficient corresponding to the frictional resistance between the mass and the surface that it rests on and the resistance exerted on the mass by air. We represent the spring constant with $k > 0$; the spring constant is associated with Hooke's Law [3, p.375]. The position of the mass' center of gravity is $y(t)$, and we measure $y(t)$ as the displacement of the center of gravity from the equilibrium position, where we say $y(t) = 0$. When the spring is compressed, we designate $y(t) < 0$, and $y(t) > 0$ when the spring is stretched. We say that a harmonic oscillator is underdamped if it oscillates about its equilibrium position with a displacement that decreases in magnitude as it oscillates. We will see that the decreasing magnitude is caused by decaying exponential functions. Below is a sample graph of an underdamped harmonic oscillator.

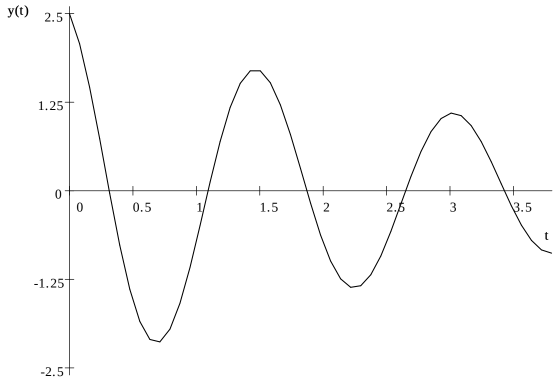

Figure 2

From the graph we see that the mass was released from a position where the spring was stretched, or $y(t) > 0$. Notice that as the mass oscillates, or as $y(t)$ changes polarity, the amplitude of $y(t)$ is decreasing.

It is well-known that given m, c, and k for a simple harmonic oscillator, we can find a general solution that expresses all possible solutions for the oscillator. It is interesting to consider this inversely: given the ability to take distinct measurements of a physical harmonic oscillator's position over time, can we use these measurements to determine m, c, and k for the oscillator? C.W. Groetsch presented this question in *Inverse Problems: Activities for Undergraduates* [2, pp.111-4], and it will be the focus of our attention.

2. Background

Consider the harmonic oscillator of Figure 1. We will use Newton's second law to derive the harmonic oscillator's equation. Newton's law states $F = ma$, where F is force, m is mass, and a is acceleration [1, pp.155-6]. Since $a = y''(t)$, the second derivative of the position measurement, we can rewrite Newton's equation as

$$F = my''(t). \tag{14}$$

Now we consider the force exerted on the mass by the spring, F_s, and the damping force, F_d, which represents resistance due to air and frictional forces. Both F_s and F_d are in the direction of the mass's rest position because they oppose the direction of acceleration. From Hooke's Law, F_s is proportional to $y(t)$ [3, p.140], and we assume F_d is proportional to $y'(t)$. Since $F = F_s + F_d$ we express equation 14 as $-cy'(t) - ky(t) = my''(t)$, or

$$my''(t) + cy'(t) + ky(t) = 0. \tag{15}$$

Because the function $y(t) = e^{rt}$ has derivatives that look much like the original function, it is a good guess for a solution to equation 15. To test this idea, we substitute $y(t) = e^{rt}$ and its derivatives into the equation:

$$\begin{aligned} my''(t) + cy'(t) + ky(t) &= m\frac{d^2(e^{rt})}{dt^2} + c\frac{d(e^{rt})}{dt} + k(e^{rt}) \\ &= (mr^2 + cr + k)e^{rt}. \end{aligned}$$

Our guess works as long as r satisfies $mr^2 + cr + k = 0$, the differential equation's characteristic equation. Using the quadratic formula, we find that $y(t) = e^{rt}$ is a solution to equation 15 when $r = \frac{-c}{2m} \pm \frac{\sqrt{c^2-4mk}}{2m}$.

For our purposes, we are interested specifically in harmonic oscillators that have oscillatory motion. The characteristic equations of such differential equations have complex roots with non-zero imaginary parts. It is easy to show that the constants in our oscillator's equation of motion must satisfy $c < 2\sqrt{mk}$ in order to have such roots. We define

$$\alpha \equiv \frac{c}{2m} \text{ and } \beta \equiv \frac{\sqrt{4mk - c^2}}{2m}, \tag{16}$$

and obtain the complex valued solutions $y(t) = e^{(-\alpha \pm i\beta)t}$.

We would like to have real-valued solutions to equation 15. We let $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$ be our two complex-valued solutions. Applying Euler's Formula,

$$\begin{aligned}\tilde{y}_1(t) &= e^{(-\alpha+i\beta)t}\\ &= e^{-\alpha t}e^{i\beta t}\\ &= e^{-\alpha t}\left(\cos(\beta t)+i\sin(\beta t)\right)\end{aligned}$$

and

$$\begin{aligned}\tilde{y}_2(t) &= e^{(-\alpha-i\beta)t}\\ &= e^{-\alpha t}e^{-i\beta t}\\ &= e^{-\alpha t}\left(\cos(\beta t)-i\sin(\beta t)\right).\end{aligned}$$

We can construct linear combinations of $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$ to obtain the following real-valued solutions:

$$\begin{aligned}y_1(t) &= \frac{\tilde{y}_1(t)+\tilde{y}_2(t)}{2}\\ &= e^{-\alpha t}\cos(\beta t)\end{aligned}$$

and

$$\begin{aligned}y_2(t) &= \frac{\tilde{y}_1(t)-\tilde{y}_2(t)}{2i}\\ &= e^{-\alpha t}\sin(\beta t).\end{aligned}$$

Combining the two solutions,

$$y(t)=e^{-\alpha t}\left(A\cos(\beta t)+B\sin(\beta t)\right) \tag{17}$$

is our general solution, where A and B are constants determined by the harmonic oscillator's initial conditions, or values of $y(0)$ and $y'(0)$ [1, pp.284-8]. We can use the general solution to find the particular solution of any initial-value problem [1, p.23].

3. Development of Problem

We now return to the task of solving our inverse problem. Suppose, for a moment, that we can take n measurements of position, velocity, and acceleration. Then it seems plausible that we could use the following system of n equations, where $t_{n+1}=t_n+h$, to find m, c, and k:

$$\begin{bmatrix} y''(t_1) & y'(t_1) & y(t_1)\\ y''(t_2) & y'(t_2) & y(t_2)\\ \vdots & \vdots & \vdots\\ y''(t_n) & y'(t_n) & y(t_n)\end{bmatrix}\begin{bmatrix} m\\ c\\ k\end{bmatrix}=\begin{bmatrix}0\\0\\ \vdots\\ 0\end{bmatrix}.$$

Notice, however, that the system of equations is homogeneous. We know that the system is consistent because the zero solution will solve the system. By an existence and uniqueness theorem for linear systems, the system will either have a unique solution or infinitely many solutions [4, p.24]. Infinitely many solutions do not make sense in our physical situation. Also, we are not interested in the unique zero solution.

Hence, we can hope to find at most two of the unknowns. We assume a unit mass, or $m = 1$. Reflecting this assumption, we rewrite 15 as

$$cy'(t) + ky(t) = -y''(t). \tag{18}$$

Notice that assuming $m = 1$ allows us to rewrite α and β as $\alpha \equiv \frac{c}{2}$ and $\beta \equiv \frac{\sqrt{4k-c^2}}{2}$.

Equations of the form 18 will have characteristic equations with complex roots that have non-zero imaginary parts when $c < 2\sqrt{k}$. Assuming $m = 1$ allows us to construct a non-homogeneous system. We can solve the following system of two linear equations for c and k when $i \neq j$:

$$\begin{bmatrix} y'(t_i) & y(t_i) \\ y'(t_j) & y(t_j) \end{bmatrix} \begin{bmatrix} c \\ k \end{bmatrix} = - \begin{bmatrix} y''(t_i) \\ y''(t_j) \end{bmatrix}. \tag{19}$$

Because we are only taking measurements of position, we will need to approximate the derivatives $y'(t)$ and $y''(t)$ using only values of $y(t)$. We use the following approximations:

$$y''(t_i) \approx \frac{y(t_{i+1}) - 2y(t_i) + y(t_{i-1})}{h^2} \tag{20}$$

$$y'(t_i) \approx \frac{y(t_{i+1}) - y(t_{i-1})}{2h}. \tag{21}$$

Notice that each approximation depends on the position of the mass at three distinct times. We now demonstrate that these approximations are good for small values of h by using Taylor expansions. Expanding $y(t_{i+1})$ and $y(t_{i-1})$ around t_i yields

$$\begin{aligned} y(t_{i+1}) &= y(t_i + h) \\ &= y(t_i) + y'(t_i)h + \frac{y''(t_i)}{2}h^2 + \frac{y'''(c_1)}{6}h^3 \end{aligned}$$

and

$$\begin{aligned} y(t_{i-1}) &= y(t_i - h) \\ &= y(t_i) - y(t_i)h + \frac{y"(t_i)}{2}h^2 - \frac{y'''(c_2)}{6}h^3 \end{aligned}$$

where $c_1, c_2 \in (0, h)$. Substituting these expansions into equation 20,

$$\begin{aligned} y''(t_i) &\approx \frac{y(t_{i+1}) - 2y(t_i) + y(t_{i-1})}{h^2} \\ &= ((y(t_i) + y'(t_i)h + \frac{y''(t_i)}{2}h^2 + \frac{y'''(c_1)}{6}h^3) - 2y(t_i) \\ &\quad +(y(t_i) - y'(t_i)h + \frac{y''(t_i)}{2}h^2 - \frac{y'''(c_2)}{6}h^3))/h^2 \\ &= \frac{y''(t_i)h^2 + \frac{y'''(c_1) - y'''(c_2)}{6}h^3}{h^2} \\ &= y''(t_i) + \frac{y'''(c_1) - y'''(c_2)}{6}h. \end{aligned}$$

So the approximation 20 is equal to $y''(t_i)$ plus a term involving the remainders of the Taylor expansions. Notice that

$$\lim_{h \to 0} \left(y''(t_i) + \frac{y''(c_1) - y'''(c_2)}{6}h \right) = y''(t_i).$$

So as $h \to 0$, our approximation converges to $y''(t_i)$. The proof that the approximation 21 converges to $y'(t_i)$ as $h \to 0$ is similar.

Combining our equation of motion 18 with the approximations 20 and 21, we get an approximate equation for the motion of the harmonic oscillator, where $\tilde{c}$ and $\tilde{k}$ are approximations to the constants c and k:

$$\tilde{c}\left(\frac{y(t_{i+1}) - y(t_{i-1})}{2h}\right) + \tilde{k}y(t_i) = -\left(\frac{y(t_{i+1}) - 2y(t_i) + y(t_{i-1})}{h^2}\right). \tag{22}$$

Multiplying through both sides by h^2, equation 22 becomes

$$\frac{h}{2}\left(y(t_{i+1}) - y(t_{i-1})\right)\tilde{c} + h^2 y(t_i)\tilde{k} = -y(t_{i-1}) + 2y(t_i) - y(t_{i+1}). \tag{23}$$

Notice that four distinct measurements of $y(t_i)$ allow us to construct two equations of the form in equation 23. We now use these two equations to construct a system of linear equations:

$$\begin{bmatrix} \frac{1}{2}h(y(t_2) - y(t_0)) & h^2 y(t_1) \\ \frac{1}{2}h(y(t_3) - y(t_1)) & h^2 y(t_2) \end{bmatrix} \begin{bmatrix} \tilde{c} \\ \tilde{k} \end{bmatrix} = \begin{bmatrix} -y(t_0) + 2(y(t_1) - y(t_2)) \\ -y(t_1) + 2(y(t_2) - y(t_3)) \end{bmatrix}. \tag{24}$$

We can solve system 24 for the approximations $\tilde{c}$ and $\tilde{k}$. We find

$$\begin{bmatrix} \tilde{c} \\ \tilde{k} \end{bmatrix} = \begin{bmatrix} \frac{2(-y(t_1)^2 - y(t_1)y(t_3) + y(t_2)^2 + y(t_2)y(t_0))}{h(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0))} \\ \frac{2(y(t_1)y(t_2) - y(t_0)y(t_3) + y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0))}{h^2(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0))} \end{bmatrix}. \tag{25}$$

It seems that our approximations to c and k should become better as our approximations to $y'(t_i)$ and $y''(t_i)$ become better. This corresponds to the case of small h.

Theorem 1 *As $h \to 0$, the system in 25 will converge to* $\begin{bmatrix} c \\ k \end{bmatrix}$.

Proof. Let $f(h)$ and $g(h)$, respectively, be the numerator and denominator of $\tilde{c}$. We can show that $\lim_{h\to 0} f^{(n)}(h) = 0$ and $\lim_{h\to 0} g^{(n)}(h) = 0$ for $n = 0, 1, 2$. Therefore, applying L'Hospital's Rule [5, p.138], we have

$$\begin{aligned}
\lim_{h\to 0} \tilde{c} &= \lim_{h\to 0} \frac{f^{(3)}(h)}{g^{(3)}(h)} \\
&= \frac{f^{(3)}(0)}{g^{(3)}(0)} \\
&= \frac{-24\alpha\beta^2 A^2 - 24\alpha\beta^2 B^2}{-12\beta^2 A^2 - 12\beta^2 B^2} \\
&= 2\alpha.
\end{aligned}$$

Recalling our definition for α, $\lim_{h\to 0} \tilde{c} = 2\alpha = 2\left(\frac{c}{2}\right) = c$.

Similarly with $\tilde{k}$, we let the numerator and denominator be, $p(h)$ and $q(h)$, respectively. Here, $\lim_{h\to 0} p^{(n)}(h) = 0$ and $\lim_{h\to 0} q^{(n)}(h) = 0$ for $n = 0, 1, 2, 3$. Using L'Hospital's Rule again,

$$\begin{aligned}
\lim_{h\to 0} \tilde{k} &= \lim_{h\to 0} \frac{p^{(4)}(h)}{q^{(4)}(h)} \\
&= \frac{p^{(4)}(0)}{q^{(4)}(0)} \\
&= \frac{-48\beta^4 A^2 - 48\beta^4 B^2 - 48\alpha^2\beta^2 A^2 - 48\alpha^2\beta^2 B^2}{-48\beta^2 A^2 - 48\beta^2 B^2} \\
&= \alpha^2 + \beta^2.
\end{aligned}$$

Using the definitions for α and β,

$$\lim_{h\to 0} \tilde{k} = \alpha^2 + \beta^2 = \left(\frac{c}{2}\right)^2 + \left(\frac{\sqrt{4k - c^2}}{2}\right)^2 = k.$$

■

So we find that our approximations, $\tilde{c}$ and $\tilde{k}$, are closer to the true values when h is small. It will be useful to have some way to express the error in approximating c and k. To do so, we will need the following two lemmas.

Lemma 2 *Consider functions of the form*

$$y(t) = e^{-\alpha t}(A\cos(\beta t) + B\sin(\beta t))$$

with constants $\alpha > 0$, β, A, and B. There exists some maximum M such that $M \geq |y(t)|$ when $t \geq 0$.

Proof. Notice that $\left|e^{-\alpha t}\right| \leq 1$, $|\cos(\beta t)| \leq 1$, and $|\sin(\beta t))| \leq 1$ for $t \geq 0$. Applying the Triangle Inequality [5, p.4] and these observations,

$$\begin{aligned} |y(t)| &= \left|e^{-\alpha t}\right| |(A\cos(\beta t) + B\sin(\beta t))| \\ &\leq \left|e^{-\alpha t}\right| (|A\cos(\beta t)| + |B\sin(\beta t)|) \\ &\leq |A| + |B|. \end{aligned}$$

We let $M \geq |A| + |B|$. ■

Lemma 3 *Consider functions of the form*

$$y(t) = e^{-\alpha t}(A\cos(\beta t) + B\sin(\beta t))$$

with constants α, β, A, and B. For the nth derivative of $y(t)$, there exist constants A_n and B_n such that

$$y^{(n)}(t) = e^{-\alpha t}(A_n\cos(\beta t) + B_n\sin(\beta t)).$$

Proof. We will prove this using induction. First we show that our lemma holds for the first derivative.

$$\begin{aligned} y'(t) &= -\alpha e^{-\alpha t}(A\cos(\beta t) + B\sin(\beta t)) \\ &\quad +\beta e^{-\alpha t}(-A\sin(\beta t) + B\cos(\beta t)) \\ &= e^{-\alpha t}[-\alpha(A\cos(\beta t) + B\sin(\beta t)) \\ &\quad +\beta(-A\sin(\beta t) + B\cos(\beta t))] \\ &= e^{-\alpha t}[(-\alpha A\cos(\beta t) + \beta B\cos(\beta t)) \\ &\quad +(-\alpha B\sin(\beta t) - \beta A\sin(\beta t))] \\ &= e^{-\alpha t}[(-\alpha A + \beta B)\cos(\beta t) + (-\alpha B - \beta A)\sin(\beta t)] \\ &= e^{-\alpha t}(A_1\cos(\beta t) + B_1\sin(\beta t)) \end{aligned}$$

In the above, we've let $A_1 = -\alpha A + \beta B$ and $B_1 = -\alpha B - \beta A$.

Now we assume that our conclusion is also true for the nth derivative so that $y^{(n)}(t) = e^{-\alpha t}(A_n\cos(\beta t) + B_n\sin(\beta t))$ for some constants A_n and B_n. Taking the derivative of $y^{(n)}(t)$, we find, analogously to the case of $n = 1$, that there exists $A_{n+1} = -\alpha A_n + \beta B_n$ and $B_{n+1} = -\alpha B_n - \beta A_n$ such that $y^{(n+1)}(t) = e^{-\alpha t}(A_{n+1}\cos(\beta t) + B_{n+1}\sin(\beta t))$. ■

Let $E(c)$ and $E(k)$ be the error in using the system in 25 to approximate c and k, respectively. Then $E(c) = \tilde{c} - c$ and $E(k) = \tilde{k} - k$. Using Lemma 1 and 2, we can show that $E(c)$ and $E(k)$ have order-two convergence. To state this result, we will define $g(h)$ and $q(h)$ to be the denominators of $E(c)$ and $E(k)$, respectively:

$$\begin{aligned} g(h) &= h(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0)); \\ q(h) &= h^2(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0)). \end{aligned}$$

Since $g^{(4)}(h)$ and $q^{(5)}(h)$ will be combinations of derivatives of $y(t)$, it follows from Lemma 1, Lemma 2, and the Triangle Inequality that there exist constants M_4 and M_5 such that $|g^{(4)}(h)| \leq M_4$ and $|q^{(5)}(h)| \leq M_5$ for all $h \in [0, \infty)$.

Theorem 4 *Suppose $y(t)$ is a function of the form in 17 and that it solves equation 18. Let $\tilde{c}$ and $\tilde{k}$ be approximations to the constants c and k, as in 25. If*

$$h < \min\left[\frac{2\beta^2(A^2+B^2)4!}{M_4}, \frac{2\beta^2(A^2+B^2)5!}{M_5}\right],$$

then there exist constants $\kappa, \sigma > 0$ such that

$$|E(c)| \leq \kappa(h^2)$$

and

$$|E(k)| \leq \sigma(h^2).$$

Proof. From 25 we get

$$\begin{aligned} |E(c)| &= |\tilde{c} - c| \\ &= |2(-y(t_1)^2 - y(t_1)y(t_3) + y(t_2)^2 + y(t_2)y(t_0)) \\ &\quad -c(h(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0)))| \\ &\quad /|h(y(t_1)y(t_3) - y(t_1)^2 - y(t_2)^2 + y(t_2)y(t_0))|. \end{aligned}$$

Using Taylor's Theorem [5, p.126] we can rewrite this as

$$\begin{aligned} |E(c)| &= \left|\frac{\frac{4}{3}\alpha^3\beta^2(A^2+B^2)h^5 + \frac{f^{(6)}(d_1)}{6!}h^6}{-2\beta^2(A^2+B^2)h^3 + \frac{g^{(4)}(d_2)}{4!}h^4}\right| \\ &= \left|\frac{h^5\left(\frac{4}{3}\alpha^3\beta^2(A^2+B^2) + \frac{f^{(6)}(d_1)}{6!}h\right)}{h^3\left(-2\beta^2(A^2+B^2) + \frac{g^{(4)}(d_2)}{4!}h\right)}\right| \\ &= h^2\left|\frac{\frac{4}{3}\alpha^3\beta^2(A^2+B^2) + \frac{f^{(6)}(d_1)}{6!}h}{-2\beta^2(A^2+B^2) + \frac{g^{(4)}(d_2)}{4!}h}\right| \end{aligned}$$

where $d_1, d_2 \in (0, h)$ and $f(h)$ and $g(h)$ are the numerator and denominator of $E(c)$, respectively.. Using Lemma 1 and Lemma 2, we know that there exists some max, M_6, such that $\left|f^{(6)}(h)\right| \leq M_6$ on the interval $[0, \infty)$. Likewise there exists a maximum, M_4, such that $\left|g^{(4)}(h)\right| \leq M_4$ on $[0, \infty)$. Using the Triangle Inequality on the numerator and the Reverse Triangle Inequality [5, p.5] on the denominator,

$$
\begin{aligned}
|E(c)| &\le h^2 \left| \frac{\left|\frac{4}{3}\alpha^3\beta^2(A^2+B^2)\right| + \left|\frac{M_6}{6!}h\right|}{\left|2\beta^2(A^2+B^2)\right| - \left|\frac{M_4}{4!}h\right|} \right| \\
&\le h^2 \left| \frac{\frac{4}{3}\alpha^3\beta^2(A^2+B^2) + \frac{M_6}{6!}h}{2\beta^2(A^2+B^2) - \frac{M_4}{4!}h} \right| \\
&\le h^2(\kappa).
\end{aligned}
$$

where $\kappa \ge \max\limits_{[0,h]} \left| \frac{\frac{4}{3}\alpha^3\beta^2(A^2+B^2)+\frac{M_6}{6!}h}{2\beta^2(A^2+B^2)-\frac{M_4}{4!}h} \right|$.

The proof that $E(k) \le h^2(\sigma)$ is similar. ∎

4. A Numerical Example

We now turn our attention towards an application of our system for approximating m, c, and k. We choose the following harmonic oscillator for consideration:

$$4y'(t) + 5y(t) = -y''(t). \tag{26}$$

Since $4 < 2\sqrt{5}$, we know that the oscillator is underdamped, and we find that $\alpha = -2$ and $\beta = 1$. It is easy to show that the following particular solution will solve equation 26:

$$y(t) = e^{-2t}\cos(t) + 2e^{-2t}\sin(t). \tag{27}$$

So we assume that we are taking position measurements of the oscillator whose equation is 27 at a fixed interval of h apart in time. After taking four measurements of $y(t)$, we construct a system like that in 25 using our position measurements and value of h. However, since we have the solution, we can simply find $y(t)$ at four distinct times that occur h apart. Starting at $t_0 = 0$ with $h = 0.1$ and an accuracy of fifteen digits we find $\tilde{c} = 3.94751$ and $\tilde{k} = 4.91391$ where

i	t_i	$y(t)$
0	0	1
1	0.1	0.978113886
2	0.2	0.923302344
3	0.3	0.848669638

Our approximation seems to work well. From equation 26, we see that $c = 4$ and $k = 5$, and the approximations are near these values. Since there is order-two convergence to c and k, approximations should become much better as $h \to 0$. Using the same system with $h = 0.005$ the error decreases to less than 1×10^{-3} in both $\tilde{c}$ and $\tilde{k}$.

5. Further Questions

Notice that constructing our approximations as we have results in two approximate equations (see 24) that become increasingly similar to each other as $h \to 0$. Perhaps we can approximate c and k better by creating approximations as in 22 that are centered around two non-adjacent times, or two times that differ by more than $2h$. If we construct our approximate equations, using six measurements of $y(t)$ centered about our non-adjacent $t_i's$, will our approximations be more accurate than those produced by our original system?

It turns out that constructing our system in this manner does not offer anything over our original system for approximation. We can show that the approximations found with such a system approach c and k at order-two convergence as $h \to 0$. Because our original system requires fewer measurements of $y(t)$ and offers the same rate of convergence, we find that the original method of approximation is more efficient.

We are also interested in whether or not using more than two equations to approximate c and k will offer advantages. Notice that when the number of equations in system 19 exceeds two, the system becomes overdetermined. Therefore the least-squares method must be introduced to find the $\tilde{c}$ and $\tilde{k}$ that best fit our multiple equations. Since further approximation is required as the number of equations in our system increases, it seems that our approximations will become less accurate as more equations are used. Using numerical calculations, we conjecture that this method was also less attractive than our original system of approximation. Not only did the overdetermined system require more equations, but it also produced less accurate approximations.

We have constructed a system that will approximate constants for a horizontally-aligned oscillator, or those modeled by a homogeneous differential equation. However, it seems that with a little modification, our system could be used to approximate constants for a vertical spring-mass system that is modelled with a non-homogeneous differential equation, such as $my''(t) + cy'(t) + ky(t) = w$, where w is a constant. Notice that such a system will require three equations constructed from measurements of $y(t)$, but it will be able to solve for m, in addition to c and k, because it is non-homogeneous.

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Primes and Primality Testing: A Technological/Historical Perspective

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1. Introduction

A number p greater than one is prime if its only divisors are 1 and itself. Although the concept of primality is easily understood by grade school-aged children, the theory behind prime numbers has meant a lifetime of (often fruitless) labor for a number of mathematicians since at least the third century B.C. The Pythagoreans believed that primes had some sort of mystical property and studied these primes as keys to unlocking the mysteries of the universe ([5]). Today, mathematicians study primes as a chiefly recreational interest, even though primes do have modern application through methods of data encryption. See [4] for more information.

Throughout history, methods for primality testing have evolved within the constantly developing field of number theory. In present times, with the use of computers, primality testing has been transformed into a number of highly technical algorithms. These algorithms are not unlike the traditional methods used before the information age: they are merely more efficient, streamlined, and automated versions of their predecessors. How efficient are these primality tests when compared to the original tests used throughout history? More specifically, when using a particular computer algebra system, such as *Mathematica*, what are the limitations of such primality tests when the performance of such tests is limited to functions using only existing *Mathematica* commands and evaluating these tests on commonly available equipment?

2. Early Results

The first known test for determining primality was given by Eratosthenes around 200 B.C. To find all prime numbers less than n, the test removed the nonprime numbers (less than n) in the following way:

List all numbers from 2 to n. Begin with the number 2. For all numbers larger than 2, cross out those having 2 as a factor (i.e. 4, 6, 8, 10, etc.) up to n. Next, find the first number larger than 2 that is not crossed out, namely 3. Cross out all multiples of 3, up to n. Continue in such a way until reaching $\sqrt{n}$. Because any number greater than $\sqrt{n}$ must necessarily be multiplied by a number smaller than $\sqrt{n}$ in order for its product to be less than n, we know that we have eliminated all numbers less than n that are composite. (`http://www.utm.edu/research/primes`)

This "sieving" method is one of the processes used today (in modified form) to determine the primality of a special type of number, called a Mersenne number, by the Great Internet Mersenne Prime Search (`www.mersenne.org/math.htm`). This test can be easily duplicated using *Mathematica*, although its evaluation for large numbers having unknown primality would be nearly impossible without some advanced programming background.

How do we know that the list of primes will never end? The infinitude of the primes has been proven in many different ways by many different mathematicians, including Euclid (whose proof is most well-known), Washington, Schorn, Euler, and Kummer. Kummer's proof reads as follows:

Suppose there exists only a finite number of primes p_n, where

$$p_1 < p_2 < \cdots < p_r.$$

Let N be the product

$$p_1 p_2 \cdots p_r.$$

Since $N > 2$, the integer $N-1$, being the product of primes, will have as a factor at least one prime p_i; this prime must be a factor of N as well, since N is the product of all possible primes. Since p_i divides both N and $N-1$, it necessarily divides their difference, so p_i divides $N-(N-1)=1$. This is clearly impossible! Thus the number of primes is infinite. ([7], p. 4)

Because there exist infinitely many primes, the discovery of new primes will continue regardless of how many primes have already been found, provided that available technology is capable of considering increasingly large numbers. The largest known prime number to date, for example, has 7,235,733 digits (`www.mersenne.org`)!

Although the sieve of Eratosthenes is efficient and useful in looking at the broad scheme of prime numbers, in order to test a particular large number n for primality, this test would require a greater period of time than is available in any one person's life. It is necessary, then, to find and use tests for which a particular number n can be proven prime without the need to consider all numbers less than $\sqrt{n}$ as potential factors. Among these tests are Pepin's test for Fermat numbers, the primality test for Mersenne numbers, and the Lucas Test. Each one of these will be examined in detail in the following pages.

3. Congruences and Quadratic Residues

Two concepts fundamental to the understanding of these three primality tests are modular congruence and quadratic residues. We say that a is *congruent* to b modulo m, denoted by

$$a \equiv b (\operatorname{mod} m),$$

if and only if

$$a - b = km$$

for some integer k. For example, $7 \equiv 2 (\operatorname{mod} 5)$, since $7 - 2 = (1)(5)$. On the other hand, $5 \equiv 7 (\operatorname{mod} 8)$ can not possibly be true since there does not exist an integer k such that $5 - 7 = k(8)$. The computer algebra system *Mathematica* uses the command `Mod[a,n]` to solve $x \equiv a (\operatorname{mod} n)$; that is, `Mod[a,n]` gives an integer x, where $0 \leq x < n$ and

$$a - kn = x$$

for some integer k. When dealing with a raised to some power, it is more efficient to use the command `PowerMod[a,k,n]`, which solves

$$x \equiv a^k (\operatorname{mod} n).$$

The other concept fundamental to the understanding of the three primality tests being examined, the quadratic residue, is defined in the following way: If the greatest common divisor of a and p is 1 (that is, a and p are relatively prime), and if there exists an integer b such that

$$a \equiv b^2 (\operatorname{mod} p),$$

then a is a *quadratic residue* modulo p. (More details on quadratic residues can be found in [2], p. 90.) Having established these preliminary notions, we may now examine the primality tests.

4. Pepin's Test

Fermat numbers are special numbers of the form $2^{2^n} + 1$, where n is a natural number. Some examples of Fermat numbers are 3, 5, 17, 257, and 65537. In 1877, Pepin provided a method for determining the primality of Fermat numbers; the following is an adaptation of his test ([6], p. 71):

Let $F_n = 2^{2^n} + 1$ for some $n \geq 2$. Then, for $k \geq 2$, the following are equivalent:

i)F_n is prime and $L(k, F_n) = -1$, and

ii)$k^{(F_n - 1)/2} \equiv -1 \, (\mathrm{mod}\, F_n)$,

where $L(k, F_n)$ is equivalent to the Legendre symbol $\left(\dfrac{k}{F_n}\right)$; that is,

$$L(a, b) = \begin{cases} 1 & \text{if } a \text{ is a quadratic residue modulo } p \\ -1 & \text{otherwise} \end{cases}.$$

The L-notation is used here to distinguish it from the common operation division. It is not imperative that one has a full understanding of the Legendre symbol in order to execute Pepin's test. We need only concern ourselves with ii) of Pepin's test, since ii) is equivalent to i). Let us evaluate, by hand, Pepin's Test for $F_2 = 17$. Let us try $k = 3$. Then

$$3^{(17-1)/2} = 3^8 = 3^2 \cdot \left(3^3\right)^2.$$

But

$$3^2 = 9 \equiv 9 (\mathrm{mod}\, 17),$$

and

$$3^3 = 27 \equiv 10 (\mathrm{mod}\, 17),$$

so that

$$\left(3^3\right)^2 \equiv 10^2 = 100 \equiv 100 - 6 \cdot 17 \equiv -2 (\mathrm{mod}\, 17).$$

Thus,

$$3^{(17-1)/2} \equiv 9 \cdot (-2) = -18 \equiv -1 (\mathrm{mod}\, 17).$$

So $F_2 = 17$ is prime. The largest known prime Fermat number is $F_4 = 65537$. Using *Mathematica*, it is easy to show that 65537 is prime by taking $k = 3$:

```
Fer[n_] := 2^(2^n) + 1;
PowerMod[3, (Fer[4] - 1)/2, Fer[4]] == Fer[4] - 1
```

Mathematica's response:

```
True
```

We have therefore verified with reasonable certainty that $F_4 = 65537$ is prime. This was originally done by Fermat himself in the 17th century, long before the advent of the computer ([5]). What about Fermat numbers larger than F_4? Our test can be evaluated up to F_{19}, but its usefulness is limited, since we already know that all of the Fermat numbers up to F_{19} are composite. (See the Appendix for more information on this and other results from tests in *Mathematica*.) Is our test able to show that these composite Fermat numbers are, indeed, composite? Due to a property of the Legendre symbol as it relates to Fermat numbers, the latter part of i) is always true using $k = 3$. As a result, generating "False" on part ii) implies that the given Fermat number is composite. A major limitation of Pepin's test is that it merely asserts that a given Fermat number is prime or composite; it does not give the factorization of composite Fermat numbers ([7], p. 62). Let us then, move on to the Mersenne numbers.

5. Mersenne Numbers

A Mersenne number is of the form $2^n - 1$ for some known prime number n. The first 10 Mersenne numbers are: 3, 7, 31, 127, 2047, 8191, 131071, 524287, 8388607, and 536870911. Interest in Mersenne numbers stemmed from mathematics involving perfect numbers ([7], p. 65). A perfect number is a number that is equal to the sum of its divisors (excluding itself). An example of a perfect number is

$$28 = 1 + 2 + 4 + 7 + 14.$$

A theorem proven partially by Euclid and partially by Euler gives: A number n is perfect if and only if it has the form $n = 2^{q-1}M_q$ for some q, where $M_q = 2^q - 1$ is prime (i.e. a Mersenne prime) ([2], p. 22). The most efficient algorithm for determining primality of a given Mersenne number was given by Lucas and later modified by Lehmer. Although the test itself has limited usefulness (it cannot give factors of composite Mersenne numbers), it is the best test currently available for determining primality of Mersenne numbers.

The test for primality of Mersenne numbers can be stated in the following way:

Let $P = 2$, $Q = -2$, and (U_m), (V_m), $m \geq 0$ be the associated Lucas sequences, having the discriminant $D = 12$, where the discriminant is $D = P^2 - 4Q$. Then $N = M_n$ is a prime if and only if N divides $V_{(n+1)/2}$.

Lucas numbers are given by the recursive sequence $\{v_n\}$, where

$$v_n = v_{n-1} + v_{n-2}$$

and

$$v_1 = 1, v_2 = 3.$$

However, it is not necessary to know the particulars of the Lucas sequences; for ease of calculation, the sequence can be replaced by the following:

$$\begin{aligned} s_0 &= 4 \\ s_k &= (s_{k-1})^2 - 2. \end{aligned}$$

Here are the first few terms of the new sequence:

$$\begin{aligned} s_0 &= 4 \\ s_1 &= 4^2 - 2 = 14 \\ s_2 &= 14^2 - 2 = 194 \\ s_3 &= 194^2 - 2 = 37634. \end{aligned}$$

The theorem is then restated as follows: $M_n = 2^n - 1$ is prime if and only if M_n divides s_{n-2}. Examine the Mersenne number $M_5 = 2^5 - 1 = 31$. The associated term of the sequence s_n is $s_3 = 37634$. Does 31 divide 37634? Yes, since $1214 \cdot 31 = 37634$. Thus 31 must be prime. Although this test is fairly simple computationally for small Mersenne numbers, such as M_5, it is difficult to compute values of the sequence s_{n-2} as n increases. Here are the next couple of terms of s_n:

$$\begin{aligned} s_4 &= 37634^2 - 2 = 1416317954 \\ s_5 &= 1415317954^2 - 2 = 2005956546822746114. \end{aligned}$$

Imagine squaring s_5 by hand! It is easy to see here that *Mathematica* will be very valuable in calculating the terms of this sequence. One way to perform the primality test for Mersenne numbers in *Mathematica* is the following:

```
s[0]:=4;
s[n]:=(s[n-1])^2-2;
m[k_]=2^k-1;
MerTest[n_]:=Mod[s[n-2],m[n]]==0.
```

Evaluated at M_5, this test is executed in the following way:

```
k=5;
MerTest[5]
True
```

The `MerTest` function works very well for Mersenne numbers up to

$M_{31} = 2^{31} - 1$; however, at M_{31}, this test fails to evaluate because of the large size of s_{29}. Thus it is necessary to modify our test to the following:

```
m[k_]=2^k-1;
s[0]= 4;
s[n_]:= Mod[s[n - 1]^2-2,m[k]]
s[k-2]== 0.
```

This modification allows *Mathematica* to evaluate the `Mod` function as it is calculating the terms of the recursive sequence, making the algorithm simpler by limiting the size of the terms of s_n. Not only can this new test determine the primality of M_{31}; (upon modification of a pre-set limit called the recursion limit) it can determine the primality of Mersenne numbers up to M_{11213}, the largest known Mersenne prime until 1971! When given M_{19937}, the largest known Mersenne prime until 1978, the test fails to evaluate. The kernel automatically quits, and *Mathematica* gives no output. Calculating the value of M_{19937}, however, is no simple task; M_{19937}, remember, is $2^{19937} - 1$, a massive number thousands of digits long! In addition, the associated term of the sequence s_n has to be calculated recursively; finding s_{19935} requires calculation of the first 19935 terms of our original recursive sequence and then finding x, $0 \leq x < M_{19937}$ such that $x \equiv s_{19935} (\operatorname{mod} M_{19937})$.

6. The Lucas Primality Test

The final primality test examined here is the Lucas test for primality. A major advantage of this test is that, unlike the previous two tests, the Lucas test does not only apply to one particular type of number; any number of unknown primality can be used in this test. In order to understand the workings of this test, it is necessary to understand an important theorem by Pierre de Fermat, known as Fermat's Little Theorem:

If p is a prime and a is an integer, then $a^p \equiv a (\operatorname{mod} p)$. In particular, if p does not divide a, then $a^{p-1} \equiv 1 (\operatorname{mod} p)$ ([7], p. 13).

The Lucas test for primality is a modification of Fermat's Little Theorem. It was developed by Lucas in 1891. It reads as follows:

Let $N > 1$. Assume that there exists some integer $a > 1$ where

i)$a^{N-1} \equiv 1 (\operatorname{mod} N)$, and

ii)a^m is not congruent to $1 (\operatorname{mod} N)$ for every $m < N - 1$ such that m divides $N - 1$.

Then N must be prime.

Let us evaluate Lucas' test for $N = 7$ (which we already know is prime). Choose $a = 3$. Then

$$3^{7-1} = 729 \equiv 729 - 7 \cdot 100 = 29 \equiv 1 (\bmod 7).$$

Now, for part ii) of the test: We must verify that for any divisor q of 6, a^q is not congruent to $1(\bmod 7)$. Now,

$$3^2 = 9 \equiv 2 (\bmod 7)$$

and

$$3^3 = 27 \equiv -1 (\bmod 7).$$

We have just verified that 7 is prime. Writing a program in *Mathematica* to perform the Lucas test is fairly simple. Let us attempt to determine the primality of 71887 using $a = 3$:

```
n=71887;
LTest[a_,n_]:=Mod[a^(n-1),n]=1
LTest[3,71887]
True
```

We have just verified i). Now, for ii), we must show that for any m that divides $N - 1$, a^m is not congruent to 1. To do this, we must know the factors of $N - 1$; we can factor $N - 1$ with *Mathematica* using the `FactorInteger` command:

```
FactorInteger[71886]
{{2,1},{3,1},{11981,1}}
```

The factors of 71886 are 2, 3, and 11981. Now for ii), we should perform a modified version of the `LTest` on each of 2, 3, and 11981, and on every possible combination of the three. However, this process becomes more and more time-consuming as n increases. Let's consider, then, an abbreviated approach – the Lucas test as modified by Brillhart & Selfridge ([7], p.32):

A number n is prime if there exists an integer a such that a^{n-1} is congruent to $1(\bmod n)$, where $a^{(n-1)/q}$ is not congruent to $1(\bmod n)$ for any prime q that divides $n - 1$.

Then we only need to test each of p, q, and r using our modified Lucas test and $a = 3$:

```
LTest2[a_,n_,k_]:=Mod[a^k,n]=1
p=2;
q=3;
r=11981;
LTest2[3,n,(n-1)/p]
LTest2[3,n,(n-1)/q]
LTest2[3,n,(n-1)/r]
False
False
False
```

Thus 71887 is prime. Although this test has a major advantage in that it can be performed on any positive integer greater than one, the Lucas test has two major shortcomings. The first is that the prime factors of $n - 1$ must be known. Determining these factors is difficult without the use of the *Mathematica* command `FactorInteger`, albeit possible with an infinite span of time in which to work. Actually using `FactorInteger` makes the assumption that this command is definitely accurate for large values of n. In addition, the limitations of the `FactorInteger` command itself limit the capabilities of our `LTest`.

Another major problem is that of having to choose a specific a to run the test. Because it is not possible to test all potential values of a, there is not a specific point after a number of repetitions of the Lucas test, that one can say, "This number is definitely composite." Of course, after attempting a few thousand different choices of a without being able to determine the primality of a, it is a valid assumption that n is very likely prime. Again, however, we are bound by the constraints of time. Even performing the Lucas test for a few thousand choices of a and checking to see if each prime factor q of $n - 1$ resulted in

$$a^{(n-1)/q} \equiv 1 (\bmod n)$$

would be a laborious task for a programming novice.

Nonetheless, using our `LTest` and `LTest2`, we are able to show that the largest known prime until 1951, $n = 2^{127} - 1$, is indeed prime. (Note that $2^{127} - 1$ also happens to be a Mersenne number.) To do so, we must use the `PowerMod` command instead of the `Mod` command. This is the new test:

```
n=2^127 - 1;
LTest3[a_,c_,n_]:=PowerMod[a,c,n]==1
LTest3[2,n-1,n]
True
```

Of course, then we must test all factors of $n-1$ to see if ii) holds. The prime factors of $n - 1$ are 2, 3, 7, 19, 43, 73, 127, 337, 5419, 92737, 649657, and 77158673929. When `LTest3` is performed, however, it turns out that our choice of $a = 2$ makes the test inconclusive. In addition, the use of $a = 4, 5, 7, 8, 9, 10, 11, 13, 17$, and 19 also makes the test inconclusive. In the end, as often happens in mathematics, a seemingly arbitrary choice of $a = 127$ actually showed that $2^{127} - 1$ is prime.

The Lucas test is less restricted in its list of potential primes than are the other two tests. It is actually capable of testing the primality of both Mersenne numbers and Fermat numbers. The actual worth of these evaluations, however, is questionable. Our version of Lucas' test can be evaluated for large Mersenne numbers, at least using $a = 2$. Attempting to evaluate the Lucas test for Mersenne numbers using larger values of a results in our function continuing to evaluate over a span of days, and still with no results. In addition, in order to know the prime factors of $n - 1$, we must either compute them by hand, or rely on the `FactorInteger` command. The `FactorInteger` command itself is limited in its capacity; attempting to use this command to factor causes *Mathematica* to run for days on end with no results. As previously mentioned, our Lucas test is also capable of testing the primality of Fermat numbers. Using our test in *Mathematica*, we are capable of performing the Lucas test up to F_{28}. The results of performing these tests, however, are inconclusive. Because the Lucas test is incapable of guaranteeing that a given number is composite, evaluation of the Lucas test for Fermat numbers larger than F_4 carries very little meaning; the Lucas test is not even capable of verifying what is already known – that the Fermat numbers greater than F_4 and less than F_{28} are all composite!

7. Project GIMPS

One modern attempt to find large unknown primes is the Great Internet Mersenne Prime Search, also known as project GIMPS. As mentioned previously, project GIMPS uses a modified sieve of Eratosthenes along with the test for Mersenne numbers mentioned previously, and a method developed by Pollard for testing potential factors. The future of the theory of prime numbers seems to lie within project GIMPS. Indeed, the largest known prime to date, the 41st Mersenne number, was found by a participant in project GIMPS. Anyone with the proper computing equipment can join project GIMPS and join the search for new large primes! For more information, see `www.mersenne.org`.

8. Conclusion

Although all three tests examined have proven useful and somewhat easily programmable in *Mathematica* using our given resources, each has its strengths and its shortcomings. Pepin's test is able to handle thousands of digits, but it is unable to actually factor a composite Fermat number. The test for Mersenne numbers works quickly for Mersenne numbers and is able to verify the primality of record Mersenne primes up to 1978; however, it depends on the recursively defined sequence s_n, which eventually fails to evaluate due to an internal memory specification. The Lucas Test, on the other hand, is able to evaluate the primality of any type of positive integer; it works well for Fermat numbers and Mersenne numbers as well, up to a point. The major shortcoming of the Lucas test with respect to the Mersenne numbers is finding the prime factors of $N - 1$. Moreover, while the Lucas test can be evaluated for larger Fermat numbers than Pepin's test, the results are inconclusive and thus of limited usefulness. Although Pepin's test can actually be evaluated for larger values than the Mersenne test, the results we obtained using Pepin's test carry little meaning as well; thus the most useful and efficient test, given current commonly available resources, is the Mersenne test.

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The Problem Corner

Edited by Pat Costello and Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before January 1, 2007. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring, 2007 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)622-3051)

CONTINUING PROBLEMS 585, 587, 589

Problem 585. *(Corrected) Proposed by José Luis Diaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Suppose that the roots $z_1, z_2, ..., z_n$ of

$$z^n + a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \cdots + a_1 z^1 + a_0 = 0$$

are in arithmetic progression with difference d. Prove that

$$d^2 = \frac{12\left[(n-1)a_{n-1}^2 - 2na_{n-2}\right]}{n^2(n^2-1)}.$$

Problem 587. *Proposed by José Luis Diaz-Barrero, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Show that if A, B, C are the angles of a triangle, and a, b, c its sides, then

$$\prod_{\text{cyclic}} \sin^{1/3}(A-B) \le \sum_{\text{cyclic}} \frac{\left(a^2+b^2\right)\sin(A-B)}{3ab}.$$

Problem 589. *Proposed by the editor.*

Find a, b, c, d, and e so that the number

$$a8b2cd7e3$$

is divisible by both 73 and 137, where a, b, c, d, and e are distinct integers chosen from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $a > 0$.

NEW PROBLEMS 597-603

Problem 597. *Proposed by Bangteng Xu, Eastern Kentucky University, Richmond KY.*

Determine the following limit.

$$\lim_{n\to\infty} \frac{1 \cdot n + 3\,(n-1) + 5\,(n-2) + \cdots + (2n-3) \cdot 1}{n^3}$$

Problem 598. *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA.*

Let C be a unit circle centered at the point $(3, 4)$. Let $O = (0, 0)$, and let $A = (1, 0)$. Let P be a variable point on C, and let $a = PA$ and $b = PO$. Find a non-constant polynomial $f(x, y)$ such that

$$f(a, b) = 0$$

for all points P on C.

Problem 599. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, MO.*

Primes of the form $3n^2 + 3n + 1$ are called *Cuban primes*. Find necessary and sufficient conditions for $3n^2 + 3n + 1$ to be divisible by 7.

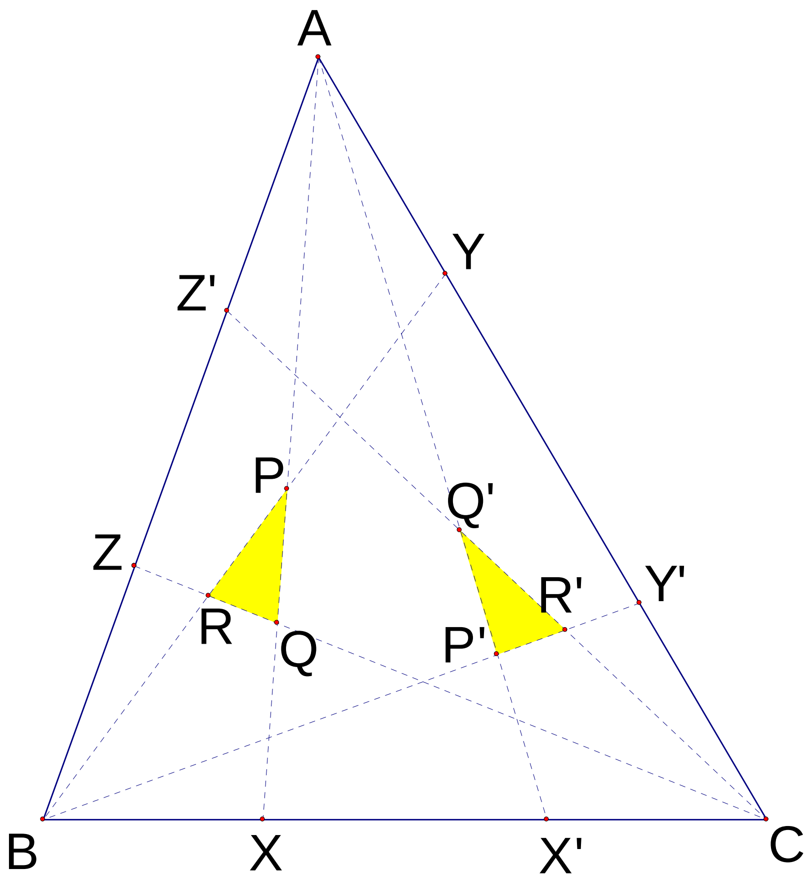
Problem 600. *Proposed by Stanley Rabinowitz, MathPro Press, Chelmsford, MA.*

In ΔABC, let X, Y, and Z be points on sides BC, CA, and AB, respectively. Let

$$x = \frac{BX}{XC},\ y = \frac{CY}{YA},\ \text{and } z = \frac{AZ}{ZB}.$$

The lines AX, BY, CZ bound a central triangle PQR. Let X', Y', and Z' be the reflections of X, Y, and Z, respectively, about the midpoints of the sides of the triangle upon which they reside. These give rise to a central triangle $P'Q'R'$. Prove that the area of ΔPQR is equal to the area of $\Delta P'Q'R'$ if and only if either

$$x = y \text{ or } y = z \text{ or } z = x.$$

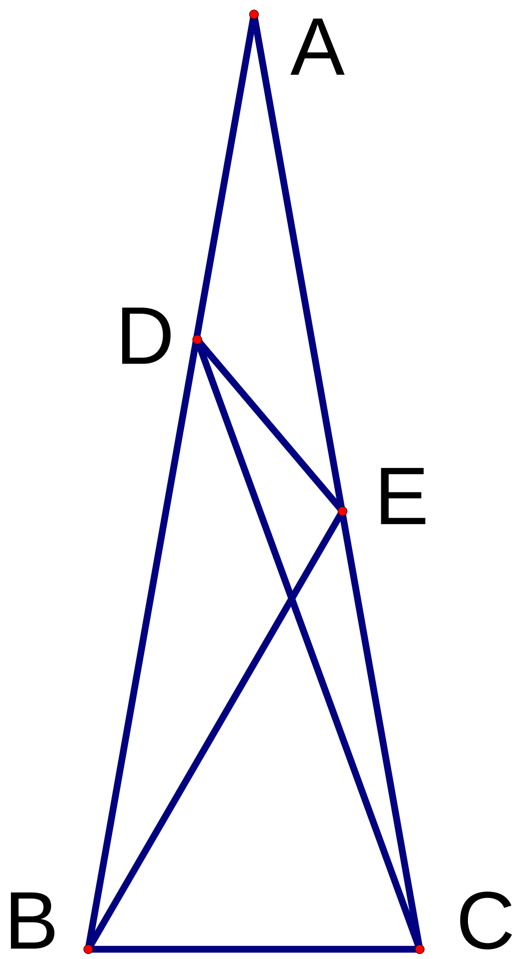


Problem 601. *Proposed by Johannas Winterink, Albuquerque, NM.*

You are given the following information about the drawn triangle:

- Point A, D, and B are collinear;
- Points A, E, and C are collinear;
- $\angle DAE = 20°$, $\angle ADE = 130°$, $\angle AEB = 140°$, $\angle ADC = 150°$.

Prove that $AB = AC$.



Problem 602. *Proposed by the editor.*

Consider the sequence of polynomials defined recursively by

$$\begin{cases} p_1(x) = (x-2)^2 \\ p_2(x) = [p_1(x)-2]^2 \\ \vdots \\ p_n(x) = [p_{n-1}(x)-2]^2 \end{cases}.$$

If

$$p_n(x) = x^{2n} + a_{2n-1}x^{2n-1} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_1x + 4,$$

find closed formulas for the coefficients a_{2n-1}, a_{2n-2}, a_2, and a_1.

Problem 603. *Proposed by the editor.*

Consider the following variant on Pascal's triangle. Start with the top two rows the same as in Pascal's triangle. For the remaining rows, put 1 at each end. For each interior entry, add the two diagonal values above the position plus the value in the row above which is between the two summed values. This means an interior entry is a sum of values in the equilateral triangle above the position. Rows 0, 1, 2, 3, and 4 of the triangle are:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 3 & & 1 & & \\ & 1 & & 5 & & 5 & & 1 & \\ 1 & & 7 & & 13 & & 7 & & 1 \end{array}.$$

One notable fact about this triangle is that all entries in the triangle are odd. Another fact is that the second diagonal is the set of odd numbers. Find a closed formula for the sum of the entries across the m^{th} row.

Please help your editor by submitting problem proposals.

SOLUTIONS 579, 586, 588

A late solution for Problem 575 was received from David Ritter, student, Messiah College, Grantham, PA.

Problem 579. *Proposed by M. Khoshnevisan, Griffith University, Gold Coast, Queensland, Australia. (Corrected)*

A Generalized Smarandache Palindrome (GSP) is a concatenated number of the form

$$a_1a_2\cdots a_na_n\cdots a_2a_1$$

or

$$a_1a_2\cdots a_{n-1}a_na_{n-1}\cdots a_2a_1,$$

where $a_1, a_2, ..., a_n$ are positive integers of various numbers of digits. Find the number GSP of four digits which are not palindromic numbers.

Solution *by Carl Libis, University of Rhode Island, Kingston, RI.*

The GSP of four digits that are not palindromic numbers are either of the form a_1a_1, where a_1 is a two-digit number or of the form $a_1a_2a_1$, where a_2 is a two-digit number. First we calculate the GSP of four digits that are not palindromic numbers of the form a_1a_1, where a_1 is a two-digit number. Of the ninety two-digit numbers between 10 and 99, there are eighty-one two-digit numbers not including 11, 22, 33, 44, 55, 66, 77, 88 and 99. Thus there are eighty-one GSP of form a_1a_1 consisting of four digits and which are not palindromic. Next we calculate the GSP of four digits that are not palindromic numbers of the form $a_1a_2a_1$, where a_2 is a two-digit number. There are one hundred numbers a_2 between 00 and 99. Ninety of them are not palindromic numbers. Since there are nine choices for a_1, there are 810 GSP of the form $a_1a_2a_{12}$ having four digits which are not palindromic numbers. Combining these results, there are 891 GSP of four digits which are not palindromic numbers.

Also solved by Russell Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri, and the proposer. One incorrect solution was received.

Problem 586. *Proposed by Pat Costello, Eastern Kentucky University, Richmond, KY.*

Let f_n denote the n^{th} Fibonacci number: i.e.,

$$f_1 = 1, f_2 = 1,$$

and for all integers $n > 2$,

$$f_n = f_{n-1} + f_{n-2}.$$

Find the exact value of the infinite series

$$\sum_{n=1}^{\infty} \frac{f_n (\bmod 3)}{3^n}.$$

Solution *by Martina Bray, student, Eastern Kentucky University, Richmond, KY. (Revised by the editor.)*

Looking at the residues of the first several terms of the Fibonacci series $(\bmod 3)$, we find the cycle of eight numbers

$$1, 1, 2, 0, 2, 2, 1, 0$$

which repeats indefinitely. Thus the sum can be found by grouping. Since

$$F_{1+8t} \equiv F_1 \equiv 1 (\bmod 3),$$

one can use the formula for the sum of an infinite geometric series $S = \frac{t_1}{1-r}$, with $t_1 = \frac{1}{3}$ and $r = \frac{1}{3^8}$ to get $S_1 = \frac{3^7}{3^8 - 1}$, where S_1 denotes the sum of the terms of the form $\left(F_{1+8t}/3^{1+8t}\right)$. Proceeding similarly, we obtain $S_2 = \frac{3^6}{3^8-1}$, $S_3 = \frac{2 \cdot 3^5}{3^8-1}$, $S_4 = 0$, $S_5 = \frac{2 \cdot 3^3}{3^8-1}$, $S_6 = \frac{2 \cdot 3^2}{3^8-1}$, $S_7 = \frac{3^1}{3^8-1}$, and $S_8 = 0$. Thus the desired sum is

$$S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8 = \frac{3477}{6560}.$$

Also solved by Matthew Glenn Dawson, student, Union University, Jackson, TN; Clayton W. Dodge, University of Maine, Orono, ME; Ceyhun Ferik, student, Eastern Kentucky University, Richmond, KY; Yongbok Lee, student, Eastern Kentucky University, Richmond, KY; April Spears, student, Eastern Kentucky University, Richmond, KY, and the proposer.

Problem 588. *Proposed by the editor.*

Prove that the first 2005 digits after the decimal point in the decimal expansion of

$$\left(7+\sqrt{48}\right)^{2005}$$

are nines.

Solution *by Messiah College Problem Solving Group, Messiah College, Grantham, PA.*

Let $\alpha = 7+\sqrt{48} = 7+4\sqrt{3}$. First we note that

$$\frac{1}{\alpha} = 7-4\sqrt{3} < 0.1.$$

By the Binomial Theorem, we know that

$$\alpha^{2005} = x + y\sqrt{3}$$

and

$$\alpha^{-2005} = \left(7-4\sqrt{3}\right)^{2005} = x - y\sqrt{3},$$

where x and y are positive integers. So we have

$$0 < \left(7-4\sqrt{3}\right)^{2005} = \alpha^{-2005} < 10^{-2005},$$

that is,

$$0 < x - y\sqrt{3} < 10^{-2005}.$$

Thus

$$0 < 2x - \left(x+y\sqrt{3}\right) < 10^{-2005},$$

i.e.,

$$0 < 2x - \alpha^{2005} < 10^{-2005}.$$

Since $2x$ is an integer, this establishes our desired result. The integer part of α^{2005} is $2x-1$, and the fractional part is greater than $0.9999...0$ (2005 nines).

In Memoriam: Merle Mitchell

At its November, 2005 initiation ceremony, the New Mexico Alpha Chapter of Kappa Mu Epsilon at the University of New Mexico paid tribute to retired colleague Professor Merle Mitchell, presenting her with a plaque in recognition for her lifetime work in mathematics and in KME. Merle was the first woman professor of mathematics at UNM and had a lifetime involvement with KME. The chapter has minutes of hers from KME meetings held at UNM going as far back as 1944. Merle also knew one of the first presidents of KME, C. V. Newsom.

At the time Merle received the plaque she was eighty-four years old and in perfect health of body and mind. Merle was very happy to receive this recognition. Sadly, Merle passed away on February 28, 2006 due to a lymphoma. At the time, she was visiting her sister in Atlanta, Georgia.

Thank You Referees!

The editor wishes to thank those individuals who refereed papers submitted to *The Pentagon* during the last two years. Since the number of refereed papers was small, the names of referees will not be listed, so that the anonymity of the reviewing process can be maintained. Thanks also to the many other individuals who volunteered to serve as referees but were not used during the past two years. Referee interest forms will be sent again soon so that interested faculty may volunteer. If you wish to volunteer as a referee, feel free to contact the editor to receive a referee interest form.

Announcement of the Thirty-Sixth Biennial Convention of Kappa Mu Epsilon

The Thirty-Sixth Biennial Convention of Kappa Mu Epsilon will be hosted by the Missouri Alpha chapter at Missouri State University in Springfield, Missouri. The convention will take place April 12-14, 2007. Each attending chapter will the usual travel expense ($.35/mile) reimbursement from the national office as described in Article VI, Section 2, of the Kappa Mu Epsilon Constitution.

A significant feature of our national convention will be the presentation of papers by student members of Kappa Mu Epsilon. The mathematical topic selected by each student should be of interest to the author and of such scope that it can be given adequate treatment in a timed oral presentation. Senior projects and seminar presentations have been a popular way for faculty to get students to investigate suitable topics. Student talks to be judged at the convention will be chosen prior to the convention by the Selection Committee on the basis of submitted written papers. At the convention, the Awards Committee will judge the selected talks on both content and presentation. The rankings of both the Selection and Awards Committees will determine the top four papers.

Who may submit a paper?

Any undergraduate or graduate student member of Kappa Mu Epsilon may submit a paper for consideration as a talk at the national convention. A paper may be coauthored. If a paper is selected for presentation at the convention, the paper must be presented by one or more of its authors.

Presentation topics

Papers submitted for presentation at the convention should discuss material understandable by undergraduates who have completed only calculus courses. The Selection Committee will favor papers that satisfy this criterion and which can be presented with reasonable completeness within the time allotted. Papers may be original research by the student(s) or exposition of interesting but not widely known results.

Presentation time limits

Papers presented at the convention should take between 15 minutes and 25 minutes. Papers should be designed with these limits in mind.

How to prepare a paper

The paper should be written in the standard form of a term paper. It should be written much as it will be presented. A long paper (such as an honors thesis) must not be submitted with the idea that it will be shortened at presentation time. Appropriate references and a bibliography are expected. Any special visual aids that the host chapter will need to provide (such as computer, overhead projection system, etc.) should be clearly indicated at the end of the paper.

The first pages of the paper must be a cover sheet giving the following information:

- Title
- Name of the author or authors (These names should not appear elsewhere in the paper.)
- Student status (undergraduate or graduate)
- Author's permanent and school addresses and phone numbers
- Name of the local KME chapter and school
- Signed statement giving approval for consideration of the paper for publication in The Pentagon (or a statement about submission for publication elsewhere)
- Signed statement of the chapter's Corresponding Secretary attesting to the author's membership in Kappa Mu Epsilon.

How to submit a paper

Five copies of the paper, with a description of any charts, models, or other visual aids that will be used during the presentation, must be submitted. The cover sheet need only be attached to one of the five copies. The five copies of the paper are due by February 1, 2007. They should be sent to:

Dr. Ron Wasserstein, KME President-Elect
262 Morgan Hall
Washburn University
1700 SW College Avenue
Topeka, KS 66621

Selection of papers from presentation

A Selection Committee will review all papers submitted by undergraduate students and will choose approximately fifteen papers for presentation and judging at the convention. Graduate students and undergraduate students whose papers are not selected for judging may be offered the opportunity to present their papers at a parallel session of talks during the convention. The President-Elect will notify all authors of the status of their papers after the Selection Committee has completed its deliberations.

Criteria used by the Selection and Awards Committees

Judging criteria used by both the Selection Committee and Awards Committee will include

- Choice and originality of topic
- Literature sources and references
- Depth, significance, and correctness of content
- Clarity and organization of materials
- Overall effect

In addition to the above criteria, the Awards Committee will judge the oral presentation of the paper on

- Adherence to the time constraints
- Effective use of graphs and/or visual aids.

The rubric used for judging is available from the President-Elect.

Prizes

All authors of papers presented at the convention will be given two-year extensions of the subscription to The Pentagon. Authors of the four best papers presented by undergraduates, as decided by the Selection and Awards Committees, will each receive a cash prize.

Publication

All papers submitted to the convention are generally considered as submitted for publication in the Pentagon. Unless published elsewhere, prize-winning papers will be published in The Pentagon after any necessary revisions have been completed. All other papers will be considered for publication. The Editor of The Pentagon will schedule a brief meeting with each author during the convention to review his or her manuscript.

Kappa Mu Epsilon News

Send news of chapter activities and other noteworthy KME events to
Connie Schrock, KME Historian
Department of Mathematics, Computer Science, and Economics
Emporia State University
1200 Commercial Street
Campus Box 4027
Emporia, KS 66801
or to
schrockc@emporia.edu

Installation Report

Maryland Epsilon
Villa Julie College, Stevenson, Maryland

The Maryland Epsilon chapter of Kappa Mu Epsilon was installed at Villa Julie College in Stevenson, Maryland on Saturday, December 3, 2005. The installation ceremony was held at 7:00 p.m. on the college campus in the St. Paul Companies Pavilion. Don Tosh, National President of KME, was the installing officer.

Maryland Epsilon has eighteen charter members. Dr. Christopher Barat, who was already a member of KME, was instrumental in starting the chapter and served as conductor during the installation ceremony. He is now the Corresponding Secretary of the chapter. Charter members initiated during the ceremony are: Rachel Bauer, Jennifer Cooper, Elizabeth Crosland, Anthony Debraccio, Tamara Ford, Nichole Hammerbacher, Elizabeth Hand, Richard Haney, Melissa Kline, Jeanette Krug, Katie McClean, Ashley Ritz, Pamela Smith, Kimberly Still, Stephanie Walsh, Philip Williams, and Justin Young.

The officers installed during the ceremony were: Richard Haney, President; Rachel Bauer, Vice President; Pamela Smith, Secretary; Tamara Ford, Treasurer; Christopher Barat, Corresponding Secretary and Faculty Sponsor.

The ceremony was concluded with remarks from Dr. Susan Gorman, Chair of the Science and Math Division, and Dr. Susan Slattery, Chair of the Mathematics Department.r.

Following the ceremony the initiates and guests were treated to a buffet. The good food and pleasant conversation concluded a very enjoyable evening.

Chapter News

AL Alpha – Athens State University

Chapter President– Holly Gasque, 30 Current Members, 0 New Members
Other fall 2005 officers: Michelle Gist, Vice–President; Mimi Cook, Secretary; Elizabeth Pylant, Treasurer; Dottie Gasbarro, Corresponding Secretary.

KME members of Alabama Alpha worked a service/food booth at the Athens Ole Time Fiddlers Convention in October to raise funds for local projects and meetings/socials. KME members and officers participated in New Student Day in September, 2005 for all new ASU students.

AL Zeta – Birmingham Southern College

Chapter President– Gardner Moseley, 8 Current Members, 3 New Members
Other fall 2005 officers: Kelly Bragan, Vice–President; David Ray, Secretary; Jill Stupiansky, Treasurer; Mary Jane Turner, Corresponding Secretary.

Fall Initiation was held on Nov. 30, 2005. There were three new members.

CO Delta – Mesa State College

Corresponding Secretary-Erik Packard.

New Initiates – Kandice H. Abramson, Nicholas S. Bingham, Robert R. Miller, Adam C. Myers, Camella A. Nielsen, Katherine A. Stadelman, Christopher A. Turiano.

CT Beta – Eastern Connecticut State University

Current Members, 0 New Members.
Fall 2005 officers: Mizan R. Khan, Treasurer; Christian L. Yankov, Corresponding Secretary.

New Initiates - Nichole L. Caisse, Sheryl L. Garcia, Katie M. Hardy, Meaghan E. Kehoegreen, Erin I. Quinn, Heather L. Souza, Michelle D. Sposato, Jeffrey B. Weber, Gordon T. Willey.

GA Alpha – University of West Georgia

Chapter President–Ginger Jones, 26 Current Members, 0 New Members.
Other fall 2005 officers: Matt Jones, Vice–President; Max Perkins, Secretary; Dmitiry Plaks, Treasurer; Dr. Joe Sharp, Corresponding Secretary.

Once again the Georgia Alpha Chapter of KME conducted its annual Food and Clothing Drive for the Needy with the proceeds going to the Salvation Army and the Community Food Kitchen. On December 1, 2005, we held our Fall Social at a local Pizza Restaurant with KME members, quests, and mathematics faculty in attendance. A fine time was had by all!

GA Gamma – Piedmont College

Corresponding Secretary-Raul Brooks.

New Initiate – Andrew Ryan Mingledorff, Joanna Meryl Kilburn, Kimberly Freeman, Daniel Paul Funt, Brandon H. Cash, Ashley Dunson, Rob Brown, Ashley Parker, Meredith Ray.

IA Alpha – University of Northern Iowa

Chapter President– Lynne Dieckman, 41 members, 4 New Members.

Other fall 2005 officers: Joyce Boike, Vice–President; Miki Mead, Secretary; Paul Grammens, Treasurer; Mark D. Ecker, Corresponding Secretary.

Student member Miki Mead presented her paper "Technology Use in the Mathematics Classroom" at our first fall meeting on September 20, 2005 at Professor Mark Ecker's residence. The University of Northern Iowa Homecoming Coffee was held at Professor Suzanne Riehl's residence where student member Paul Grammens presented his paper on "Economics and the Olympics". Our third meeting was held on November 15, 2005 at Professor Russ Campbell's residence where student member Joyce Boike presented her paper on "The History of the Abacus". Student members Beth Nanke and Mitch Huppenbauer addressed the fall initiation banquet with "Presidential Elections: A Foundation in Probability". Our banquet was held at Peppers restaurant in Cedar Falls on December 6, 2005 where four new members were initiated.

New Initiates – Erin Conrad, Brenda Funke, Megan Spooner, Brad Schoening.

IA Delta – Wartburg College

Chapter President– Justin Peters, 21 Current Members, 0 New Members

Other fall 2005 officers: Brian Borchers, Vice–President; Jill Seeba, Secretary; Joee Williams, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

At the Wartburg Homecoming Renaissance Fair, our club successfully ran our traditional annual fundraiser by selling egg-cheese. During November we invited a graduate student from Iowa State University to present on her research.

IL Iota – Lewis University

Corresponding Secretary-Margaret M. Juraco.

New Initiates – Steven Berger, Ellen Deinzer, Thomas Dupre, Tiffany Eischen, Paul J. Kaiser, Ray Klump, Natalie Kremer, Elaina Ktistou, Jack Lelko, Rachel Oesterreicher, Joel Pommier, Stephen Weierman.

KS Alpha – Pittsburg State University

Corresponding Secretary-Tim Flood.

New Initiates – Blake Allen, Jeffrey Beckwith, Eric Clawson, Derek Hooper, Jason Knight, Robert McFadden, Katherine Raymaker, Adam Bennett, Heather Haselrick, Lauren Kaminski, Casey Kuhn, Bryan Peters, Shelley Sarwinski, Ernest Vaughn, Erin Wells, Daniel Cormode, Ashley Buckner, Anni McCoy.

KS Delta –Washburn University

Chapter President– Kristin Ranum, 25 Current Members, 13 New Members

Other fall 2005 officers: Fai Ng, Vice–President; Carolyn Cole, Secretary; Carolyn Cole, Treasurer; Kevin Charlwood, Corresponding Secretary.

The Kansas Delta chapter of KME met for three luncheon meetings with the Math Club during the Fall semester. The meeting typically featured speakers or mathematics presentations. We have one student actively pursuing a project for the regional meeting at Northern Iowa in April 2006.

KY Beta – University of the Cumberlands

Chapter President – Kyle Harris, 35 Current Members, 0 New Members

Other fall 2005 officers: Lane Royer, Vice–President; Kelly Schnee, Secretary; James D. Roaden, Treasurer; Jonathan Ramey, Corresponding Secretary.

On September 20, the Kentucky Beta chapter helped to host an ice cream party for the freshman math and physics majors. Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on October 18. On December 8, the entire department, including the Math and Physics Club, the Kentucky Beta chapter, and Sigma Pi Sigma had a Christmas party with about 45 people in attendance.

New Initiates – Dr. Don Good, Brittany Beeler, Angela Calchera, Jennifer Dillon, Kyle Harris, Rachel Morrin, Erin Peters, Ada Christina Rickett, James D. Roaden, Lane Royer, Kelly Schnee, Sarah Strunk, Stephen Taylor, Allison Tyler, Shaina West.

MA Alpha – Assumption College

Chapter President– Brent Hager, 11 Current Members, 0 New Members

Other fall 2005 officers: Heather Leaman, Vice–President; Kathryn Sullivan, Secretary; Maura Heney, Treasurer; Allison Cerulo and Elise Gross, Co-Historians; Charles Brusard, Corresponding Secretary.

MD Alpha – College of Notre Dame of Maryland

Chapter President – Sarah Wassink, 18 Current Members, 0 New Members

Other fall 2005 officers: Allison Kingsland, Vice-President; Amanda Reiner, Secretary; Alka Sharma, Treasurer; Margaret Sullivan, Corresponding Secretary.

The Notre Dame KME chapter is embedded in the Hypatian Society, the campus club for students interested in mathematics, engineering, computer science, and physics. In the Fall, the activities of group included a Planetarium presentation given by Dr. Joseph Di Rienzi of the Physics Department and an astronomy observation experience on the roof of our science building under the direction of Dr. Carrie Fitzgerald of neighboring Loyola College.

New Initiates – Monique Brown, Maura O'Neill, Megan Pahr, Lauren Rea, Alka Sharma, Neeraj Sharma, Sarah Smith, Khola Tahir, Tamanika Tinsley, Vera Ulanowicz, Kimberly Wall.

MD Beta – McDaniel College

Corresponding Secretary-Harry Rosenzweig.

New Initiates – Bruce Chappell, Paul Hugus, Carolanne Bianco.

MD Delta – Frostburg State University

Chapter President – Kimberly Embrey, 20 Current Members, 0 New Members.

Other fall 2005 officers: Matt Crawford, Vice-President; Terry Apple, Secretary; Jeff Meyer, Treasurer; Dr. Mark Hughes, Corresponding Secretary.

The Maryland Delta Chapter had a number of interesting and fun activities during the fall semester. We commenced the term with an organizational meeting in September. Plans were made to have a picnic for math majors/minors (or those thinking of becoming such). The picnic was held in mid – October and the weather cooperated wonderfully! In early November, a number of our KME student and faculty members traveled down to Germantown, Maryland to attend the fall sectional meeting of the Mathematical Association of America. Finally, a lecture entitled "Solving the Quintic Equation" was presented by Dr. Mark Hughes during our last meeting on November 30.

MD Epsilon – Villa Julie College

Corresponding Secretary-Dr. Christopher E. Barat.

New Initiates – Rachel Bauer, Jennifer Cooper, Elizabeth Crosland, Anthony Debraccio, Tamara Ford, Nicole Hammerbacher, Elizabeth Hand, Richard Haney, Melissa Kline, Jeanette Krug, Katie McClean, Ashley Ritz, Pamela Smith, Kimberly Still, Stephanie Walsh, Philip Williams, Justin Young.

MI Beta – Central Michigan University

Corresponding Secretary-Arnie Hamel.

New Initiates – Steven T. Briggs, Jenna L. French, Eric J. Hall, Sarah M. Karl, Naomi A. Kassner, Brian W. Keinath, John C. Koehn, Alan M. Lamielle, Elizabeth A. Wascher, Brian D. Wyzlic.

MI Delta – Hillsdale College

Chapter President – Christin Alford, 13 Current Members, 0 New Members

Other fall 2005 officers: Hannah Mahan, Vice-President; Katherine von Heiland, Treasurer; Dr. John H. Reinoehl, Corresponding Secretary.

MO Alpha – Missouri State University

Chapter President– April Williams, 33 Current Members, 3 New Members

Other fall 2005 officers: Samantha Cash, Vice–President; Chad Gripka, Secretary; Uriah Williams, Treasurer; John Kubicek, Corresponding Secretary.

This semester Missouri Alpha hosted the annual Fall Mathematics department picnic, held 4 monthly meetings with two faculty presentations and 2 student presentations.

New Initiates – James Black, Katie Schmidt, Rachel Schatz.

MO Beta – Central Missouri State University

Chapter President– Missy Libbert, 20 Current Members, 5 New Members

Other fall 2005 officers: Jonathan Petersen, Vice–President; Spencer Louden, Secretary; Jennifer Delana, Treasurer; Charissa Eichman, Historian; Rhonda McKee, Corresponding Secretary.

Missouri Beta Chapter held meetings during the fall semester. In September, watched two short videos – "The Shape of Space," and "Donald in Mathemagic Land," In October, Dr. Baeth gave a talk on the "15 Puzzle" and in November, three students presented reports they had done for a class. The end of semester party consisted of a chili supper and games at Dr. McKee's house.

New Initiates – Sarah Baker, Ashley Dial, Travis Overfelt, Francis Schmitz, Rachel Utrecht.

MO Gamma – William Jewell College

Chapter President– Michelle Richards, 11 Current Members, 0 New Memebers

Other fall 2005 officers: Jake Wyllie, Vice–President; Dr. Mayumi Sakata, Treasurer; Dr. Mayumi Sakata, Corresponding Secretary.

MO Lambda – Missouri Western State College

Chapter President– Robert Smith, 40 Current Members, 0 New Members

Other fall 2005 officers: Whitney Lowrey, Vice–President; Heather Goforth, Secretary; Daniel Cassity, Treasurer; Don Vestal, Corresponding Secretary.

MO MU – Harris Stowe State College

Corresponding Secretary-Jack Behle.

New Initiates – Alois Hoog, Daniel Romanelli, Daryl Smith, Lawrence White, Taliah Whitfield, Daren Wolf.

MO Nu – Columbia College

Chapter President – Jamie Netherton, 11 Current Members, 7 New Members

Other fall 2005 officers- Michael Perkins, Vice-President; Jenna Holdmeyer, Secretary; Thureen Khan, Treasurer; Dr. Ann Bledsoe, Corresponding Secretary

Our Columbia College Chapter brought two different speakers, Dr. Tanya Taksar and Dr. Zara Girnius, in October. They spoke to the KME members about different job opportunities in the field of mathematics other than teaching or actuarial science. KME members opened this up to all students in an upper level Mathematics or Computer Science courses.

On November 30th , Columbia College Chapter of KME invited Dr. Jan Segert, Director of Mathematic Graduate Studies at the University of Missouri Columbia to Speak to KME members about graduate school. Again we extend an invitation to any student of Columbia College who was interested in possible continuing their education in Mathematics of related fields.

We felt that for the spring semester we should do something for those students who are not interested in mathematics. Our projects include a wallet size tip table that will be sitting near our KME bulletin board. These will have table of 10%, 15%, and 20% for different values. Also we hope to be making an instructional worksheet that will show (1) how to enter grades and weights into lists on the TI-83 Calculators and (2) how to compute your GPA using these lists.

KME meetings are generally held once a month and each student member of KME takes turns tutoring at an after school program for students in the Columbia area for grades K-12.

MO Theta – Evangel University

Chapter President– Andrew Reed, 18 Current Members, 0 New Members. Other fall 2005 officers: Tiffany Brown, Vice–President; Don Tosh, Corresponding Secretary.

Meetings were held monthly. Attendance was good, helped along by the free pizza. The final meeting was held at the home of Don Tosh and featured banana splits. In November two students, Andrew Reed and Dianne Henry, presented papers at a local math conference which was co-hosted by Evangel. It was good practice for the regional convention this spring.

MO Zeta – University of Missouri

Corresponding Secretary-Roger Hering.

New Initiates – Angela Adams, Paul Aten, Matt Bartels, Justin Bruemmer, Kendrick Callaway, Sam Cantrell, Adam Chadek, Mark Chamberlain, Keith Connell, Dan Crahan, Adam Daniel, Eric Dixon, Andrew Draper, Kyle Duvall, Matt Flint, Joshua French, Natalie Frenz, John Fuerst, John Gantt, Melissa Giles, Christina Graham, Kyle Guinn, Emily Hackworth, Peter Hamel, Kurt Haslag, Brandon Howard, Evan LaBoube, Cara Longhenrich, Matt Lutz, Andrew Meintz, Alicia Miller, Ryan Murray, Santosh Nachu, Sara Nolte, Eric Peters, Joe Siebert, Joshua Sneller, Jason Stuckmeyer, Hannah Turner, Andrew Welter.

MS Alpha – Mississippi University for Women

Corresponding Secretary-Dr. Shaochen Yang.

New Initiates – Johanna M. Rodriquez Parra Flores, Blair Vernon, Chris WIllbanks.

MS Beta – Mississippi State University

Corresponding Secretary-Vivien Miller.

New Initiates – Spencer Arnault, Amber Russell, Rachel Hale.

NE Alpha – Wayne State College

Chapter President– Gabriel Fejfar, 12 Current Members, 0 New Members

Other fall 2005 officers: Michelle Starkjohn, Vice–President; Kristi Owens, Secretary; Melyssa Krusemark, Treasurer; John Fuelberth, Corresponding Secretary.

New Initiates – Michelle Starkjohn, Breann Parks.

NE Delta – Nebraska Wesleyan University

Chapter President– Jennifer Choutka, 4 Current Members, 0 New Members

Other fall 2005 officers: Kristen Houchin, Vice President; Zachary Brightweiser, Secretary; Zachary Brightweiser, Treasurer; Melissa Erdmann, Corresponding Secretary.

NJ Gamma – Monmouth University

Chapter President—Lisa Marchalonis, 93 Current Members, 21 New Members

Other fall 2005 officers: Catharine Russamano, Vice–President; Leslie Cordasco, Secretary; Krystle Hinds and Lauren Grobelny, Treasurer; Judy Toubin, Corresponding Secretary.

Last year NJ Gamma Chapter once again was involved in the soda tab collection for Ronald McDonald House Charity. We hosted a Praxis and graphing calculator talk presented by Professor Penge and Professor Kellmer to prepare students for the Math Praxis exam. We sponsored a talk with Jay Bennett, the author of “Curveball: Baseball, Statistics and the Role of Chance in the Game”, which talked about the role of statistics in baseball. We also had one more interesting topic presented by one of our professors, Professor Lynn Bodner, titled The Geometry of the Seven Heavens. KME also sponsored a Meet and Greet Volleyball Game where KME members got to challenge some of the faculty members. We will be selling t-shirts and are preparing for a successful induction ceremony.

NM Alpha – University of New Mexico

Corresponding Secretary-Pedro Embid.

New Initiates – Anne Fullilove, Ryan Martin, Emily Pincus, Alex Smith, Alexey Sukhinin.

OH Epsilon – Marietta College

Chapter President – Mary Kinsler, 15 Current Members, 0 New Members

Other fall 2005 officers: Kristen Martin, Vice-President; Dr. John C. Tynan, Corresponding Secretary.

OH Gamma– Baldwin-Wallace College

Chapter President – Kathleen Turk, 28 Current Members, 20 New Members

Other fall 2005 officers: Gretchen Waugaman, Vice-President; Andrew Miskimen, Secretary; Megan Saad, Treasurer; Dr. David Calvis, Corresponding Secretary.

OK Alpha – Northeastern State University

Chapter President– Josh Hamit, 74 Current Members, 15 New Members

Other fall 2005 officers: Leticia Stone, Vice–President; Jeff Smith, Secretary; Andy Hathcoat, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

We sponsored several speakers this fall. Dr. Alberto Striolo, Asst. Professor of Engineering, University of Oklahoma, spoke on "How is our research indebted to computer scientists?" Dr. Wendell Wyatt, Northeastern State University, taught us how to make stellated octahedrons. Dr. Deborah Carment spent hours selecting color coordinated scrapbook paper which our chapter provided to members to personalized their octahedrons. Our fall initiation brought 14 new members into our chapter. The annual book sale netted almost $100. We again participated in the annual NSU Halloween carnival with our "KME Pumpkin" activity. The children fished for pumpkins with meter stick poles. Our Christmas party was great! We played games and ate Christmas goodies. But the best part was the homemade pizza that our department chair, Dr. Darryl Linde, made for us.

New Initiates – Amanda Bell, Timothy Berres, Melissa D. Buchanan, Sara Danzi, Jonathan Ford, David Imwalle, Alan Keck, Christy Peters, Amy Price, Ashley Pryor, JoAnna Reedy, Alisha Smith, Mara Stoica, Douglas Ward, Justin Watts.

OK Gamma – Southwest Oklahoma State University

Corresponding Secretary-Bill Sticka.

New Initiates – Tim Gaston, Sulav Regmi, Cammi Valdez.

PA Alpha – Westminster College

Chapter President – Lauren Beichner, 19 Current Members, 0 New Members

Other fall 2005 officers: Sarah Spardy, Vice-President; Christie Grewe, Secretary; Amanda Ganster, Treasurer; Carolyn Cuff, Corresponding Secretary.

The PA Alpha Chapter sponsored a pizza party for new students in September. We hosted Career Night in Mathematics and Computer Science in November. Then we provided donuts and juice for Finals during Finals week.

PA Kappa – Holy Family University

Chapter President – Patrick Heasley, 5 Current Members, 0 New Members
Other fall 2005 officers: Shawn Kane, Vice-President; Tiffany Young, Secretary; Tiffany Young, Treasurer; Sister Marcella Louise Wallowicz CSFN, Corresponding Secretary.

The PA Kappa Chapter sponsored its annual High School math competition on November 29, 2005. Thirty one students from local area high schools participated in individual contest consisting of Algebra, Geometry and Trig problems solving. There was also a "team" event – an assortment of problems from Algebra though Analysis. Trophies were awarded to the top students: Chris Ward and Mike Minner, both Fr. Judge High School, and the top school: Archbishop Ryan High School. Following the competition refreshments were served: Coca-Cola and Philadelphia soft pretzels.

On December 13, 2005 Michael James and Kangoma Tulay presented their senior seminar research papers at a forum sponsored by the PA Kappa Chapter. Mike's topic was the "Keppler Conjecture," (Hibert's 18th problem) which was proved by Dr. Tom Hales, Mellon Professor at the University of Pittsburgh. Mike corresponded regularly with Dr. Hales during the semester. Kangoma's topic was "Algorithms for Computing PI," an extensive study of various methods used from biblical times to present.

PA Pi– Slippery Rock University

Chapter President – Richard Busi, 13 Current Members, 0 New Members
Other fall 2005 officers: T. J. Deems, Vice-President; Kory Fish, Secretary; Mark Kratz, Treasurer; Elise M. Grabner, Corresponding Secretary.

PA Sigma– Lycoming College

Chapter President – Jessica Gough, 13 Current Members, 0 New Members
Other fall 2005 officers: Byron Arenella, Vice-President; Gregory Moses, Secretary; Josemar Castillo, Treasurer; Dr. S. de Silva, Corresponding Secretary.

In September 2005, Lycoming KME was requested to provide suitable graphics to be mounted beside the insignia of the other honor societies of the College in a new Honors Hall that had been opened over the summer. With help of an alumna, we were able to condense, or simplify, the KME insignia to fit the circular monochrome format that was required. Our spring induction will take place in this building.

Each semester, the Dean of the College conducts a Symposium. This semester, the theme was Einstein's Miraculous year and legacy, which gave us a rare opportunity to be involved in the symposium. We were invited to arrange for one of the Symposium events. We invited Dr. Ezra Newman, a central figure in mid-century Relativity Theory in the US. He accepted our invitation, and spoke at the final Symposium event, which was well attended and received with enthusiasm. We were pleased not only to have succeeded in planning and executing this task in our first full semester, but also to have established links with our colleagues in physics and astronomy.

This semester is at an end, and no additional activities are anticipated until the spring semester.

SC Epsilon – Francis Marion University

Corresponding Secretary-Damon Scott.

New Initiates – Joshua Kevan Croteau, Tiffany Domonique Gibson, Daniel L. Harrington, Amber M. Mabry, Michael Hay Tatum, William M. Putnam, Christopher E. Wells, Robert B. Hill.

SC Gamma – Winthrop University

Chapter President – Philip Gear, 12 Current Members, 0 New Members.

Other fall 2005 officers: Shantelle Prioleau, Vice-President; Anesha Simms, Secretary; Ian Finlayson, Treasurer; Dawn Strickland, Corresponding Secretary.

TN Delta – Carson-Newman College

Chapter President – Marsha Cox, 20 Current Members, 0 New Members.

Other fall 2005 officers: Melissa Summey, Vice-President; Philip Barger, Secretary; Philip Barger, Treasurer; B. A. Starnes, Corresponding Secretary.

TN Epsilon – Bethel College

Chapter President – Daniel Cooley, 10 Current Members, 7 New Members.

Other fall 2005 officers: Jessica Smith, Vice-President; Kamela Rogers, Secretary; Heather Brannon, Treasurer; Mr. Russell Holder, Corresponding Secretary.

New Initiates – Heather Brannon, Daniel Cooley, Francis Mumelo, Kamela Rogers, Jessica E. Smith, Chris Terry, Dr. Jesse J. E. Turner.

TN Gamma – Union University

Chapter President– Denise Baughman, 14 Current Members, 0 New Members

Other fall 2005 officers: Kendal Hershberger, Vice–President; Josh Shrewsberry, Secretary; Josh Shrewsberry, Treasurer; David Moses, Webmaster; Bryan Dawson, Corresponding Secretary.

The first chapter activity of the semester was a cookout on September 26 at the home of faculty member Chris Hail; the program consisted of a mathematical trick by "The Great Dawsoni." On our college's Day of Remembrance (on which faculty and students perform service projects in the community, November 9, several members worked together to design a spreadsheet to help analyze survey data collected by the local Boys and Girls Club; Josh Shrewsberry led the effort. A Christmas party was held at the home of faculty sponsor Math Lunsford on December 1; this year's most-stolen item during the white elephant gift exchange originally cost $4600 (a still-working 14-year-old Mac laptop). We continued our tradition of sponsoring a needy child at the annual Carl Perkins Christmas Dinner. The final event of the semester was senior seminar presentations by Denise Baughman and Tony Winkler on December 6.

TX Alpha – Texas Tech University

Chapter President– Latasha R. Smith, 38 Current Members, 0 New Members

Other fall 2005 officers: Laura A Reddy, Vice–President; Candace Cyrek, Secretary; Nancy A Gerrish, Treasurer; Dr. Anatoly B. Korchagin, Corresponding Secretary.

TX Iota – McMurry University

Corresponding Secretary-Dr. Kelly L. McCoun.

New initiates – Jennifer Codding, Adam Davidson, Joseph Glover, Brady Reeves, James Walsh, Ryan Wellhoefer, Dr. Tihon Bykov, Dr. Robert Easterling.

TX Mu – Schreiner University

Chapter President- Charmelyn Fortune, 7 Current Members, 0 New Members

Other fall 2005 officers: Aaron Mayes, Vice–President; Matthew Casey, Secretary; Matthew Casey, Treasurer; William M. Silva, Corresponding Secretary.

On Thursday, October 20, Texas Mu hosted a luncheon for math majors and prospective math majors. We watched the movie "The Right Spin", the story of the dramatic rescue in space as told by astronaut Michael Foale. Afterward, the math majors and prospective math majors participated in an informal social hour.

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KME National Website:
http://www.kappamuepsilon.org/

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
TX Nu	Texas A&M University - Corpus Christi, Corpus Christi	8 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005