THE PENTAGON

A Mathematics Magazine for Students

Volume 68 Number 1 Fall 2008

Contents

Typeset in Scientific WorkPlace Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions must include both an electronic copy and a hard copy. The electronic copy should be included as an attachment to an e-mail sent to the editor. Either a TeX file or Word document is acceptable. It is the responsibility of the author to ensure that any images are in a camera-ready form that can be scaled to the page size of *The Pentagon*. The typewritten copy should be double-spaced with wide margins on white paper. Standard notational conventions should be respected. Graphs, tables, or other materials taken from copyrighted works MUST be accompanied by an appropriate release form the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

Individual domestic subscriptions: $5.00 for two issues (one year) or $10.00 for four issues (two years); individual foreign subscriptions: $7.00 (US) for two issues (one year). Library rate: $10.00 (US) for two issues (one year) or $20.00 (US) for four issues (two years). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced. Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

Associate Editors:

The Problem Corner: Pat Costello
Department of Mathematics and Statistics
Eastern Kentucky University
521 Lancaster Avenue
Richmond, KY 40475-3102
e-mail: pat.costello@eku.edu

Kappa Mu Epsilon News: Connie Schrock
Department of Mathematics
Emporia State University
Emporia, Kansas 66801
schrockc@emporia.edu

KME Website

The national KME website can be found at

http://www.kappamuepsilon.org/

Among the items on the site:

- Contact information for national officers
- Forms
- How to start a KME chapter
- Information on KME conventions

When you design a chapter homepage, please remember to make it clear that your page is for your chapter, and not for the national organization. Also, please include a link to the national homepage and submit your local chapter webpage's URL to the national webmaster, who can be contacted from the national website.

Subscription Renewals and Change of Address

Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 67 Number 2, if the code 67-2 appears on your label then this is your last issue!

Early renewals save us the cost of mailing out renewal notices. To renew, please send your check - just $10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subscriptions) - together with your name and address and a copy of your old address label to:

Don Tosh, *The Pentagon* Business manager
Department of Science and Technology
Evangel University
1111 N. Glenstone Ave.
Springfield, MO 65802-2191 USA

Please renew promptly to avoid gaps in your journal collection. **If you have a change of address,** please don't forget *The Pentagon*! Send your change of address to the business manager at the above address. Failure to do so may cost KME additional postage fees!

Back Issues

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for $5.00 per copy. Please send inquiries to:

Don Tosh, *The Pentagon* Business manager
Department of Science and Technology
Evangel University
1111 N. Glenstone Ave.
Springfield, MO 65802-2191 USA

Shortest Way Home

Natalie McCall, *student*

KS Epsilon

Fort Hays State University
Hays, KS 67601

Presented at the 2008 North Central Regional Convention and awarded "top four" status by the Awards Committee.

1. Introduction

This past summer on the 4th of July, I attended the fireworks display in Hays, KS. This event takes place near Gross Memorial Coliseum making the adjacent parking lot a prime viewing location for many spectators, including myself. Upon its conclusion, I proceeded to my car and into the inevitable traffic jam. My mind immediately went to figuring out the quickest way out of the traffic and to my house. Unfortunately, I was not the only person with that thought and eventually circled my way around to where I began. My path is outlined in Figure 1. From this frustrating journey came the question of how to find the most efficient path out of a packed parking lot. As it turns out, this question can get very complicated very fast so the solution presented is a simplified version of the question I first posed on that explosive, exhaust-filled night.

On the way to the final, hopefully time-saving answer, I will introduce Dijkstra's algorithm for finding the shortest path. This algorithm finds the shortest path in a weighted graph, by labeling vertices through multiple iterations.

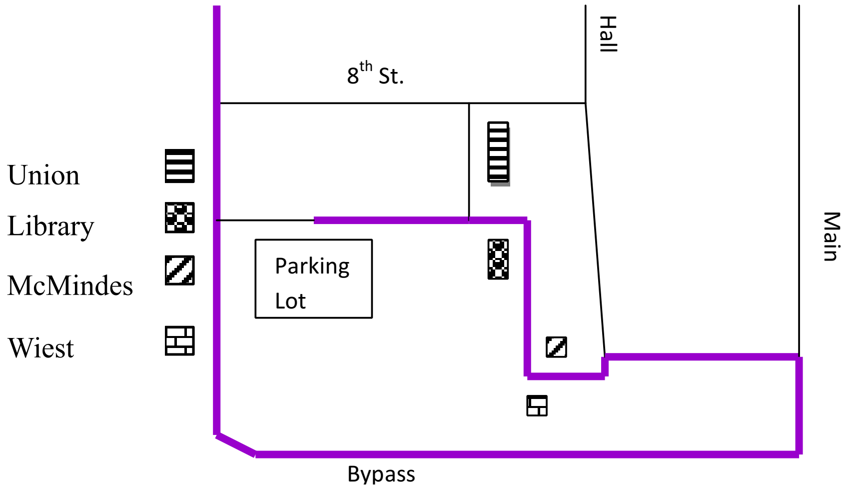


Figure 1

This process makes it a label-setting algorithm. Through these iterations, the shortest path from one vertex to another is determined. Dijkstra's algorithm sets the stage for the Floyd shortest path algorithm, which determines the shortest path between each pair of vertices in a graph. Use of this algorithm allows the possibility of choosing any vertex as the destination vertex, which is essential in the given problem.

After an introduction to Dijkstra's algorithm, I will look at an example of the process and then expand on that example with the Floyd shortest path algorithm. Several variables will be introduced, including the different weights given to things such as stop signs and two-lane roads. I will then apply the latter algorithm to the original question posed and discover which path I should have taken. Finally I will discuss what adjustments can be made so this algorithm can be used in a more complicated situation.

2. Dijkstra's Algorithm

Dijkstra's shortest path algorithm starts with a source, from which each path begins. This vertex is often labeled s. Suppose we also know the k vertices that are closest in total length to vertex s and a shortest path from s to each of these vertices. This algorithm is formally stated with the following steps:

Step 1: Initially, all arcs and vertices are unlabeled. Assign a number $d(x)$ to each vertex x. This is the tentative length of the shortest path from s to x with only labeled vertices used as intermediate vertices. Since the shortest path from the initial vertex s to itself is zero, set $d\left(s\right) = 0$, and for all other vertices, set $d\left(x\right) = \infty$. Let y be the last vertex that was lableled. Label vertex s and let $y = s$.

Step 2: For each unlabeled vertex x, redefine $d(x)$ as

$$d\left(x\right) = \min\left\{d\left(x\right), d\left(y\right) + a\left(y, x\right)\right\},$$

where $a\left(y, x\right)$ is the arc length from vertex y to vertex x. This can be done by observing the vertices directly connected to vertex y (no intermediate vertices are used). If $d\left(x\right) = \infty$ for all vertices, stop since there are no paths from vertex s to any other vertex. Otherwise, label vertex x with the smallest value of $d(x)$. Let $y = x$.

Step 3: If the last vertex, t, is labeled, stop because the shortest path from s to t is labeled. If t is not labeled, repeat Step 2.. (Evans and Minieka, p.85).

For a better understanding of the above steps, consider the following example.

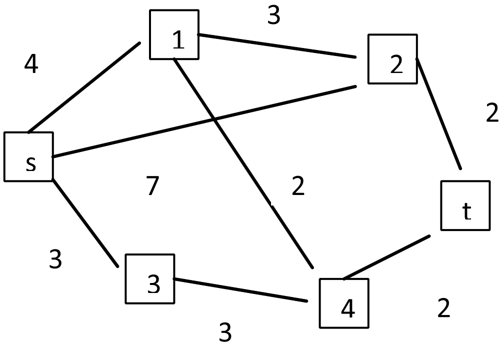


Figure 2

Apply Dijkstra's algorithm to find the shortest path from node s to node t.

Step 1: Only node s is permanently labeled, $d(s) = 0$, and all other arcs are given the length $d\left(x\right) = \infty$ for all $x \neq s$. Let $y = s$.

Step 2: Redefine $d(x)$ with the formula

$$d\left(x\right) = \min\left\{d\left(x\right), d\left(y\right) + a\left(x, y\right)\right\}.$$

$$\begin{aligned} d(1) &= \min\{d(1), d(s) + a(s,1)\} = \min\{\infty, 0+4\} = 4 \\ d(2) &= \min\{d(2), d(s) + a(s,2)\} = \min\{\infty, 0+7\} = 7 \\ d(3) &= \min\{d(3), d(s) + a(s,3)\} = \min\{\infty, 0+3\} = 3 \end{aligned}$$

The minimum distance from node s to any other node with no intermediate vertices is $d(3) = 3$. Now label node 3 and arc $(s,3)$. Let $y = 3$.

Step 3: Node t has not yet been labeled, so return to step 2.

Step 2: Node 4 is the only choice from node 3, so the equation gives

$$d(4) = \min\{d(4), d(3) + a(3,4)\} = \min\{\infty, 3+3\} = 6.$$

Since we previously found $d(1) = 4$, and this is less than $d(4) = 6$, label node 1 and arc $(s,1)$ and let $y = 1$.

Step 3: Note that node t is not yet labeled, so repeat step 2.

Step 2: From node 1 are the choices of node 2 or node 4. It follows from the equation that

$$\begin{aligned} d(2) &= \min\{d(2), d(1) + a(1,2)\} = \min\{7, 4+3\} = 7 \\ d(4) &= \min\{d(4), d(1) + a(1,2)\} = \min\{6, 4+2\} = 6. \end{aligned}$$

The minimum distance is at $d(4) = 6$, so label node 4 and either arc (1,4) or (3,4). Let $y = 4$.

Step 3: Node t has still not been labeled, so repeat step 2.

Step 2: The only node to choose from when leaving node 4, is node t. So

$$d(t) = \min\{d(t), d(4) + a(4,t)\} = \min\{\infty, 6+2\} = 8.$$

But $d(2) = 7$ is the minimum tentative distance from s, so label node 2 and arc $(s,2)$, and let $y = 2$. Repeating step 2, since node t is not yet labeled, with the given equation gives

$$d(t) = \min\{d(t), d(2) + a(2,t)\} = \min\{8, 7+2\} = 8.$$

Step 3: Node t is labeled using $d(4) = 8$ from the previous step, so label arc $(4,t)$. ([1], p. 87)

The shortest path from s to t contains the arcs $(s,3)$, (3,4), and $(4,t)$. This algorithm definitely allowed the shortest path to be found, but there appeared to be a lot of backtracking. What if we were able to look at all the arc lengths simultaneously? Is there a way to find the shortest path with any number of intermediate vertices? As it turns out, that is where Floyd's algorithm improves upon Dijkstra's algorithm.

3. Floyd's Algorithm

This algorithm introduces $m \times m$ matrices to determine the shortest path from one vertex to another. The size of the matrix is determined by the number of vertices in the graph; $1, 2, 3, \ldots, m$. Let d_{ij}^{k} be the length of the shortest path from vertex i to vertex j, where only the first k vertices are used as intermediate vertices. If this path does not exist, then let $d_{ij}^{k} = \infty$. It follows that a direct path from vertex i to vertex j, with no intermediate vertices is denoted as d_{ij}^{0}. In Dijkstra's algorithm the arc length from node s to itself was given by $d(s) = 0$. The same is true in Floyd's algorithm and is denoted as $d_{ii} = 0$. It is also true that the shortest path from i to j is represented as d_{ij}^{m}. From this we get D^k is the $m \times m$ matrix, whose i, j element is d_{ij}^{k}. Matrix D^1 is determined by using a recursive formula on D^0. Likewise, each matrix is determined from the preceding matrix until D^m is determined from D^{m-1}. (Evans and Minieka, p.95)

The steps for finding the shortest path using Floyd's algorithm are as follows:

Step 1: Number the vertices of the graph $1, 2, ..., m$. Create the matrix D^0 whose i, j element is the length of the path from vertex i to vertex j with no intermediate vertices used. With the above example

$$D^0 = \begin{bmatrix} 0 & 4 & 7 & 3 & \infty & \infty \\ \infty & 0 & 3 & \infty & 2 & \infty \\ \infty & \infty & 0 & \infty & \infty & 2 \\ \infty & \infty & \infty & 0 & 3 & \infty \\ \infty & \infty & \infty & \infty & 0 & 2 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix}.$$

Since the paths from one node to the next are only one direction, the bottom diagonal of the matrix is infinity.

Step 2: Determine the elements of D^k from the elements of D^{k-1} for $k = 1, 2, 3, ..., m$, using the recursive formula $d_{ij}^{k} = \min \left\{ d_{ik}^{k-1} + d_{kj}^{k-1}, d_{ij}^{k-1} \right\}$. In matrix D^m, the i, j-element represents the length of the shortest path from vertex i to vertex j, so as each element is calculated, it is important to record the path through which it was determined.

Remember that $D_{ii}^{k} = 0$ for all i and k, so the main diagonal of each matrix should be zero. Looking at the formula, notice that

$$d_{ik}^{k} = \min \left\{ d_{ik}^{k-1} + d_{kk}^{k-1}, d_{ik}^{k-1} \right\} = d_{ik}^{k-1} \text{ for all } i = 1, 2, ..., m.$$

It may be easier to see with actual numbers, so without loss of generality consider

$$d_{21} = \min\left\{d_{21}^0 + d_{11}^0, d_{21}^0\right\} = \min\left\{d_{21}^0, d_{21}^0\right\} = d_{21}^0,$$

since $d_{ii}^k = 0$. Using the same argument, it is also true that $d_{ki}^{k-1} = d_{ki}^k$.

As an example, let's apply Floyd's algorithm to Figure 3 below.

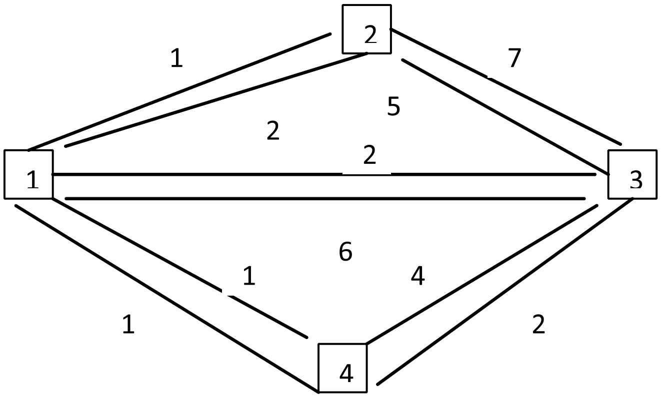


Figure 3

There are four vertices in Figure 3 so D^k will consist of 4×4 matrices. For step one we determine

$$D^0 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 7 & \infty \\ 6 & 5 & 0 & 2 \\ 1 & \infty & 4 & 0 \end{bmatrix}.$$

As expected, the main diagonal is zero since the length of the path from one vertex to itself is zero. There are no paths connecting vertices 2 and 4, likewise the distance of the paths between them is infinity. Since D^0 has been determined, it is now possible to calculate D^1 through D^4 using the formula $d_{ij}^k = \min\left\{d_{ik}^{k-1} + d_{kj}^{k-1}, d_{ij}^{k-1}\right\}$. Remember to make note of the path that each element represents.

Since $D_{ik}^{k-1} = d_{ik}^k$, the first row of D^1 remains the same. The following table lists the calculations for the elements of D^1 along with the corresponding paths.

Path Length	Corresponding Path
$d_{22}^1 = 0$	
$d_{23}^1 = \min\left\{d_{21}^0 + d_{13}^0, d_{23}^0\right\} = \min\{2+2, 7\} = 4$	$(2,1), (1,3)$
$d_{24}^1 = \min\left\{d_{21}^0 + d_{14}^0, d_{24}^0\right\} = \min\{2+1, \infty\} = 3$	$(2,1), (1,4)$
$d_{31}^1 = d_{31}^0 = 6$	$(3,1)$
$d_{32}^1 = \min\left\{d_{31}^0 + d_{12}^0, d_{32}^0\right\} = \min\{6+1, 5\} = 5$	$(3,2)$
$d_{33}^1 = 0$	
$d_{34}^1 = \min\left\{d_{31}^0 + d_{14}^0, d_{34}^0\right\} = \min\{6+1, 2\} = 2$	$(3,4)$
$d_{41}^1 = d_{41}^0 = 1$	$(4,1)$
$d_{42}^1 = \min\left\{d_{41}^0 + d_{12}^0, d_{42}^0\right\} = \min\{1+1, \infty\} = 2$	$(4,1), (1,2)$
$d_{43}^1 = \min\left\{d_{41}^0 + d_{13}^0, d_{43}^0\right\} = \min\{1+2, 4\} = 3$	$(4,1), (1,3)$
$d_{44}^1 = 0$	

The matrix form for the above table is

$$D^1 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 4 & 3 \\ 6 & 5 & 0 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}.$$

Follow the same process to get

$$D^2 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 4 & 3 \\ 6 & 5 & 0 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}.$$

The corresponding paths for these elements can also be put in a matrix. The corresponding path matrix for D^2 is

$$\begin{bmatrix} & (1,2) & (1,3) & (1,4) \\ (2,1) & & (2,1),(1,3) & (2,1),(1,4) \\ (3,1) & (3,2) & & (3,4) \\ (4,1) & (4,1),(4,2) & (4,1),(1,3) & \end{bmatrix}.$$

Continuing, we have

$$D^3 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 4 & 3 \\ 6 & 5 & 0 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

with the corresponding paths

$$\begin{bmatrix} & (1,2) & (1,3) & (1,4) \\ (2,1) & & (2,1),(1,3) & (2,1),(1,4) \\ (3,1) & (3,2) & & (3,4) \\ (4,1) & (4,1),(4,2) & (4,1),(1,3) & \end{bmatrix}.$$

Finally,

$$D^4 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 4 & 3 \\ 3 & 4 & 0 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix},$$

with corresponding paths

$$\begin{bmatrix} & (1,2) & (1,3) & (1,4) \\ (2,1) & & (2,1),(1,3) & (2,1),(1,4) \\ (3,4),(4,1) & (3,4),(4,1),(1,2) & & (3,4) \\ (4,1) & (4,1),(1,2) & (4,1),(1,3) & \end{bmatrix}.$$

So the shortest path from vertex 2 to vertex 3 is element $d^4_{2,3}$, which has length 4 and passes through vertex 1. Likewise, the shortest path from vertex 3 to vertex 2 has a length of 4 and goes through the vertices 4 and 1 respectively.

4. Application of Floyd's Algorithm to the Proposed Problem

As mentioned previously, the proposed problem can become very complicated so certain variables have been assumed to be true. First, the roads of the city have been simplified. Cars exiting the parking lot are allowed access only to the streets shown in Figure 4. Since the proposed question involves traffic dispersing from a single parking lot, the length of the paths from one vertex to another is a rough estimate of the mileage multiplied by ten, as measured by the odometer in my car.

The weight given to streets consisting of two lanes heading in the same direction has been reduced by one. The weight of the arc length is increased by two if that arc is heading toward a 4-way stop at the next immediate vertex. The weight of the arc length is increased by three if that arc is heading toward a 2-way stop, cross traffic does not stop, at the next immediate vertex. In the same fashion, a stop light will increase the weight of the arc length by one. These weight adjustments are arbitrarily chosen according to how the flow of traffic is affected at the intersections. Since a car approaching a 2-way stop will have to wait longer for cross-traffic to clear than a car heading toward a 4-way stop, the path heading into that 2-way stop vertex is assigned a heavier weight.

For various reasons, not all intersections of the city are marked in Figure 4. If police patrol forces traffic to turn only one direction, that intersection is not counted as a vertex. For example, notice that the intersection of Main St. and Bypass 181 is not marked in Figure 4. Due to the Highway Patrol, cars already on the Bypass could not turn onto Main St. and cars on Main St. could only turn west onto the Bypass. For the same reason, some roads are given paths that only travel in one direction, like the path from vertex 8 to vertex 9. Not all variables are taken into consideration. Things such as weather conditions and construction, among many other variables, are not accounted for, and are thus assumed to have no affect on the situation.

Since the proposed question involves determining the best route out of the parking lot and to a specific destination, I broke the city into quadrants. It is assumed that once a car gets to a vertex on the edge of a quadrant and crosses into that quadrant, traffic is no longer an issue. Figure 4 is quadrant three with vertex 17 being the origin. If a car's destination is in quadrant 1, it should use the shortest path to vertex 17. It is also assumed that approximately the same number of cars is heading for each of the different quadrants.

The map in Figure 4 shows the weights of the arcs adjusted according to the stipulations above. All 18 vertices are shown and labeled. The arrows shown indicate the direction of the path.

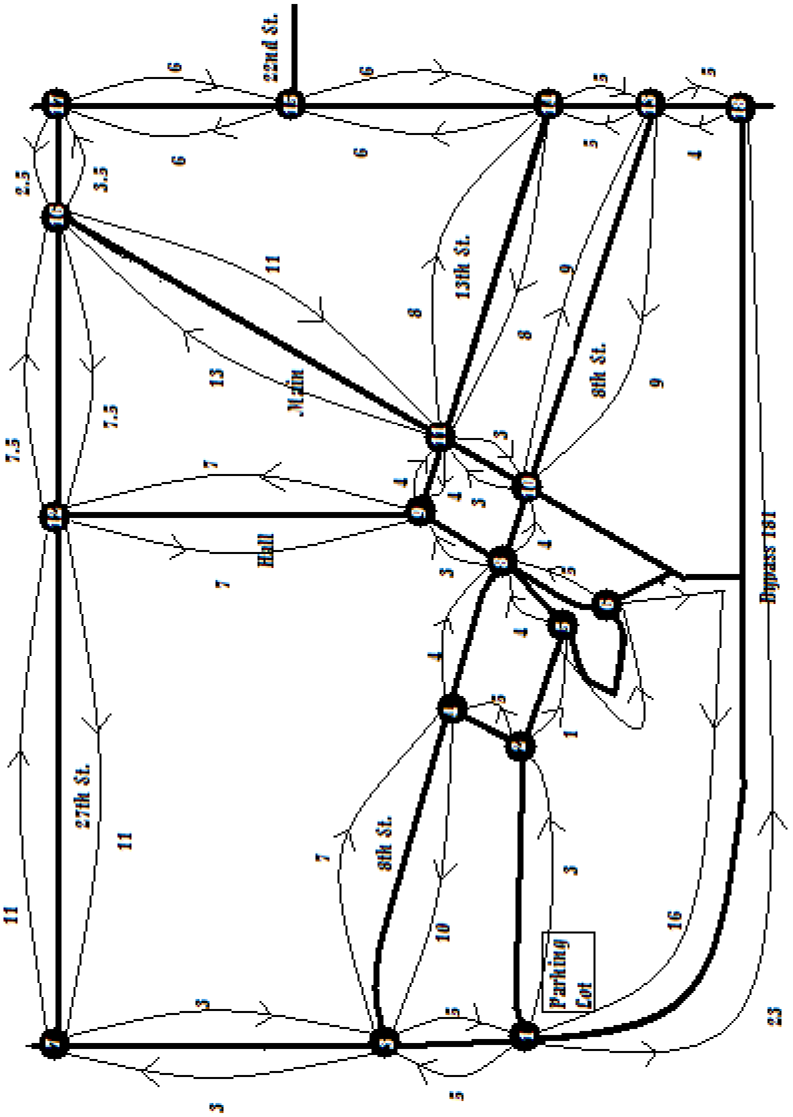

22nd St.
13th St.
8th St.
Main
Hall
27th St.
8th St.
Bypass 181
Parking Lot

Figure 4

Using the weights in Figure 1, our initial matrix is the matrix D^0 shown on the next page. Each element d_{ij}, with an actual value in this matrix, represents the weight of the arc length from vertex i to vertex j with no intermediate vertices. There is a direct path from vertex 1 to vertex 2 of length 3. Now, for step 2, we apply the formula

$$d_{ij}^k = \min\left\{d_{ik}^{k-1} + d_{kj}^{k-1}, d_{ij}^{k-1}\right\}$$

to each element of D^0 to determine D^1. The first three elements in row one are found as follows.

$$\begin{aligned}
d_{11}^1 &= \min\left\{d_{11}^0 + d_{11}^0, d_{11}^0\right\} = 0 \\
d_{12}^1 &= \min\left\{d_{11}^0 + d_{12}^0, d_{13}^0\right\} = \min\{0+3, 3\} = 3 \\
d_{13}^1 &= \min\left\{d_{11}^0 + d_{13}^0, d_{13}^0\right\} = \min\{0+5, 5\} = 5.
\end{aligned}$$

Likewise, the first three elements of the next two rows are:

$$\begin{aligned}
d_{21}^1 &= \min\left\{d_{21}^0 + d_{11}^0, d_{21}^0\right\} = \min\{\infty+0, \infty\} = \infty \\
d_{22}^1 &= 0 \\
d_{23}^1 &= \min\left\{d_{21}^0 + d_{13}^0, d_{23}^0\right\} = \min\{\infty+5, \infty\} = \infty \\
d_{31}^1 &= \min\left\{d_{31}^0 + d_{11}^0, d_{31}^0\right\} = \min\{5+0, 5\} = 5 \\
d_{32} &= \min\left\{d_{31}^0 + d_{12}^0, d_{32}^0\right\} = \min\{5+3, \infty\} = 8 \\
d_{33}^1 &= 0
\end{aligned}$$

The only element thus far to take on a new value is $d_{32}^1 = 8$. The corresponding path for this arc length is now (3,1), (1,2). Filling in the rest of the matrix we get the matrix D^1 shown on a following page. The elements $d_{32}^1 = 8$, $d_{3,18}^1 = 28$, $d_{6,2}^1 = 19$, $d_{6,3}^1 = 21$, $d_{6,18}^1 = 39$ are the only elements to take on new values. Only one intermediate vertex is used for each of these elements. Notice that there is also a path with one intermediate vertex from vertex 1 to vertex 4, but remember that only the first k vertices can be used as intermediate vertices. Since the intermediate vertex from 1 to 4 is vertex 2, it is not yet taken into account. In the same manner, the paths of the elements listed above all pass through vertex one.

After continuing to apply this formula to the elements of matrices D^2 through D^{18}, we end up with the matrix of shortest paths. Each matrix was calculated in Excel using the MIN function. I used the number 100 to represent infinity in Excel.

$$D^0 = \begin{bmatrix}
0 & 3 & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 23 \\
\infty & 0 & \infty & 5 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
5 & \infty & 0 & 7 & \infty & \infty & 3 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 10 & 0 & \infty & \infty & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 0 & 5 & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
16 & \infty & \infty & \infty & \infty & 0 & \infty & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 3 & 10 & \infty & \infty & 0 & \infty & \infty & \infty & \infty & 11 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 3 & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & \infty & 4 & 7 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 3 & \infty & 9 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4 & 3 & 0 & \infty & \infty & 8 & \infty & 13 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & 11 & \infty & 7 & \infty & \infty & 0 & \infty & \infty & \infty & 7.5 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 9 & \infty & \infty & 0 & 4 & \infty & \infty & \infty & 4 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 8 & \infty & 4 & 0 & 5 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 5 & 0 & \infty & 5 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 11 & 7.5 & \infty & \infty & \infty & 0 & 2.5 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 5 & 1.5 & 0 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 3 & \infty & \infty & \infty & \infty & 0
\end{bmatrix}$$

$$D^1 = \begin{bmatrix}
0 & 3 & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 23 \\
\infty & 0 & \infty & 5 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
5 & 8 & 0 & 7 & \infty & \infty & 3 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 28 \\
\infty & \infty & 10 & 0 & \infty & \infty & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 0 & 5 & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
16 & 19 & 21 & \infty & \infty & 0 & \infty & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 39 \\
\infty & \infty & 3 & 10 & \infty & \infty & 0 & \infty & \infty & \infty & \infty & 11 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 3 & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & \infty & 4 & 7 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 3 & \infty & 9 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4 & 3 & 0 & \infty & \infty & 8 & \infty & 13 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & 11 & \infty & 7 & \infty & \infty & 0 & \infty & \infty & \infty & 7.5 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 9 & \infty & \infty & 0 & 4 & \infty & \infty & \infty & 4 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 8 & \infty & 4 & 0 & 5 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 5 & 0 & \infty & 5 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 11 & 7.5 & \infty & \infty & \infty & 0 & 2.5 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 5 & 1.5 & 0 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 3 & \infty & \infty & \infty & \infty & 0
\end{bmatrix}$$

$$D^{18} = \begin{bmatrix}
0 & 3 & 5 & 8 & 4 & 9 & 8 & 8 & 11 & 12 & 15 & 18 & 21 & 23 & 28 & 26 & 28 & 23 \\
20 & 0 & 15 & 5 & 1 & 6 & 18 & 5 & 8 & 9 & 12 & 15 & 18 & 20 & 25 & 23 & 25 & 22 \\
5 & 8 & 0 & 7 & 8 & 14 & 3 & 11 & 14 & 15 & 18 & 14 & 24 & 26 & 29 & 22 & 24 & 28 \\
15 & 18 & 10 & 0 & 19 & 24 & 13 & 4 & 7 & 8 & 11 & 14 & 17 & 19 & 24 & 22 & 24 & 21 \\
21 & 24 & 26 & 29 & 0 & 5 & 25 & 4 & 7 & 8 & 11 & 14 & 17 & 19 & 24 & 22 & 24 & 21 \\
16 & 19 & 21 & 24 & 20 & 0 & 24 & 5 & 8 & 9 & 12 & 15 & 18 & 20 & 25 & 23 & 25 & 39 \\
8 & 11 & 3 & 10 & 12 & 17 & 0 & 14 & 17 & 18 & 21 & 11 & 27 & 29 & 26 & 19 & 21 & 31 \\
29 & 32 & 24 & 31 & 33 & 38 & 21 & 0 & 3 & 4 & 7 & 10 & 13 & 15 & 20 & 18 & 20 & 17 \\
26 & 29 & 21 & 28 & 30 & 35 & 18 & 32 & 0 & 7 & 4 & 7 & 16 & 12 & 17 & 15 & 17 & 20 \\
33 & 36 & 28 & 35 & 37 & 42 & 25 & 39 & 7 & 0 & 3 & 14 & 9 & 11 & 16 & 16 & 19 & 13 \\
30 & 33 & 25 & 32 & 34 & 39 & 22 & 26 & 4 & 3 & 0 & 11 & 12 & 8 & 13 & 13 & 16 & 16 \\
19 & 22 & 14 & 21 & 23 & 28 & 11 & 25 & 7 & 14 & 11 & 0 & 23 & 19 & 15 & 7.5 & 10 & 27 \\
42 & 45 & 37 & 44 & 46 & 51 & 34 & 48 & 16 & 9 & 12 & 23 & 0 & 4 & 9 & 16 & 14 & 4 \\
38 & 41 & 33 & 40 & 42 & 47 & 30 & 44 & 12 & 11 & 8 & 19 & 4 & 0 & 5 & 12 & 10 & 8 \\
33 & 36 & 28 & 35 & 37 & 42 & 25 & 39 & 17 & 16 & 13 & 14 & 9 & 5 & 0 & 6.5 & 5 & 13 \\
27 & 30 & 22 & 29 & 31 & 36 & 19 & 33 & 15 & 14 & 11 & 7.5 & 17 & 13 & 7.5 & 0 & 2.5 & 21 \\
28 & 31 & 23 & 30 & 32 & 37 & 20 & 34 & 16 & 16 & 13 & 9 & 14 & 10 & 5 & 1.5 & 0 & 18 \\
45 & 48 & 40 & 47 & 49 & 54 & 37 & 51 & 19 & 12 & 15 & 26 & 3 & 7 & 12 & 19 & 17 & 0
\end{bmatrix}$$

The elements of matrix D^{18} are the lengths of all possible shortest paths. There were no value changes from D^{17} to D^{18}. We can now use matrix D^{18} and the corresponding paths to determine the shortest path from one vertex to another. For example, the element $d^{3}_{7,2}$ represents the shortest path from vertex 7 to vertex 2 and is of length 11. The corresponding path is (7,3),(3,1),(1,2), meaning this path passes through vertices 3 and 1 respectively. Since Floyd's algorithm finds the shortest path from every vertex to every other vertex, it makes sense that all the elements have an actual value and none of them remained infinity.

My destination from the parking lot was just off Main St. and north of vertex 11. For simplicity, I will say my final destination was vertex 11. Matrix D^{18} has the length from vertex 1 to vertex 11 as 15. Looking at the map, notice that the greatest intermediate vertex is 9. This is important in finding the corresponding path, which is (1,2),(2,5),(5,8),(8,9),(9,11). Refer back to Figure 4 to follow the path I should have taken.

The application of Floyd's algorithm to my proposed problem seems trivial since one could easily add up the weights in Figure 4. I only split the city into four quadrants and then assumed that once a car crossed into one of those quadrants, traffic would no longer be an issue. I also assumed that the great number of cars had different destinations that were evenly dispersed across the city. For this algorithm to be more necessary and accurate, the city could be split into more sections, perhaps even using all possible streets and intersections.

With Floyd's algorithm, we have found the shortest paths from every vertex to every other vertex. To improve on this, we could determine how many cars should take each of the shortest paths according to their final destinations. This could also minimize the time it takes to get all cars to their destinations.

Dijkstra provided the basis for finding the shortest path from an initial source to one destination. From this algorithm, Floyd was able to develop his algorithm which introduced matrices. These matrices can be used to follow the development of the shortest paths, as the intermediate vertices expand from using only vertex 1 up to vertex $k - 1$. It is important to document the paths used when calculating the next matrix so we know how the final length is determined.

After setting the groundwork by introducing these algorithms, it was possible to determine the path I should have taken. Calculating all the matrices and keeping track of the corresponding paths of each element made it possible to trace the more efficient path in Figure 4. Comparing that path to the path outlined in Figure 1 shows that I did not choose the optimal way home.

It would not be too difficult to just directly calculate the paths between each of the vertices in my proposed problem. If all streets in the city were considered, it would be more difficult. Then adding in all the different variables such as one-way streets and stop signs would make direct calculations overly time consuming. This is where the application of Floyd's algorithm shows its benefits by doing that work for you.

References

[1] J. R. Evans and E. Minieka, *Optimization Algorithms for Networks and Graphs*, Marcel Dekker, Inc, New York, New York, 1992.

[2] E. G. Goodaire and M. M. Parmenter, *Discrete Mathematics with Graph Theory*, Prentice-Hall, Upper Saddle River, New Jersey, 1998.

[3] L. Goodman, A. Lauschke, and E. W. Weisstein, *Dijkstra's Algorithm*. From MathWorld–A Wolfram Web Resource, `http://mathworld.wolfram.com/DijkstrasAlgorithm.html`

Characterization of the Vertex-Reinforced Random Walk and Trapping Subgraphs

Thomas Alden Gassert, *student*

Bowdoin College
Brunswick, ME 04011

and

Aaron Yeager, *student*

MO Alpha
Missouri State University
Springfield, MO 65806

Presented at the 2008 North Central Regional Convention.

1. Introduction

A *vertex-reinforced random walk* (VRRW) as defined by Pemantle in [1] is a random walk defined on a locally finite graph G that is more likely to move to vertices which it has previously visited. We note that in the use of locally finite we mean that every vertex of G has finite degree. Let the walk be described by the sequence $\{X_t\}_{i=0}^{\infty}$, where X_t is the location of the walk at time t. We define a counting function

$$Z(t,v) = 1 + |\Phi(t,v)|$$

where $\Phi(t,v) = \{t' : X_{t'} = v, t' \leq t\}$ is the set of times the walk visits a vertex v by time t. For a subset $V \subseteq G$, we define

$$Z(t,V) = \sum_{v \in V} Z(t,v).$$

The probability of moving from X_t to $x \in G$ is given by

$$\mathsf{P}(X_{t+1} = x : X_1, \ldots, X_t) = \frac{Z(t,x)}{\sum_{y \sim X_t} Z(t,y)}$$

if $x \sim X_t$ and 0 otherwise. If a vertex x is adjacent to vertex y we write $x \sim y$. Similarly if x is adjacent to all the vertices in $S \subseteq G$ we write $x \sim S$.

Definition 1 The *boundary* of a set of vertices $S \in G$ is the set

$$\partial S = \{b \in G \backslash S : b \sim S\}.$$

Definition 2 A subset $S \subseteq G$ is a *complete n-partite graph* if it is a disjoint union of nonempty sets $V_1, ..., V_n$, also called *pseudo-vertices*, such that if $x \in V_i$, $y \in V_j$, then $x \sim y \Longleftrightarrow i \neq j$.

Definition 3 The *essential range* of a random walk is the set of points the walk hits infinitely often.

With positive probability, the essential range of the VRRW on an infinite connected graph will be a finite, non-empty set of vertices called a trapping subgraph.[1]

Definition 4 A subgraph $G' \subseteq G$ is a *trapping subgraph* if is is composed of a complete n-partite graph $S = \bigcup\limits_{i=1}^{n} V_i$ union its boundary $B = \partial S$ and for any $b \in B$ there exists $V_i \subseteq S$ and $x \in S \backslash V_i$ such that $b \nsim V_i \cup \{x\}$.

Remark 1 Since G is a locally finite graph, $|G'| < \infty$.

2. Survey of Previous Results Concerning the VRRW

By grouping vertices within the trapping subgraph in this manner, Volkov [3] obtains a partial result concerning the time distribution of the walk in G'.

Theorem 2 Let $G' = S \cup B$, $S = \bigcup\limits_{i=1}^{n} V_i$, be a trapping subgraph of G. Then for the VRRW which originates on G', with a positive probability there exist a set of positive numbers $\{\alpha_v, v \in S\}$ with $\sum\limits_{v \in S} \alpha_v = 1$ such that the following are fulfilled:

1. VRRW never leaves G'.
2. $Z(t, v)/t \rightarrow \alpha_v$ for all $v \in S$ as $t \rightarrow \infty$.
3. $\sum\limits_{v \in V_i} \alpha_v = \frac{1}{n}$ for all $i \in \{1, ..., n\}$.
4. $\log Z(t, b) / \log t \rightarrow \frac{n}{n-1} \sum\limits_{x \in S, x \sim b} \alpha_x$ for all $b \in B$. [3]

Notice that (3) states that the sum of the limiting probabilities of the vertices in each pseudo-vertex adds to the limiting probability of the pseudo-vertex that contains them. Also take notice that imposing the additional assumption that if there exists a vertex in each pseudo-vertex that is not adjacent to any boundary point, (4) implies that the empirical weight of each boundary point is zero.

In this paper we provide an alternate proof of (3), and with the additional assumption stated above we also give an alternate proof of (4).

3. Survey of Previous Results Concerning Locating a Trapping Subgraph in an Arbitrary Graph

The following algorithm, as given by Volkov in [3], can be used to locate a trapping subgraph on an arbitrary, locally finite graph G. This algorithm will be successful on any finite graph and on graphs with bounded degree. However, this algorithm will not always be successful. We will look at some of the cases where the algorithm will fail.

Definition 5 A *clique* is a complete subgraph C which is not a subset of a larger complete subgraph.

ALGORITHM FOR LOCATING A TRAPPING SUBGRAPH

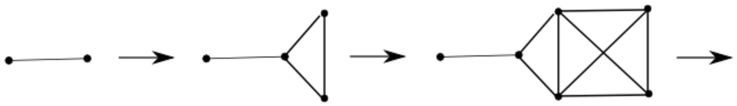
1. Locate a clique $C = \bigcup_{i=1}^{n} \{v_i\} \subseteq G$. If there does not exist a vertex $b \in \partial C$ connected to more than $n-2$ vertices in C, then C fulfills definition 4, and the clique is itself a trapping subgraph.
2. If there exists a $b \in \partial C$ that has an edge to $n-1$ vertices in C, assume without loss of generality that $b \nsim v_1$. Construct the pseudo-vertices V_i such that $v_i \in V_i$. Then fill out each pseudo-vertex in order beginning with V_1 according to the following:
$$V_i = \left\{ y \in G : y \sim x \forall x \in \bigcup_{j \neq i} V_j \right\}.$$
3. If for some V_i there exist $x, y \in V_i$ such that $x \sim y$, then we can find a new complete subgraph of order $n+1$, namely,
$$\{v_1, v_2, ..., v_{i-1}, x, y, v_{i+1}, ..., v_n\}.$$
We then let C be a clique containing this set and restart the algorithm. Otherwise, let $S = \bigcup_{i=1}^{n} V_i$, which is a complete n-partite graph by construction.

4. If there exists $b \in \partial S$ such that $b \sim V_i$ for all i, then we can again find a complete subgraph of order $n+1$ containing b and a representative from each V_i that is adjacent to b. Again we find a clique containing this subgraph and restart the algorithm. If this algorithm terminates, then $S \cup \partial S$ is a trapping subgraph.

The following are examples of graphs where the algorithm will not terminate.

Example 3 Define a strictly increasing sequence of integers $\{n_i\}_{i=1}^{\infty}$, $n_1 > 1$. Let $A_1 = K_{n_1}$, and construct A_i by attaching $K_{n_{i+1}}$ to $K_{n_i} \subseteq A_{i-1}$ along a common subset $K_{n_i - 1} \subseteq K_{n_{i+1}}$. Let $m_j = i$, where A_i is the graph of least index such that $K_{n_{i+1}} \cap A_i \neq \varnothing$. Continue the construction such that $m_1, m_2, ...$ is a non-terminating, strictly increasing series.

We call $G = \bigcup\limits_{i=1}^{\infty} A_i$ the *glue graph*. The glue graph is an infinite, but locally finite graph for which the algorithm will never terminate. In particular, the algorithm will always result in a pseudo-vertex containing two connected vertices.



An example of the first three steps of a glue graph construction

Theorem 4 The glue graph does not contain a trapping subgraph.

Proof Let G, $\{n_i\}$, and A_1, A_2,... be defined as above. If G contains a trapping subgraph G', then it must be that $G' \subseteq A_n$ for some n. We will show that any trapping subgraph in A_i is not a trapping subgraph in A_{i+1} and therefore is not a trapping subgraph in G.

Consider A_i for some $i \in \mathbb{N}$. By construction, this graph only contains one clique, K_{n_i}. Thus the only trapping subgraph in A_i is the clique of order n_i and its boundary points.

Now consider $A_{i+1} = A_i$ with $K_{n_{i+1}}$ attached. Attaching $K_{n_{i+1}}$ to A_i introduces $n_{i+1} - n_i + 1$ boundary points to the trapping subgraph, each of which borders $n-1$ vertices in the clique. These border points violate the properties of a trapping subgraph. Thus, the trapping subgraph in A_n is not a trapping subgraph in A_{n+1}, and therefore is not a trapping subgraph in G. ■

The following is an example of a different shortcoming of the algorithm. On this graph, the algorithm always terminates, but there are subgraphs which the algorithm is never able to locate.

Example 5 There are two types of trapping subgraphs on the square lattice: one with the n-partite portion in the shape of a box and all the neighbors, and the other with the n-partite portion in the shape of a cross and with all the boundary points.

Beginning with any clique in the lattice, the first step of the algorithm will add all boundary points of v_2 to the first pseudo-vertex, immediately finding a trapping subgraph in the shape of a cross. The algorithm never results in a box-shaped trapping subgraph.

An algorithm which cycles through the pseudo-vertex adding only a single vertex to each pseudo-vertex at each pass and terminates when no additional points can be added to any of the pseudo-vertices would be able to find each trapping subgraph containing the clique depending on the order adjacent vertices were added.

Definition 6 An ordering $(v_1, ..., v_n)$ on the set of vertices of a clique contained in a trapping subgraph is called *good* if, when using the ordering, the same trapping subgraph is found when the algorithm is applied.

In Example 5, there is no good ordering for cliques contained in a box-shaped trapping subgraph. Every clique contained in a cross-shaped trapping subgraph has a good ordering.

Let C_n be a clique of order n contained in a trapping subgraph G'. Define $B' = \{b \in \partial S : b \text{ is adjacent to } n-1 \text{ vertices in } C_n\}$.

If $|B'| > 0$, define $B_i = \{x \in C_n : x \sim b_i \in B'\}$.

Conjecture 1 Every $C \subseteq G'$, where $|C| > 2$, has a good ordering. Furthermore, $v_1 \in \bigcup B_i$.

4. Limit Probabilities for Trapping Subgraphs

We give simplified proofs of the results of theorem 2 in the case where G is itself a trapping subgraph.

Definition 7 The *empirical weight* of a subset $T \subseteq G$ is the ratio of time spent in T to the total number of steps in the walk.

The empirical weight of a vertex v is given by

$$\frac{Z(t,v)}{t}.$$

The empirical weight of a pseudo-vertex V is

$$\frac{Z(t,V)}{t}.$$

For all points in the trapping subgraph, we are interested in the distribution of their empirical weights in the limit as $t \to \infty$. Assume that the empirical weights converge in the limit. Then for any $x \in G$ there exists a corresponding probability $p(x)$ which is the limit of the empirical weight of x.

Theorem 6 For the VRRW on an n-partite graph G with limiting distribution $p\left(x\right) > 0$, we have the fixed point condition

$$\sum_{y\sim x} \frac{p\left(y\right)}{\sum\limits_{z\sim y} p\left(z\right)} = 1,$$

and all the pseudo vertices of G have empirical weight $\frac{1}{n}$.

Proof Note that the probability of being at vertex x at the current step is $p\left(x\right)$. The probability of going to x on the next step is given by the transform probability

$$Tp\left(x\right) = \sum_{y\sim x} p\left(y\right) \frac{p\left(x\right)}{\sum\limits_{z\sim y} p\left(z\right)}.$$

In the limiting distribution

$$Tp\left(x\right) = p\left(x\right).$$

Since each $p\left(x\right) > 0$, we achieve the fixed point condition

$$\sum_{y\sim x} \frac{p\left(y\right)}{\sum\limits_{z\sim y} p\left(z\right)} = 1.$$

This shows that

$$\sum_{x\in G} p\left(x\right) = 1.$$

Recall that $G = \bigcup_{i=1}^{n} V_i$. Let $p_i = \sum_{x\in V_i} p\left(x\right)$ be the probability of each pseudo-vertex in G. Since all probabilities must sum to one,

$$\sum_{k\neq j} p_k = 1 - p_j.$$

For any $x \in V_i$, by the fixed point condition we have

$$1 = \sum_{y\sim x} \frac{p\left(y\right)}{\sum\limits_{z\sim y} p\left(z\right)} = \sum_{j\neq i}\sum_{y\in V_j} \frac{p\left(y\right)}{\sum\limits_{k\neq j}\sum\limits_{z\in V_k} p\left(z\right)} = \sum_{j\neq i} \frac{p_j}{\sum\limits_{k\neq j} p_k} = \sum_{j\neq i} \frac{p_j}{1-p_j}.$$

Now let $u_j = \frac{p_j}{1-p_j}$, so that $\sum\limits_{j\neq i} u_j = 1$. If we expand this summand we have the following array

$$\begin{aligned}
0 + u_2 + u_3 + \cdots + u_n &= 1\\
u_1 + 0 + u_3 + \cdots + u_n &= 1\\
u_1 + u_2 + 0 + \cdots + u_n &= 1\\
&\vdots\\
u_1 + u_2 + u_3 + \cdots + 0 &= 1.
\end{aligned}$$

If we multiply the j^{th} equation by $(2-n)$ to any one of the equations and sum over all of them we see that we have a telescoping sum that yields $(n-1)\,u_j = 1$. Solving for u_j shows that

$$u_j = \frac{1}{n-1}.$$

Solving for p_j gives

$$p_j = \frac{u_j}{1+u_j} = \frac{\frac{1}{n-1}}{1+\frac{1}{n-1}} = \frac{1}{n}. \qquad \blacksquare$$

Our next theorem regards the empirical weights of the boundary points.

Theorem 7 Let G be a trapping subgraph. Let S be the complete n-partite portion of G, and suppose that S has any number of boundary points. So long as there exists a vertex in each pseudo-vertex that is not adjacent to any boundary point, the empirical weight of each boundary point is zero.

Proof For any boundary point $b \sim V_i$, we can find $x, y \in V_i$ such that x$\sim$ b and y is not adjacent to any point in the boundary. Evaluating the transform equation for y gives

$$\sum_{z\sim y} \frac{p(z)}{\sum\limits_{w\sim z} p(w)} = 1,$$

and for x

$$\sum_{z\sim x} \frac{p(z)}{\sum\limits_{w\sim z} p(w)} = 1.$$

Notice that the neighbors of y are a subset of the neighbors of x, which is also adjacent to m boundary points $\{b_i\}_{i=1}^m$. Thus we have

$$1 = \sum_{z\sim x} \frac{p(z)}{\sum\limits_{w\sim z} p(w)} = 1 = \sum_{z\sim y} \frac{p(z)}{\sum\limits_{w\sim z} p(w)} + \sum_{i=1}^{m} \frac{p(b_i)}{p(x)} = 1 + \sum_{i=1}^{m} \frac{p(b_i)}{p(x)},$$

which implies that each $p(b_i) = 0$. ■

Acknowledgements. This research was initiated under the auspices of a Research Experience for Undergraduates program hosted at Penn State University during Summer, 2007, under the advice of Vaughn Climenhaga. We are grateful to all for the opportunities provided.

References

[1] R. Pemantle and S. Volkov, Vertex-Reinforced Random Walk on Z Has Finite Range, *The Annals of Probability,* **27**(3) (July 1999), 1368-1388.

[2] P. Tarr, Vertex-Reinforced Random Walk on Z Eventually Gets Stuck on Five Points, *The Annals of Probability,* **32**(3B) (July 2004), 2650-2701.

[3] S. Volkov, Vertex-Reinforced Random Walk on Arbitary Graphs, *The Annals of Probability,* **29**(1) (Jan. 2001), 66-91.

A Group-Theoretic Description of an N-Tone Equal Temperament Pitch System

Carmen Weddell, *student*

MI Alpha

Albion College
Albion, MI 49224

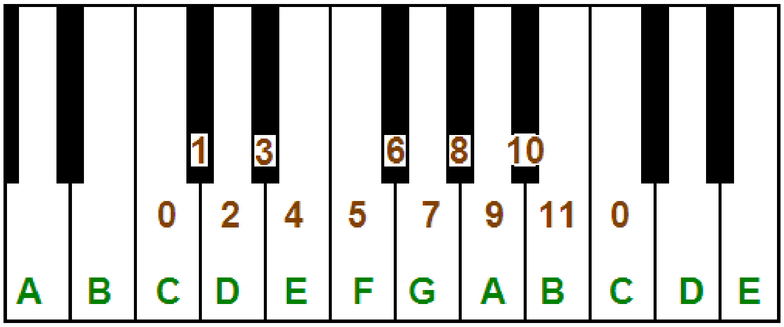
1. Introduction

The musical system employed in all western concert music consists of twelve tones. To any lover of music, it is evident that this system of pitches is coherent and enjoyable, but what is it that makes the number 12 succeed? Are there other divisions of the octave that give rise to acceptable tonal systems? To explore these questions, I undertake a mathematical investigation of our musical system. My goal is to identify desirable traits of the 12-tone system and apply them to a higher-ordered N-tone system.

In this exploration, I expand upon the basic principles underlying previously published work on this topic. Several mathematicians and music theorists have made significant contributions to the field of group-theoretic representations of musical principles; most notable to my research are Gerald Balzano [1], Frank J. Budden [2], and Paul F. Zweifel [3].

2. A Generalized Description of Equal-Temperament Pitch Systems

The common musical system consists of twelve pitch classes. A pitch class can be understood as the set of all pitches of the same name. Thus, all notes named D belong to one pitch class and all notes labeled A$\flat$ (pronounced "A flat") to another. In general, X$\flat$ is the black key directly to the left of X on the keyboard. We assign each pitch class to an element of $\mathbb{Z}_{12} = \{0, 1, 2, \ldots, 11\}$ as shown in figure 1; all C's are 0, all D's are 2, and all A$\flat$'s are 8.



Standard Keyboard Relating Note Names and Elements of $\mathbb{Z}_{12}$

In a theoretical system, we can split the octave into N pitch classes. We model this system with $\mathbb{Z}_N = \{0, 1, 2, \ldots, N-1\}$. Our goal is to determine which selections of N result in sufficiently interesting systems with properties similar to those of the 12-tone system.

As noted by Balzano [1], the group $\mathbb{Z}_{12}$ has two possible—and equally necessary—interpretations. The first is static: 0 corresponds to C, 5 to F, 8 to G♯ (or A♭) , and so on.[1] The second interpretation of our set is dynamic; rather than a pitch class, each element of $\mathbb{Z}_{12}$ represents an interval: 0 is a perfect unison, 5 is a five-semitone interval (a perfect fourth), and 8 is an eight-semitone interval (a minor sixth).

I have implied that $\mathbb{Z}_N$ forms a mathematical group. A group is a mathematical object defined by a set of elements and an operation with four properties: closure, associativity, inverses, and an identity. In the case of $\mathbb{Z}_N$, the operation is addition (mod N); addition of group elements is carried out as usual, but the result is reduced by N as necessary to remain in the set. In $\mathbb{Z}_{12}$, for example, $9 + 5 = 14 = 2 \pmod{12}$. $\mathbb{Z}_N$ is closed by the description of addition $(\text{mod } N)$, addition is associative, the identity element is 0, and the inverse of a given element a is $N - a$ since $a + (N - a) = N = 0(\text{mod } N)$. Since all four properties are satisfied, $\mathbb{Z}_N$ is a group under addition $(\text{mod } N)$.

To understand addition of group elements in a musical sense, one must understand the static/dynamic duality of the system. The equation $6 + 7 = 1 \pmod{12}$ makes sense in mathematical terms, but replacing the numbers with the pitch classes gives us musical gibberish: F♯ + G = C♯. Rather than interpreting each element of $\mathbb{Z}_{12}$ as a pitch class, however, we can interpret it as an ascending interval of a given number of semi-tones. Using this interpretation of the system, we have two plausible interpretations: (1) we

---

[1] $G\sharp$ and $A\flat$ are enharmonic; they are simply two names for the same note. Likewise, $D\flat = C\sharp$, $E\flat = D\sharp$, $G\flat = F\sharp$, and $B\flat = A\sharp$.

can start on a pitch, ascend a specific number of semitones, and end on a new pitch (F♯ + 7 semitones = C♯), or (2) we can represent both group elements as intervals and combine them into a new interval (6 semitones + 7 semitones = 1 semitone).

3. The Circle of Semitones

Mathematically, $\mathbb{Z}_N$ is cyclic group because it is produced by multiples of a single group element called a generator. In this case, we use the notation $\langle 1\rangle$ to indicate the group generated by multiples of 1. Since every element of $\mathbb{Z}_N$ is a multiple of 1, we write

$$< \langle 1\rangle = \{0(1), 1(1), 2(1), \ldots, (N{-}1)(1)\}^2 = \{0, 1, 2, \ldots, N{-}1\} = \mathbb{Z}_N,$$

and we call 1 a generator of $\mathbb{Z}_N$. This sequence is called the circle of semitones, which is shown in figure 2; musically $\langle 1\rangle$ represents the ascending chromatic scale. In the 12-tone system, we see that

$$\begin{aligned}\langle 11\rangle &= \{0(11), 1(11), 2(11), \ldots, 10(11), 11(11)\}\\ &= \{0, 11, 22(\operatorname{mod} 12), \ldots, 110(\operatorname{mod} 12), 121(\operatorname{mod} 12)\}\\ &= \{0, 11, 10, \ldots, 2, 1\}\end{aligned}$$

represents the descending chromatic scale, which is the circle of semitones read counterclockwise.[3] Since $\langle 11\rangle$ contains every element of $\mathbb{Z}_{12}$, 11 is another generator of the group; $\langle 11\rangle = \mathbb{Z}_{12}$. In $\mathbb{Z}_N$, $\langle N-1\rangle$ is the descending chromatic scale.

---

[2] It is important to recognize that in this case, the successive integers $0, 1, 2, ..., 11$ are not considered group elements. I use the notation $3\,(1)$ to represent $1+1+1$, where 1 is a group element. For clarity, I will always place the group element inside the parentheses and the multiplier in front of them.

[3] Technically speaking, mathematicians consider $\langle 1\rangle$ and $\langle 11\rangle$ identical. IN traditional set theory, the order of the elements within a set is irrelevant. For the purposes of this discussion, however, the order of elements within a set can be enlightening.

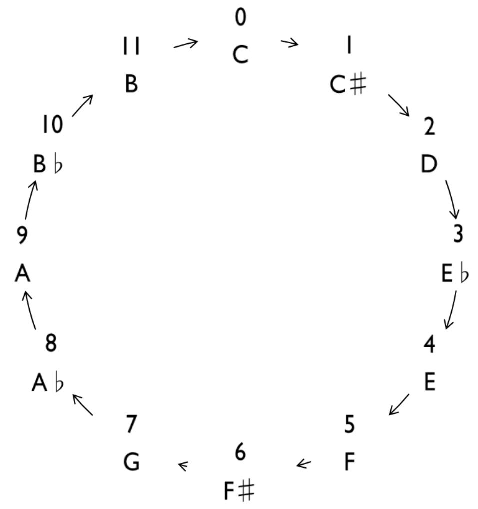


Figure 2. Circle of Semitones

4. The Circle of Fifths

The group $\mathbb{Z}_{12}$ has two generators besides 1 and 11: the elements 5 and 7. Consider

$$\begin{aligned}\langle 7\rangle &= \{0(7),1(7),2(7),3(7),4(7),5(7),\ldots,9(7),10(7),11(7)\}\\ &= \{0,7,2,9,4,11,\ldots,3,10,5\}\\ &= \mathbb{Z}_{12};\end{aligned}$$

replacement of numeric group elements with pitch classes reveals $\langle 7\rangle = \{C,G,D,A,E,B,\ldots,E\flat,B\flat,F\}$. The musician will immediately recognize the importance of this sequence; 7 generates the circle of fifths, an important sequence of pitch classes that might be considered the backbone of western classical music.

Of course, we should not be surprised that the element 7 generates the circle of fifths because its dynamic interpretation as an interval of 7 semitones corresponds to the perfect fifth itself. Likewise, 5 represents the perfect fourth, which is the inverse of 7. The elements of $\langle 7\rangle$ in reverse order correspond to $\langle 5\rangle$, which is the circle of fourths.

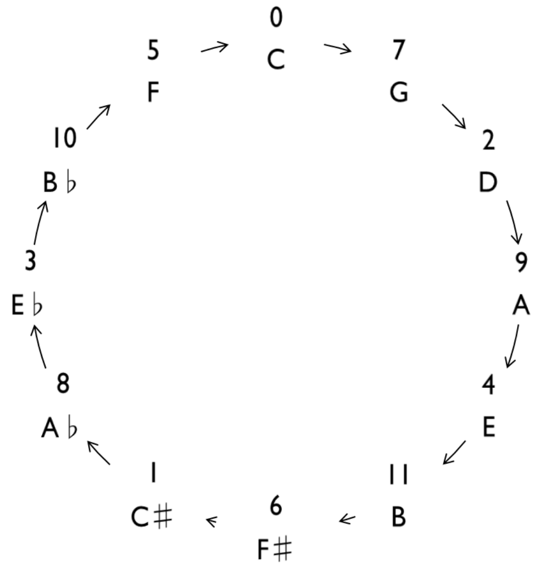


Figure 3. Circle of Fifths, 12-Tone System

In an N-tone system, we need two elements, y and $-y$ (which is $N-y$) to act as the generalized fifth and fourth. In the case $N = 12$, it is easy to determine $y = 7$ since 1, 5, 7, and 11 are the only generators of $\mathbb{Z}_{12}$, but in cases where N is larger, we have several pairs of generators, and we must determine which one should be the fourth and fifth.[4]

Systems for which $N/2+1$ and $N/2-1$ are generators of $\mathbb{Z}_N$ display properties like those of $\mathbb{Z}_{12}$, and this requirement imposes some restrictions on our choice of N. N. First, in order for $N/2$ to be an integer, N must be even. Since N is even, $N/2 \pm 1$ must be odd to generate $\mathbb{Z}_N$. Thus $N/2$ must be even, so N itself must be a multiple of 4. The smallest possible case, $N = 4$, does not have an acceptable fourth and fifth since it has only one pair of generators. Beginning with $\mathbb{Z}_8$, however, each possible system has at least two pairs of generators.

5. The Structure of Diatonic Scales

In addition to the chromatic scale, there are two other important scales used in Western concert music: the 7-tone diatonic and 5-tone pentatonic scales. Musically, diatonic scales are the most common; they are either

---

[4] It is a theorem in elementary abstract algebra that x is a generator of $\mathbb{Z}_N$ if and only if $\gcd(N, x) = 1$.

major (as in “Twinkle, Twinkle, Little Star”) or minor (as in “God Rest ye Merry Gentlemen”). The pentatonic scale, on the other hand, is less common in western concert music. One example is the tune of “Amazing Grace.” We can build both of these scales by connecting elements in the circle of fifths. Because the diatonic scale is used more often than any other scale, we will focus our attention on its construction and properties. In the 12-tone system, we construct a diatonic scale by connecting seven adjacent elements in the circle of fifths.

When we join the element 5 and the following six elements, we have the set $\{5, 0, 7, 2, 9, 4, 11\}$. Reordering the elements gives us $\{0, 2, 4, 5, 7, 9, 11\} = \{C, D, E, F, G, A, B\}$. This is the C major scale, which means that C (the group element 0) is the first note, or tonic, of the scale. Looking back to figure 1, we see that the C major scale is simply the set of white keys on the keyboard. The element 11 is called the leading tone to 0 because it is a single semitone below 0; it is the last element of the set both before and after reordering. The presence of a leading tone is an important property in every major diatonic scale.

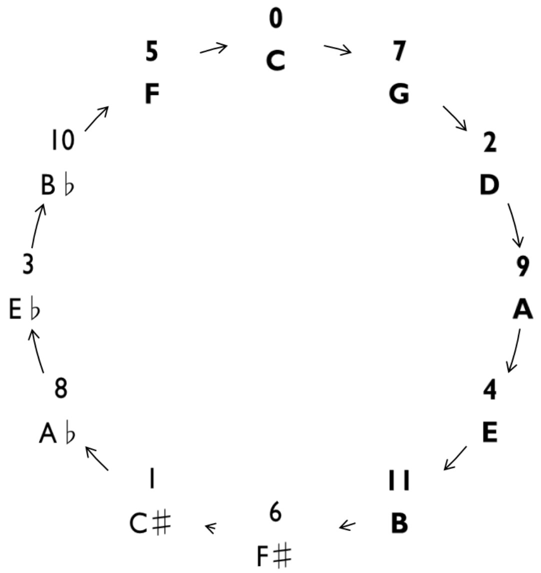


Figure 4. C Major Scale

The second important property of the diatonic scale is the $F \rightarrow F\sharp$ property.[1] This property states that the first connected element and the first non-connected element differ by a single semitone. In the case of the C major scale, the elements in question are 6 and 7. This property is universal, and, musically, it means that transitions between keys can be smooth. The pentatonic scale, built of five connected elements in the circle of fifths, also has the $F \to F\sharp$ property.

To form a diatonic scale in a generalized N-tone system, we must connect the tonic element, T, to the leading tone, $T-1$. It can be shown that this connected set contains $N/2$ elements. We must also include the element occurring directly before the tonic in the circle of fifths to ensure the $F \to F\sharp$ property. Again, this result can be proved.

Our set of connected elements begins one element before the tonic and ends with the leading tone; it spans $N/2+1$ elements of the circle of fifths. Our conclusion is that the diatonic scale with tonic 0 in an N-tone system where $N = 4k$ is $\{0, 2, 4, \ldots, N/2-2, N/2-1, N/2+1, N/2+3, \ldots, N-1\}$. More generally, the diatonic scale with tonic T in the same system is $\{T, T+2, T+4, \ldots, T+N/2-2, T+N/2-1, T+N/2+1, T+N/2+3, \ldots, T+N-1\}$. Note that the number of elements in the diatonic scale is the same as the number of semitones in the generalized fifth.

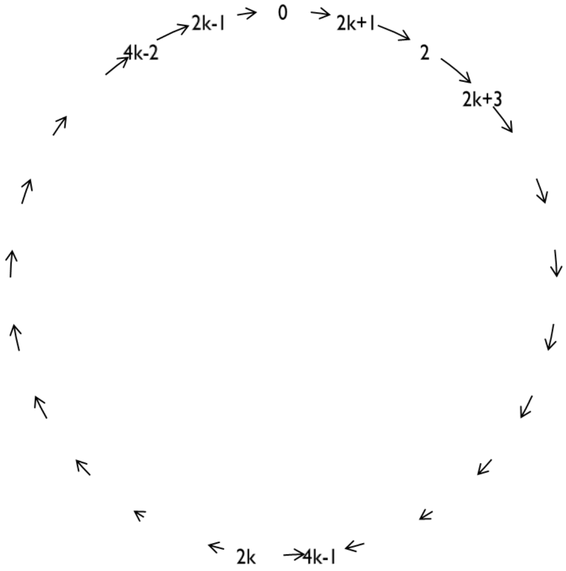


Figure 5. Diatonic Scale for N-tone System, $N = 4k$

In an N-tone system, the pentatonic scale is $N/2-1$ connected elements in the circle of fifths. As is the case in the common system, the pentatonic scale is the second connected subset of the circle of fifths to exhibit the $F \to F\sharp$ property.

6. Diatonic Chords

At this point, we have developed a diatonic scale for a generalized N-tone system with the restriction that N is a multiple of four. The next phase of our discovery must focus on the chord structure of this scale. For which choices of N does the generalized diatonic scale have an acceptable chord structure?

In the common system, consider the C major scale, $\{0,2,4,5,7,9,11\}$, and its associated tonic chord, $\{C,E,G\} = \{0,4,7\}$. The chord spans a fifth (7 semitones) and the third of the chord (the group element 4) defines the major and minor thirds of the system; the 4-semitone interval defined by {0, 4} is a major third and the 3-semitone interval defined by {4, 7} is the minor third. The fact that the major and minor thirds differ by a single semitone is crucial to our development of chords in higher-order systems.

Using the fact that the tonic chord of a major diatonic scale must be major, it can be shown that the major third must be an even number of semitones and the minor third an odd number. Finally, since one can prove that in an N-tone system, the first $N/4$ elements of the major scale with tonic 0 are even, one can also prove that N must take the form $8k+4$ for some integer k.

Our conclusion is that any choice of N where N has the form $8k+4$ for some integer k, $k \geq 1$, provides an acceptable mathematical alternative to the 12-tone system used in western music.

Chord	Quality of Chord	Roman Numeral
$\{0,4,7\}$	Major	I
$\{2,5,9\}$	Minor	ii
$\{4,7,11\}$	Minor	iii
$\{5,9,0\}$	Major	IV
$\{7,11,2\}$	Major	V
$\{9,0,4\}$	Minor	vi
$\{11,2,5\}$	Diminished	vii°

Figure 6. Diatonic Chords of the C Major Scale

In general, a diatonic chord is a chord formed by three notes of the diatonic scale. The tonic chord is one example of a diatonic chord, and the remaining six diatonic chords of the C-major scale are listed in Figure 6. Chords with a major third as their bottom interval are major chords. Chords with a major third as their top interval are minor chords. The final chord, $\{11,2,5\}$, has two minor thirds and only spans six semitones; it is a diminished chord. Finally, a roman numeral signifying the scale degree of its root and its quality denotes each chord. For major chords, the roman numerals appear in uppercase letters, and for minor and diminished chords, they are in lowercase letters. The ° symbol also marks the diminished chord.

Gerald Balzano developed a system of visualizing the chord structure of a diatonic scale.[1] After plotting the number of 4-semitone intervals (major thirds) along the x-axis and the number of 3-semitone intervals (minor thirds) along the y-axis, each discrete point (a, b) on the plane is labeled with its corresponding element in $\mathbb{Z}_{12}$: $3a + 4b \pmod{12}$.

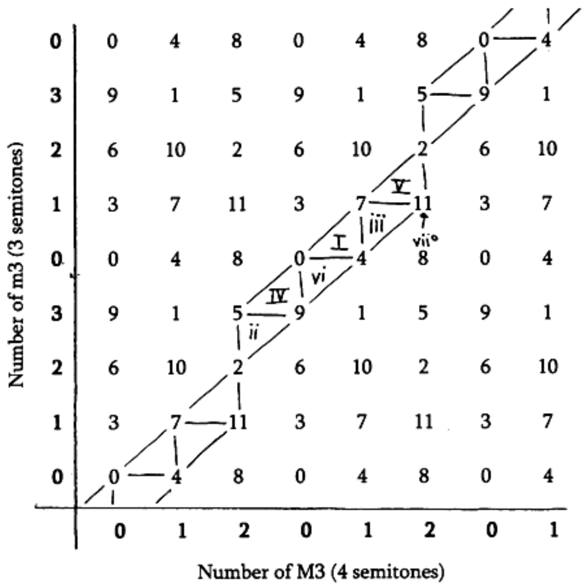


Figure 7

To view the chord structure of a given diatonic scale, simply connect the points corresponding to the elements of the specified scale. Figure 7 illustrates the structure of the C major scale, so the elements 0, 2, 4, 5, 7, 9, and 11 are connected. Each diatonic chord appears as a right triangle. Triangles corresponding to major chords have their right angle toward the lower right corner. Likewise, each minor chord appears as a right triangle with its right angle appearing in the upper left corner. Furthermore, the root of each chord appears at the lower left corner of the associated triangle. The vii° triad appears as a vertical line. Although figure 6 is specified to the case of the C major scale, the shape of the triangle chain remains unchanged for other diatonic systems.

At this point, we leave the remainder of the musical material to the musicians, and we continue our mathematical journey into the realm of higher-ordered musical systems.

7. The Case $\mathbf{N} = \mathbf{20}$

This section takes the generalizations stated above and specifies them to the case $N = 20$. We will look at the circle of fifths, diatonic scale, and chord structure for this theoretical musical system.

The case $N = 20$ is based upon an octave divided into 20 equal intervals; the octave itself is not changed, so semitones in this system are smaller than those of the common system. This system is the smallest alternative to the common system, $N = 12$. We consider the group $\mathbb{Z}_{20} = \{0, 1, 2, \ldots, 19\}$ under addition $(\bmod 20)$.

As in the case $N = 12$, the circle of semitones, shown in figure 8, is generated by the element 1.

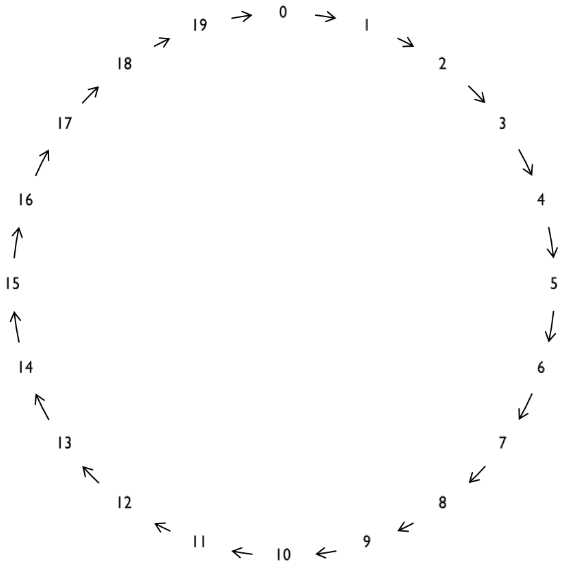


Figure 8. Circle of Semitones for 20-tone System

The circle of semitones reveals that $\mathbb{Z}_{20} = \langle 1 \rangle$. Other generators of $\mathbb{Z}_{20}$ include 3, 7, 9, 11, 13, 17, and 19. In the generalization to an N-tone system, however, we stated that the generalized fifth is the element $N/2 + 1$. The generalized fifth for the 20-tone system, then, is 11, and the generalized fourth is 9. The generalized circle of fifths appears in figure 9.

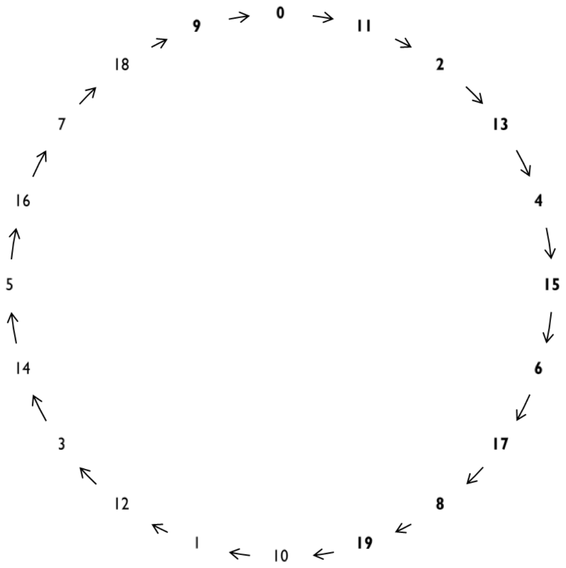


Figure 9. Circle of Fifths and Diatonic Scale for 20-tone System

Our next endeavor is to construct a diatonic scale in the 20-tone system. We do so by connecting 11 elements of the generalized circle of fifths. In particular, to construct the scale with tonic 0, we connect 11 elements beginning with 9.

By rearranging the connected elements, we have our major diatonic scale: $\{0, 2, 4, 6, 8, 9, 11, 13, 15, 17, 19\}$. Notice that replacing the first connected element, 9, with the first non-connected element, 10, results in a change of a single semitone. Thus, the 11-note scale within the 20-tone system displays the $F \rightarrow F\sharp$ property.

It was stated above that the pentatonic scale in an N-tone system is $N/2 - 1$ connected elements in the circle of fifths. In this case, the pentatonic scale should be 9 notes long. Starting at 9, then, the connected set ends with the element 17. The following element, 8, is again only a semitone away from the first element; as expected, the pentatonic scale also has the $F \rightarrow F\sharp$ property.

In the common system, the major scale with tonic 0 is the C major scale. On the keyboard, this scale is the set of white keys. Since the diatonic scale constructed above is in every other way analogous to the C major scale, it follows that the elements contained therein must determine the white keys of a generalized keyboard. Figure 10 shows a keyboard, which, with the correct tuning, would be capable of playing in the 20-tone system.

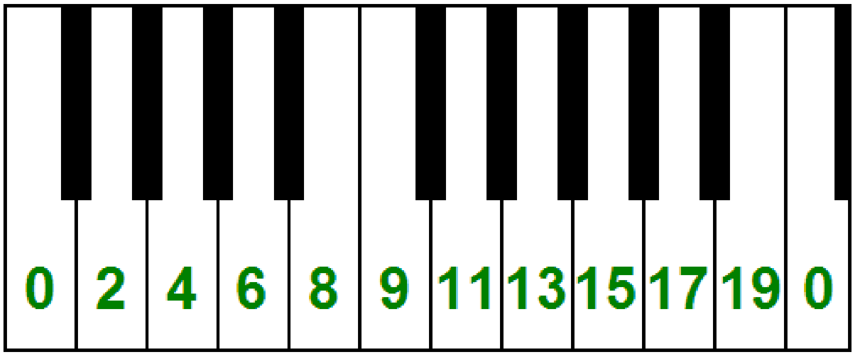


Figure 10. Generalized Keyboard for 20-tone System

The configuration of this keyboard is perfectly acceptable; like the usual key-board, black keys fall into two clusters. In the case $N = 12$, these two sets contain two and three black keys. Analogously, those of the 20-tone system come in sets of four and five. In addition, the pentatonic scale is formed by the black keys.

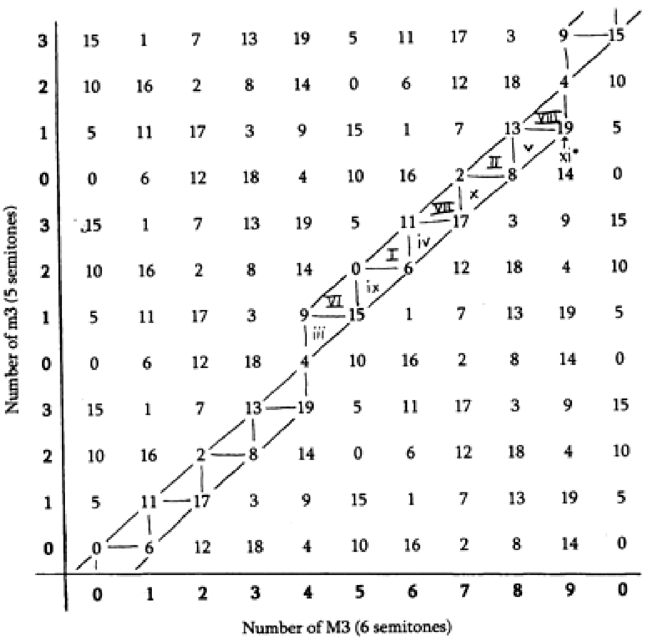
Our investigation into the 20-tone system continues with the chord structure of the system. First, we know the tonic chord has root 0 and spans a fifth, so the fifth of the chord is 11. We know the generalized major third must be an even number of semitones, and the generalized minor third one semitone smaller than the major third. Since $11 = 6+5$, it is clear that the major third must be 6 semitones and the minor third 5 semitones. Therefore, we choose the third of the chord to be 6, and the tonic chord is $\{0, 6, 11\}$. The remaining diatonic chords are summarized in figure 11.

Chord	Quality of Chord	Roman Numeral
$\{0, 6, 11\}$	Major	I
$\{2, 8, 13\}$	Major	II
$\{4, 9, 15\}$	Minor	iii
$\{6, 11, 17\}$	Minor	iv
$\{8, 13, 19\}$	Minor	v
$\{9, 15, 0\}$	Major	VI
$\{11, 17, 2\}$	Major	VII
$\{13, 19, 4\}$	Major	VIII
$\{15, 0, 6\}$	Minor	ix
$\{17, 2, 8\}$	Minor	x
$\{19, 4, 9\}$	Diminished	xi°

Figure 11. Diatonic Chords, $N = 20$

There is, of course, an analogue of the 12-tone Balzano diagram (figure 7) in the 20-tone system. We plot the number of major thirds along the x-axis and the number of minor thirds along the y-axis. In this case, then, the x- and y-axes denote the number of 6- and 5-semitone intervals, respectively.[5] Discrete points on the grid are labeled with the corresponding element of $\mathbb{Z}_{20}$; point (a, b) is $6a + 5b \pmod{20}$. In exact accordance with the case $N = 12$, the elements of the major diatonic scale are connected to reveal the corresponding major, minor, and diminished chords of the system.

As with the 12-tone system, major and minor diatonic chords are represented by triangles with right angles to the lower right and upper left, respectively. The root of each chord is again the lower left vertex of each triangle. The diminished chord, $xi^\circ = \{19, 4, 9\}$, appears as a vertical line.



3	15	1	7	13	19	5	11	17	3	9	15
2	10	16	2	8	14	0	6	12	18	4	10
1	5	11	17	3	9	15	1	7	13	19	5
0	0	6	12	18	4	10	16	2	8	14	0
3	15	1	7	13	19	5	11	17	3	9	15
2	10	16	2	8	14	0	6	12	18	4	10
1	5	11	17	3	9	15	1	7	13	19	5
0	0	6	12	18	4	10	16	2	8	14	0
3	15	1	7	13	19	5	11	17	3	9	15
2	10	16	2	8	14	0	6	12	18	4	10
1	5	11	17	3	9	15	1	7	13	19	5
0	0	6	12	18	4	10	16	2	8	14	0
	0	1	2	3	4	5	6	7	8	9	0

Figure 12. Balzano Diagram, $N = 20$

---

[5] Note that in the case $N = 12$, the major third is 4 semitones, and it is merely a coincidence that 12 is a multiple of 4; the fact that the major third in the 20-tone system is not a factor of 20 is no cause for worry. This disparity is what causes the x-axis to be numbered differently in the two diagrams.

8. Conclusion

Any choice of N where $N = 8k + 4$ for some integer $k \geq 1$ provides an acceptable mathematical alternative to the 12-tone system. Under this assumption, a generalized musical system has a generalized fourth and fifth which exhibit the same properties as those in the common 12-tone system. It is always possible to construct a diatonic and pentatonic scale with the desirable properties, and the overall diatonic chord scheme acts the same in each of these systems.

References

[1] G. Balzano, The Group-Theoretic Description of Twelve-Fold and Micro-tonal Pitch Systems, *Computer Music Journal,* **4** (1980), 66-84.

[2] F. J. Budden, *The Fascination of Groups*, Cambridge University Press, 1972, Ch. 23.

[3] P. F. Zweifel, Generalized Diatonic and Pentatonic Scales: A Group-Theoretic Approach, *Perspectives of New Music,* **34**(1).(1996), 140-61.

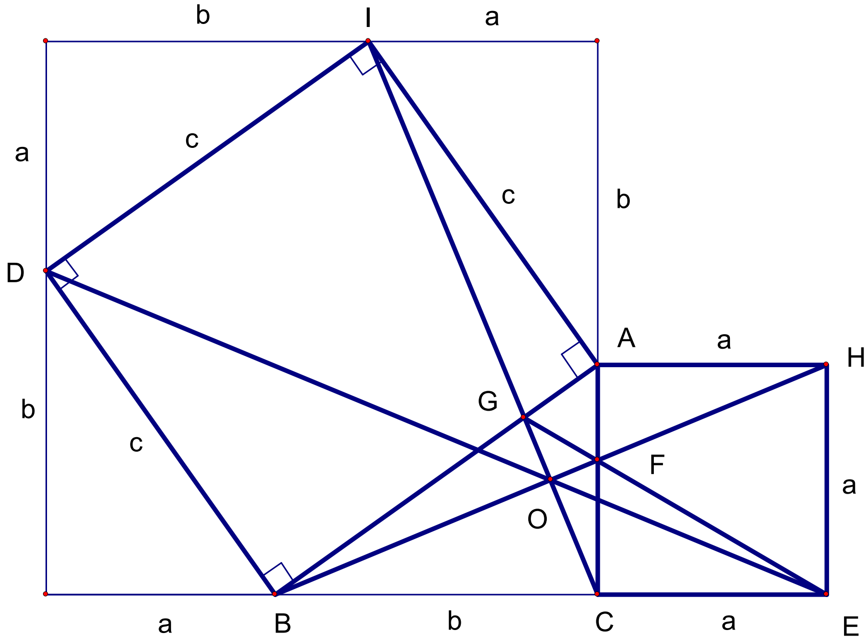
Two Short Pieces

Andrew Cusumano

Great Neck, NY 11023

1. A Trigonometric Theorem

Theorem 8 In the picture shown, line segments GF, DO, and BC all intersect at point E.



Proof:

1. Relative to point B, the equation of line IC is
$$y = a + b - \frac{(a+b)}{a}\left(x - (b-a)\right),$$
and the equation of line BH is
$$y = \frac{ax}{a+b},$$
so that for the coordintes of O, we have
$$\begin{aligned}
ay &= a^2 + ab - x\left(a+b\right) + ab - a^2 + b^2 - ab \\
\frac{a^2x}{a+b} &= a^2 + ab - x\left(a+b\right) - a^2 + b^2 \\
\frac{a^2x}{a+b} &= ab - x\left(a+b\right) + b^2 \\
a^2x &= a^2b + ab^2 - x\left(a+b\right)^2 + ab^2 + b^3 \\
a^2x + x\left(a+b\right)^2 &= a^2b + 2ab^2 + b^3 \\
x\left(a^2 + \left(a+b\right)^2\right) &= a^2b + 2ab^2 + b^3 \\
x\left(a^2 + \left(a+b\right)^2\right) &= b\left(a+b\right)^2.
\end{aligned}$$
Thus,
$$x = \frac{b\left(a+b\right)^2}{a^2 + \left(a+b\right)^2},$$
and
$$y = \frac{ab\left(a+b\right)^2}{\left(a+b\right)\left(a^2 + \left(a+b\right)^2\right)} = \frac{ab\left(a+b\right)}{a^2 + \left(a+b\right)^2}.$$

Relative to point E, the coordinates of point O are

$$\begin{aligned} x &= a+b-\frac{b\left(a+b\right)^2}{a^2+\left(a+b\right)^2} \\ &= \frac{\left(a+b\right)\left(a^2+\left(a+b\right)^2\right)-b\left(a+b\right)^2}{a^2+\left(a+b\right)^2} \\ &= \frac{a^2b+b\left(a+b\right)^2+a^3+a\left(a+b\right)^2-b\left(a+b\right)^2}{a^2+\left(a+b\right)^2} \\ &= \frac{a^3+a^2b+a\left(a+b\right)^2}{a^2+\left(a+b\right)^2} \\ &= \frac{a\left(a^2+ab+\left(a+b\right)^2\right)}{a^2+\left(a+b\right)^2}, \end{aligned}$$

and

$$y=\frac{ab\left(a+b\right)}{a^2+\left(a+b\right)^2}.$$

Thus,

$$\begin{aligned} \frac{y}{x} &= \frac{ab\left(a+b\right)}{a\left(a^2+ab+\left(a+b\right)^2\right)} \\ &= \frac{b\left(a+b\right)}{a^2+ab+\left(a+b\right)^2} \\ &= \frac{b\left(a+b\right)}{a\left(a+b\right)+\left(a+b\right)^2} \\ &= \frac{b}{a+\left(a+b\right)} \\ &= \frac{b}{2a+b}. \end{aligned}$$

Point D, relative to point E, has the coordinates $(2a+b,b)$. Thus, the slopes from E to points O and D are the same, so EO and ED are collinear, and therefore DO will meet BC at point E.

2. Relative to point B, the equation of line IC is

$$y = a + b - \frac{(a+b)}{a}\left(x - (b-a)\right),$$

and the equation of BA is

$$y = \frac{ax}{b},$$

so that for the coordinates of point G, we have

$$\begin{aligned} ay &= a^2 + ab - x\,(a+b) + ab - a^2 + b^2 - ab \\ ay &= ab - x\,(a+b) + b^2 \\ \frac{a^2x}{b} &= ab - x\,(a+b) + b^2 \\ a^2x &= ab^2 - bx\,(a+b) + b^3 \\ x\left(a^2 + b\,(a+b)\right) &= ab^2 + b^3, \end{aligned}$$

and, therefore,

$$x = \frac{b^2\,(a+b)}{a^2 + b\,(a+b)},$$

and

$$y = \frac{ab\,(a+b)}{a^2 + b\,(a+b)}.$$

Relative to point E, the coordinates of point G are

$$\begin{aligned} x &= a + b - \frac{b^2\,(a+b)}{a^2 + b\,(a+b)} \\ &= \frac{a^2\,(a+b) + b\,(a+b)^2 - b^2\,(a+b)}{a^2 + b\,(a+b)} \\ &= \frac{a^3 + a^2b + b\,(a+b)^2 - ab^2 - b^3}{a^2 + b\,(a+b)} \\ &= \frac{a^3 + a^2b + a^2b + 2ab^2 + b^3 - ab^2 - b^3}{a^2 + b\,(a+b)} \\ &= \frac{a^3 + 2a^2b + ab^2}{a^2 + b\,(a+b)} \\ &= \frac{a\left(a^2 + 2ab + b^2\right)}{a^2 + b\,(a+b)} \\ &= \frac{a\,(a+b)^2}{a^2 + b\,(a+b)}, \end{aligned}$$

and

$$y = \frac{ab\,(a+b)}{a^2 + b\,(a+b)}.$$

Relative to point E, we have

$$\begin{aligned} \frac{y}{x} &= \frac{ab\,(a+b)}{a\,(a+b)^2} \\ &= \frac{b}{a+b}. \end{aligned}$$

Point F, relative to point E, has the coordinates $\left(a, \frac{ab}{a+b}\right)$. Thus, the lines EF and EG have the same slope, so that EF and EG are collinear. Line GF will thus meet BC at point E.

2. Divergence of the Harmonic Series

Here is a quick proof that the harmonic series

$$S = \sum_{k=1}^{\infty} \frac{1}{k},$$

diverges. Let m and n be positive integers greater than one. Writing

$$\begin{aligned} S &= \left(\frac{1}{1} + \cdots + \frac{1}{n}\right) + \left(\frac{1}{n+1} + \cdots + \frac{1}{mn}\right) \\ &\quad + \left(\frac{1}{mn+1} + \cdots + \frac{1}{m^2 n}\right) + \left(\frac{1}{m^2 n+1} + \cdots + \frac{1}{m^3 n}\right) \\ &\quad + \left(\frac{1}{m^3 n+1} + \cdots + \frac{1}{m^4 n}\right) + \cdots, \end{aligned}$$

we see that

$$\begin{aligned} S &> \frac{n}{n} + \frac{mn-n}{mn} + \frac{m^2 n - mn}{m^2 n} + \frac{m^3 n - m^2 n}{m^3 n} + \frac{m^4 n - m^3 n}{m^4 n} + \cdot\cdot \\ S &> 1 + \frac{m-1}{m} + \frac{m-1}{m} + \frac{m-1}{m} + \frac{m-1}{m} + \cdots. \end{aligned}$$

Since the right-hand side of this last inequality approaches infinity, the divergence is proved.

Date Change for the Thirty-Seventh Biennial Convention of Kappa Mu Epsilon

The Thirty-Seventh Biennial Convention of Kappa Mu Epsilon will be held

March 26-28, 2009

in

Philadelphia, PA

This new date is one week earlier than previously announced. KME President-Elect Ron Wasserstein (ron@amstat.org) will send additional details to chapters soon. Each attending chapter will receive the usual travel expense ($.35/mile) reimbursement from the national office.

Students who wish to make a presentation at the national convention should submit an abstract of up to 500 words.The abstract should be accompanied by a letter from the student's project advisor, certifying that the student is doing the work specified in the abstract and the advisor's belief the student will have a fully prepared presentation by the time of the convention.

Please send the abstract and advisor letter by electronic mail not later than **February 2, 2009** to:

Dr. Ron Wasserstein, KME President-Elect
Executive Director, The American Statistical Association
ron@amstat.org

A presenter who has not prepared a formal written paper by the time of the convention is encouraged to do so soon after the convention, so that the paper can be submitted for possible publication in *The Pentagon*.

The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before August 1, 2009. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall, 2009 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051)

NEW PROBLEMS 632-640

Problem 632. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Let two rays meet at point A, and let P be a point on one ray and Q a point on the other ray. Let B be a point between A and P. Suppose the angle measure of $\angle PAQ$ is less than 60°. Show how to construct, with only compass and straightedge, points D on AP and C on AQ such that $CD = AB$ and DC makes an angle of 60° with AQ.

Problem 633. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

The integers beginning with 2008 and without spaces between them are written down:

200820092010201120122013...

Then commas are placed to form an infinite sequence of 5-digit arrangements:

$$20082, 00920, 10201, 12012, 20132, ...$$

Prove or disprove: Every 5-digit arrangement appears infinitely many times in this sequence.

Problem 634. *Proposed by Ovidiu Furdui, University of Toledo, Toledo, OH.*

Find the limit

$$\lim_{n\to\infty} \frac{1}{n}\left(\frac{n}{\frac{1}{2}+\frac{2}{3}+\cdots+\frac{n}{n+1}}\right)^n.$$

Problem 635. *Proposed by Ovidiu Furdui, University of Toledo, Toledo, OH.*

Let $k \geq 2$ be a natural number. Find the sum

$$\sum_{n_1,\ldots,n_k\geq 1} \left(\zeta\left(n_1+\cdots+n_k\right)-1\right),$$

where ζ denotes the Riemann zeta function.

Problem 636. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, MO.*

Let p be a fixed prime. Find the dimensions of all rectangles with integral side lengths and whose areas are numerically equal to p times their semiperimeters.

Problem 637. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $x, y, z \in [1, \infty)$. Prove that

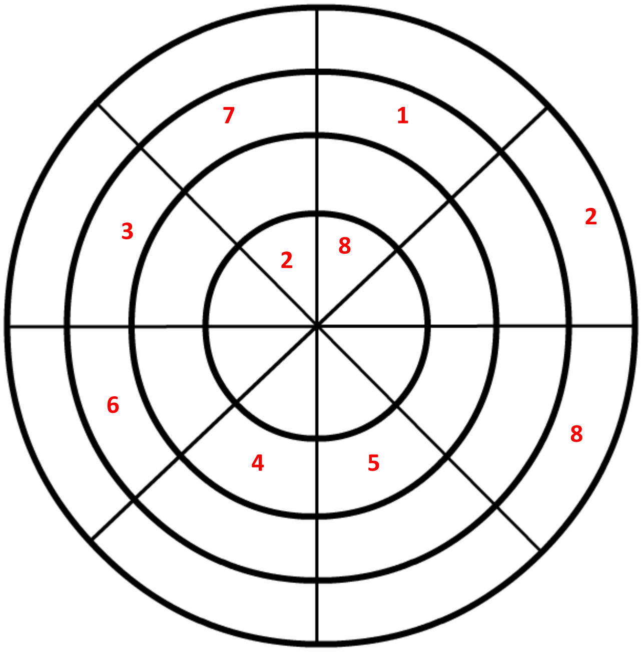
$$\frac{x}{x^2+yz}+\frac{y}{y^2+xz}+\frac{z}{z^2+xy} \leq \frac{3}{2}.$$

Problem 638. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a be a positive integer. Find the least common multiple of the number $A = a^n\left(a+1\right)^{n+1}+a$ and $B = a^{n+1}\left(a+1\right)^n+a-1$, where n is any natural number.

Problem 639. *Proposed by Peter M. Higgins and Caroline Higgins (authors of the book Circular Sudoku), Essex University, England.*

The following is a Circular Sudoku puzzle. Each of the numbers 1-8 must appear once in every ring and once in every pair of touching slices. Fill in the missing values of the puzzle.



Problem 640. *Proposed by the editor.*

The sequence 19,199,1999,... starts off with three primes; most of the numbers in the sequence, however, are composites, and there are lots of divisors of the numbers in the sequence. Prove the following:

1. The prime 19 divides infinitely many of the numbers in the sequence.
2. The composite number 551 divides infinitely many of the numbers in the sequence.
3. The composite number 323 does not divide any of the numbers in the sequence.

SOLUTIONS 616-623

Problem 616. *Proposed by Melissa Erdmann, Nebraska Wesleyan University, Lincoln, NE.*

The birthday paradox is that in a room with 23 people, the probability that two or more of them will have the same birthday (month and day) is at least 50%. Find the number of people needed so that there is a 50% probability that at least three or more of them will have the same birthday. Find the formula that will represent the probability that at least k of n total people in a room share a birthday.

Solution *by the proposer*.

In order to compute the probabilities of at least 3 people sharing a birthday in a room of n people, we compute the probability of no people sharing a birthday and the probabilities of 1 pair having the same birthday, 2 pairs having the same birthday,...,$[n/2]$ pairs having the same birthday, and subtracting all of these probabilities from 1. Using 365 days as the number of possible birthdays, the probability of no two people sharing the same birthday is

$$\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdots\frac{365-n+1}{365}.$$

The probability of one pair of people sharing the same birthday is

$$\binom{n}{2}\cdot\frac{365}{365}\cdot\frac{1}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdots\frac{365-n+2}{365},$$

where the binomial coefficient gives the number of ways to choose the pair of people, the 365 in the numerator is the number of days for this pair to have in common, the 1 after it represents that the second person in the pair has only the one choice for birthday, and then the remaining people all have distinct birthdays.

Similarly, the probability of two pair of people sharing the same birthday is

$$\binom{n}{2}\cdot\frac{365}{365}\cdot\frac{1}{365}\cdot\frac{\binom{n-2}{2}}{2!}\cdot\frac{364}{365}\cdot\frac{1}{365}\cdot\frac{363}{365}\cdots\frac{365-n+3}{365},$$

where the second binomial coefficient reflects that there are two fewer people to select from for the second pair, the 2! below it represents that the two selected pairs of people could have been chosen in the other order, and the 364 in the numerator following this represents that the second selected pair has a common birthday different from the first pair. We continue this process for $k = 2, ..., [n/2]$ pairs of people having the same birthdays. We can program this in Mathematica in the following manner:

$$N\left[1-\left(\sum_{k=0}^{\text{Floor}[n/2]}(365!/(365-(n-k))!\right.\right.$$

$$\left.\left.\cdot\left(\prod_{i=1}^{k}\text{Binomial}[n-2(i-1),2]\right)/k!\right)\right)/(365^n)\Bigg]$$

When executed with $n = 87$, we get a probability of 0.499455. When executed with $n = 88$, we get a probability of 0.511065. Hence 88 is the smallest number of people needed in a room so that the probability is greater than 50% that 3 people share the same birthday.

The formula for the smallest number of people needed in a room so that the probability is greater than 50% that k people share the same birthday gets much more complicated because of the number of cases to handle. It turns out that 187 people are needed so that 4 people share the same birthday with a probability greater than 0.5. Several additional values are given at the following website in a 1998 Ivars Peterson column for Science News entitled "Birthday Surprises." `http://www.sciencenews.org/sn_arc98/11_21_98/mathland.htm.`

Problem 617. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all triplets (x, y, z) of real numbers such that

$$\sqrt{3^x\left(4^y+5^z\right)}+\sqrt{4^y\left(3^x+5^z\right)}+\sqrt{5^z\left(3^x+4^y\right)}=\sqrt{2}\left(3^x+4^y+5^z\right).$$

Solution *by the proposer.*

Setting $a = 3^x$, $b = 4^y$, and $c = 5^z$, the equation given above becomes

$$\sqrt{a\left(b+c\right)}+\sqrt{b\left(a+c\right)}+\sqrt{c\left(a+b\right)}=\sqrt{2}\left(a+b+c\right).$$

Using the AM-GM inequality yields

$$\sqrt{a\left(b+c\right)}\leq\sqrt{2}\left(\frac{a}{2}+\frac{b+c}{4}\right),$$

$$\sqrt{b\left(a+c\right)}\leq\sqrt{2}\left(\frac{b}{2}+\frac{a+c}{4}\right),$$

and

$$\sqrt{c\left(a+b\right)}\leq\sqrt{2}\left(\frac{c}{2}+\frac{a+b}{4}\right).$$

Adding these inequalities, we obtain

$$\sqrt{a\,(b+c)}+\sqrt{b\,(a+c)}+\sqrt{c\,(a+b)}=\sqrt{2}\,(a+b+c)\,.$$

Equality holds when $a=b=c$. Therefore, the solutions to the equation of the problem are when $3^x=4^y=5^z$; that is, when $x=y=z=0$.

Problem 618. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a,b,c be real numbers such that $0<a\le b\le c<\dfrac{\pi}{2}$. Prove that

$$\frac{\sin a+\sin b+\sin c}{\cos a\,(\tan b+\tan c)+\cos b\,(\tan c+\tan a)+\cos c\,(\tan a+\tan b)}\le\frac{1}{2}.$$

Solution *by the proposer.*

Since $0<a\le b\le c<\frac{\pi}{2}$, the vectors $\langle\tan a,\tan b,\tan c\rangle$ and $\langle\cos a,\cos b,\cos c\rangle$ have components that are monotonic. The first sequence of components is non-decreasing, and the second is non-increasing. Since they have reverse order, we can apply Chebyshev's inequality. We get

$$\begin{aligned}&3\,(\cos a\tan a+\cos b\tan b+\cos c\tan c)\\ \le\ &(\cos a+\cos b+\cos c)\,(\tan a+\tan b+\tan c)\end{aligned}$$

or

$$\begin{aligned}3\,(\sin a+\sin b+\sin c)\ \le\ &\sin a+\cos a\tan b+\cos a\tan c\\ &+\cos b\tan a+\sin b+\cos b\tan c\\ &+\cos c\tan a+\cos c\tan b+\sin c.\end{aligned}$$

Therefore,

$$\begin{aligned}2\,(\sin a+\sin b+\sin c)\ \le\ &\cos a\tan b+\cos a\tan c+\cos b\tan a\\ &+\cos b\tan c+\cos c\tan a+\cos c\tan b,\end{aligned}$$

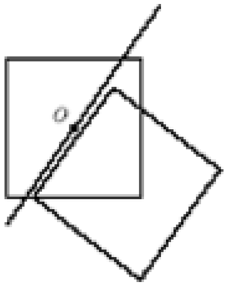
from which the statement follows. Equality holds when $a=b=c$.

Problem 619. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Several identical square napkins are placed on a table. They are placed in such a way that any two of them have a common area which is greater than half of the area of one of them. Is it always possible to pierce all the napkins with a needle going perpendicular to the plane of the table? If yes, prove it. If not, provide a counterexample.

Solution *by the proposers.*

It is always possible. We show that the center of any napkin belongs to every other napkin. Suppose, to the contrary, that one napkin has center O that does not belong to another napkin. Since the other napkin is a square, it is possible to draw a line through O which does not meet the other napkin as illustrated below. Since the line divides the napkin in two equal parts, the second napkin will overlap the first in less than half of the first napkin. This is a contradiction. Hence, in particular, the center of the top napkin belongs to every other napkin and thus a needle through the center of the top napkin will pierce all napkins.



Problem 620. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Nick chooses 81 consecutive integers, rearranges them, and concatenates them to form one long, multi-digit number N. Michael chooses 80 consecutive integers, rearranges them, and concatenates them to obtain the number M. Is it possible that $M = N$? If yes, provide an example. If not, prove it.

Solution *by the proposers.*

Nick chooses the 81 integers from 8 through 88 and Michael chooses the 80 integers 10 through 89. They can both arrange their chosen integers to form the number 891011121314...868788.
Also solved by Tian Cai (student), Pace University, New York, NY.

Problem 621. *Proposed by Lisa Hernandez, Jim Buchholz, Doug Martin (jointly), California Baptist University, Riverside, CA.*

A standard technique for showing that .999... $=1$ is to let $x =$ .999... and then $10x = 9.999...$ and so $9x = 10x - x = 9$, which gives $x = 1$. Can you derive an alternative proof that .999... $= 1$, perhaps using proof by contradiction?

Solution *by the proposer.*

Suppose that .9999... < 1. Then there exists a real number x with .9999... $< x < 1$. Since $0 < x < 1$, we can write $x = 0.x_1x_2x_3...$ Since .9999... $\neq x$, there must be at least one index i such that $x_i \neq 9$. Then $x_i \in \{0, 1, 3, ..., 8\}$. Hence $x <$ .999... But now we have a contradiction. Thus it must be that .9999...=1.

Problem 622. *Proposed by the editor.*

Prove that there are infinitely many positive, palindromic integers containing just the digits 2, 7, and 9, which are divisible by 2, 7, and 9.

Solution *by Bill Paulsen, Arkansas State University, Jonesboro, AR.*

Consider the numbers $28\,(10^n - 1)$ which look like 27999...99972 for $n > 1$. Clearly these will be a multiple of 2, 7, and 9 and also will be palindromic numbers containing only the digits 2, 7, and 9. Thus, there are an infinite number of such palindromes.

Solution *by Tian Cai (student), Pace University, New York, NY.*

The number 27972 is a number that satisfies the conditions. Therefore, there exist such numbers. Now suppose there are only finitely many numbers that satisfy the condition. Let X be the greatest. Suppose that X has n digits. Now we create the new number Y such that $Y = X + 10^n X$. Obviously, Y also satisfies the conditions, but$Y > X$. This is a contradiction. Therefore, there must be infinitely many positive, palindromic integers containing just the digits 2, 7, 9, which are divisible by 2, 7, and 9. [This proof leads to the sequence 27972, 2797227972, 27972279722797227972,... as a solution.]
Also solved by the proposer.

Problem 623. *Proposed by the editor.*

Let $f\,(n)$ be the number of rationals between 0 and 1 (noninclusive) that have denominator less than or equal to n. Prove that $f\,(n) > \dfrac{2}{3}n^{3/2}$ for all integers $n \geq 2$.

Solution *by the proposer.*

Any rational number between 0 and 1 is p/q, with p, q positive integers, $p < q$ and $\gcd\,(p, q) = 1$. Thus the Euler ϕ function can be used. Since $\phi\,(k)$ is the number of integers less than k which are relatively prime to k, $\phi\,(k)$ counts the number of rationals between 0 and 1 with denominator

k. This means that $f(n) = \sum_{k=2}^{n} \phi(k)$. Just using the values for $\phi(k)$ with $k = 2, 3, 4, 5$, and 6, we have $\sum_{k=2}^{6} \phi(k) = 11 > 10.832 = \sum_{k=1}^{6} \sqrt{k}$. By the lemma below, we know that $\phi(k) > \sqrt{k}$ for all $k > 6$. Thus, $\sum_{k=2}^{n} \phi(k) > \sum_{k=1}^{n} \sqrt{k}$. Using the type of argument from calculus involving right-hand Riemann sums, we know that $\sum_{k=1}^{n} \sqrt{k} > \int_0^n \sqrt{x}dx = \frac{2}{3}n^{3/2}$. Therefore, $f(n) > \frac{2}{3}n^{3/2}$.

Lemma 9 $\phi(k) > \sqrt{k}$ for all positive integers $k > 6$.

Proof: Let p be a prime. If $a > 1$, then $\phi(p^a) = p^{a-1}(p-1) \geq p^{a-1} \geq p^{a/2} = \sqrt{p^a}$. If p is odd, then $\phi(p) = p - 1 > \sqrt{p}$ and $\phi(2p^a) = p^{a-1}(p-1) \geq 2p^{a-1} \geq \sqrt{2p^a}$. Finally, if $p > 4$, then $p^2 + 1 > 4p$, so $(p-1)^2 = p^2 - 2p + 1 > 2p$, and we have $\phi(2p) = p - 1 > \sqrt{2p}$. Now suppose that $n = 2^a p^b q^c \cdot \cdots$. Consider the following cases.

1. If $a = 0$, then by the multiplicativity of ϕ,

$$\begin{aligned} \phi(n) &= \phi\left(p^b\right)\phi(q^c)\cdots \\ &> \sqrt{p^b}\cdot\sqrt{q^c}\cdots \\ &= \sqrt{n}. \end{aligned}$$

2. If $a = 1$ and $n > 6$, then

$$\begin{aligned} \phi(n) &= \phi\left(2p^b\right)\phi(q^c) \\ &> \sqrt{2p^b}\cdot\sqrt{q^c}\cdots \quad \text{since either } b > 1 \text{ or } p > 3 \text{ since } n > 6 \\ &= \sqrt{n}. \end{aligned}$$

3. If $a > 1$, then

$$\begin{aligned} \phi(n) &= \phi(2^a)\phi\left(p^b\right)\phi(q^c)\cdots \\ &= 2^{a-1}\phi\left(p^b\right)\phi(q^c)\cdots \\ &\geq 2^{a/2}\phi\left(p^b\right)\phi(q^c)\cdots \\ &> \sqrt{2^a}\cdot\sqrt{p^b}\cdot\sqrt{q^c}\cdots \\ &= \sqrt{n}. \end{aligned}$$

In all cases, we have the desired inequality.

Kappa Mu Epsilon News

Edited by Connie Schrock, Historian

Updated information as of August 2008

Send news of chapter activities and other noteworthy KME events to

Connie Schrock, KME Historian
Department of Mathematics, Computer Science and Economics
Emporia State University
1200 Commercial Street
Campus Box 4027
Emporia, KS 66801
or to
cschrock@emporia.edu.

Installation Report

North Carolina Epsilon
North Carolina Wesleyan College, Rocky Mount, NC

The North Carolina Epsilon Chapter of Kappa Mu Epsilon was installed on Monday, March 24, 2008, at a ceremony in the Braswell Administration Building Trustees' Room on the campus of North Carolina Wesleyan College, Rocky Mount, North Carolina. The meeting was conducted by Bill Yankosky. KME President-Elect Ron Wasserstein served as the Installing Officer. The charter members, Joshua Lee Allen, Ashley Ball, Michaela Rose Case, Kimberly Garrett, Ashley Nikole Hawkins, Joshua Randall Jenkins, Amanda Landi, Ben Lilley, Tiffany McCord, Kathleen Penrod, Susan Denise Pope, Brittany Marie Wright, L. Carol Lawrence, Mulugeta Markos, Gail T. Stafford and Bill Yankosky were inducted into the chapter. The first officers of North Carolina Epsilon, President Susan Denise Pope, Vice President Joshua Randall Jenkins, Recording Secretary Joshua Lee Allen, Treasurer Kimberly Garrett, and Corresponding Secretary/Faculty Sponsor Bill Yankosky were installed.

Chapter News

AL Alpha – Athens State University

Chapter President– Jenna O'Neal, 10 Current Members, 12 New Members Curt Merchant, Vice–President; Susan Webb, Secretary; Dottie Gasbarro, Corresponding Secretary.

Alabama Alpha Chapter had induction of 12 new members on Tuesday, April 2, 2008 in the Parlor of historic Founder's Hall on the Athens State University campus, the oldest institution of higher learning in Alabama's educational system. Founded in 1822 as Athens Female Academy, Athens State has a tradition of excellence in academics and southern charm. Inductees and their guests where honored with a reception after the Initiation Ceremony conducted by Alabama Alpha officers, Jenna O'Neal, Curtis Merchant, and Susan Webb. Dean of Arts and Sciences, Dr. Ron Fritze welcomed the attendees and Dr. Neal Fentress, Mathematics Chair, spoke about our KME tradition at Athens State.

New Initiates – Rachel Badgett, Barbara Cantrell, David Mack, Rick Martindale, James Reyer, Angela Shaddix, Bradley Sledge, Guy Wayne Stafford Jr., Joshua Alan Swindford, Kasey Danielle Taylor, Kim Whaley, Michael Williamson.

AL Epsilon – Huntingdon College

Sally Clark, Corresponding Secretary.

New initiates – Jeremy Dwain Driver, Kyle Jordan Eller, Emily Diane Hand, Caleb Allen Hartin, Tiffany Nicole Jordan, Tanja Marie McPeters, Brent Daniel Nichols.

AL Eta – University of West Alabama

Hazel Truelove, Corresponding Secretary.

New Initiates – Amanda Morgan, Henry Mosley.

AL Zeta – Birmingham Southern College

Chapter President Greg Richards –, 16 Current Members, 7 New Members John Padley , Vice–President; Manny Martinez , Secretary; Mary Jane Turner , Corresponding Secretary.

Sponsored a Spring Mathematical Colloquia presented by Dr. Wes Colley. Title " Football Rankings: The Colley Matrix Method".

New Initiates - Stephanie Kristin Barlow, Michael James Graham, Joshua Daniel Hooper, Loree Kim Killebrew, Erin Ashley Montgomery, Xinyan Yan, Margaret Ellen Baggott.

CA Delta – Cal Poly Pomona

Patricia Hale, Corresponding Secretary.

New Initiates – Konrad Aguilar, Monique Angulo, Mariano Arellano, Melissa Bilbao, Adam Eckenrode, Jairo Enciso, Regina Faye Faradineh, Charles Hale, Ken Hedrick, Erik Hieto-aho, Vanessa Jones, Knut Erik Kvarekvaal, Alex Lipp, Kristen Long, Jesus Magana, Gene Munar, Aileen Nguyen, Forrest Requarth, Sean Smith, Adrienne Spina, Robin Wilson, Phil Yates.

CA Gamma – California Polytechnic State University

Jonathan Shapiro, Corresponding Secretary.

New Initiates – Kevin McRoberts, Jeremy Van Matre, Scott Maccarone, William Taylor, Victor Meyerson, Norma Barron, Jay Hann, Hunter Glanz, Valerie Bick, Staci Pearson, Timothy Emerick, Erin Malloy, Victoria Lee, Dustin Thompson, Maddie Schroth-Miller.

CO Beta – Colorado School of Mines

Terry Bridgman, Corresponding Secretary.

New Initiates – Arianne May Dean, Matt Bolt, Ryan John Decker, Kevin James Duffy, Elise Marie Goggin, Alexsander Lopex, Jennifer Wagner.

CT Beta – Eastern Connecticut State University

Chapter President – Current Members, 19 New Members

Mizan R. Khan, Treasurer; Christian L. Yankov, Corresponding Secretary.

New Initiates – Ibiyemi Ayeni, Kelsey Bushkoff, Trevor Choleva, Matthew Cleary, Michael Culbert, Kevin Dobo, Lori Ferranti, James Foran, Michelle Forthofer, David Grey, Amy Grover, Sara Hanrahan, Jaessica Johnson, Robert Kanehl, Kristin Laterreur, Dominick Lombardozzi, Allison Plantamura, Brian Sullivan Steven Weglinski.

FL Beta – Florida Southern College

Chapter President – Alyssa Huebner, 10 Current Members, 12 New Members

Megan Beddow, Vice-President; Adam Trewyn, Secretary; Allen Wuertz, Corresponding Secretary.

New Initiates – Megan Danielle Beddow, Kristin W. Caputo, Andrei T. K. P. Grant, Laura I. Quintana, Joshua J. Samuel, Adam C. Trewyn.

GA Alpha – University of West Georgia

Scott R. Sykes, Corresponding Secretary.

New Initiates – Ashley Blasiole, David Bowling, Samantha Bullard, Joshua Hubbard, Cassie McGuire, Jessica Phillips, Michael Powers, Robert Reid, Addie Summitt, Casey Thomas.

GA Beta – Georgia College and State University

Dr. Jason Huffman, Corresponding Secretary.

New Initiates – Angel Abney, Elena E. Andreyeva, Joseph J. Arrington, Rodica Cazacu, Justin N. Cross, Russell D. Holloway, Austin P. Ladshaw, Jamie M. Nevin, Jessica C. Ollom, Claudia Ramirez, Daniel P. Smith, Edward A. Smith, Zachary P. Swilling, Christopher D. Washington.

IA Alpha – University of Northern Iowa

Chapter President – Erin Conrad, 17 Current Members, 5 New Members

Adam Schneberger, Vice–President; Kellen Miller, Secretary; Keth Kolsrud, Treasurer; Mark D. Ecker, Corresponding Secretary.

Our first spring KME meeting was held on February 7, 2008 at Professor Mark Ecker's residence where student member Maria Garcia presented her paper on “Incan Mathematics”. Our second meeting was held on March 13, 2008 at Professor Syed Kirmani’s residence where student member Beth Kolsrud presented her paper on “Risk Factors Associated with a Baby’s Low Birth Weight”. Student member Brian Curlott addressed the spring initiation banquet with "The Effect of Salary on Team Wins in Major League Baseball". Our banquet was held at Godfather’s Pizza in Cedar Falls on May 1, 2008 where six new members were initiated.

New Initiates - Reanna Collins, Corey Gevaert, Michelle Gogerty, Adam Kiefer, Rikki Shaver, Brandon Weiland.

IA Delta – Wartburg College

Chapter President – Jill Wiebke, 36 Current Members, 15 New Members
Jen Czachura, Vice–President; Sarah Danner, Secretary; Blake Haugen, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

In January we held a movie night where we watched “Flatland”. In early March we sent two teams of students to participate in the Iowa Collegiate Math Competition in Des Moines, Iowa. Carson Andorf, a Wartburg alum finishing up a Ph.D. in bioinformatics, was the speaker at our annual banquet and initiation ceremony, which was held on March 29; fifteen members were initiated.

New Initiates – Aaron Bartholmey, Andrew Bell, Stacy Berns, David Bolien, Jennifer Czachura, Larisa Greve, Blake A. Haugen, James Allen Juett, Randy Krueger, Jacob Miksell, Andrew Ott, Jordan Paulus, Kosuke Takahashi, Alicia Zimbeck, Nathaniel R. Janzen.

IA Gamma – Morningside College

Eric Canning, Corresponding Secretary.

New Initiates: Daniel Ascherl, Chantell Kuntz, Takayu Kato, Jeremy Lenz, Jeremy Letsche, Brenda Mammenga, Amanda Plemel, Michael Smith, Daniel Vennerberg.

IL Delta – University of St. Francis

Richard J. Kloser, Corresponding Secretary.

New Initiates – Meghan Bannatz, Kevin Beckman, Jason Blauw, Nicole Burke, Terrence Eakle, Aimee Eichelberger, Kelley Flaherty, David Heasley, Christina Hickey, Rachel Reyes, Teresa Spata, Cole Twitchell, Jeremy Unger, Annie Walker.

IL Eta – Western Illinois University

Boris Petracovici, Corresponding Secretary.

New Initiates – Jacob Brown, Sarah Cramsey, Timothy Gross, Allisha Langdon.

IL Iota – Lewis University

Chapter President – Trina McNamara, 45 Current Members, 13 New Members
Katherine Skurski, Vice – President; Sylvia Ciezak, Secretary; Allison Schmitz, Treasurer; Br. Tom Dupre’, Corresponding Secretary.

During the spring 2008 semester, we inducted four new members. Our total membership since our first induction stands at 45.

Our activities (aside from meetings) during the 2007-2008 academic year included the following. We made a donation of 25 scientific calculators to a needy school in the area. This was in response to our request of their needs. In addition, several of our members went to this school (K-8, about 330 students) and spoke to the older students about mathematics being not only useful but enjoyable). The students were shown some mathematical games. Our hope was that they (especially the girls) would continue to study mathematics and that some would consider careers in mathematics. On March 14 (Pi Day) our members sold individually wrapped pieces of pie. Sales amounted to about 100 individual pieces. Our society sponsored three "Math movie Monday" (MMM) video presentations from the NOVA series. They were: Infinite Secrets, the Genius of Archimedes; Galileo's Battle for the Heavens; Lost at Sea, the Search for Longitude. At the end of the academic year, our honor society sponsored a "50-50" raffle, in which the winner would receive 50% of the money collected. This money can help fund our activities.

New Initiates - Brittany DiPietro, Allison Kief, Frank Koncar, Kimberly Matysiak.

IL Theta – Benedictine University

Dr. Lisa Townsley, Corresponding Secretary.

New Initiates – Muzamil Arshad, Nicholas Dobes, Carolyn Kois, Nicolas Meyer.

IL Zeta – Dominican University

Aliza Steurer, Corresponding Secretary.

New Initiates – Carissa Boksa, Jessica Gonzalez, Nancy Gullo, Amy Lucas, Christopher Marzec, Angelina Myers, James Troken, Monika Vidmar, Jae Jun Lee, Donald Marxen.

IN Gamma – Anderson University

Stanley L. Stephens, Corresponding Secretary.

New Initiates – Alexander Erwin, Mark Gordon, Nathaniel Boggs, Raymond Kenney, Joel Bucklin, Gerardo Blanco.

KS Alpha – Pittsburg State University

Dr. Tim Flood, Corresponding Secretary.

New Initiates – Greg Clawson, Sarah Dees, Kyle Marcotte, Michelle Martin, Mark Neely, Misty Petersen, Marissa Ritter, Corey Base.

KS Beta – Emporia State University

Chapter President– Mike Moore, 26 Current Members, 5 New Members
Other spring 2008 officers:, Vice–President;, Secretary;, Treasurer; Connie Schrock, Corresponding Secretary.

The Kansas Beta chapter of KME hosted a showing of the new movie Flatland, following the movie a group of members went out for pizza. Kansas Beta, also participated in a bowling night, full of fun and laughter.

There was one seminar this semester given by Chris Imm, from Johnson County Community College, over "Infinity". We closed the year with a cookout and initiations.

KS Delta –Washburn University

Chapter President– Tamela Bolen, 30 Current Members, 10 New Members
Brandy Mann, Vice–President; Richard Nelson, Secretary; Richard Nelson, Treasurer; Kevin Charlwood, Corresponding Secretary.

The Kansas Delta chapter of KME met for three luncheon meetings with the Washburn Math Club during the semester. Our chapter's annual initiation banquet and ceremony was held on February 25, 2008 with 10 new initiates inducted, including two faculty honorees. Three students, Tamela Bolen, Alexandria Jeannin and Brandy Mann, prepared and presented papers at the KME Regional Convention hosted at Pittsburg State April 4 – 5. Brandy won a "top 4" prize for her presentation. Faculty Kevin Charlwood, Sarah Cook, Bill Gahnstrom, Hwa Chi Liang, Gaspar Porta, Jennifer Wagner and students Sarah Butler, Riley Harrington, Richard Nelson, and Sean VanDyke also attended the Regional Convention.

New Initiates – Derek Brown, William "Bill" Brichacek, Holly Kennedy, Jessica Luse, Amanda McCullough, Celeste Rojero, Shannon Spangler, Sean VanDyke.

KS Gamma – Benedictine College

Chapter President – Chris G'Sell , 15 Current Members, 5 New Members
Erica Goedken , Vice-President; Josie Villa, Secretary; Dr. Linda Herndom, Corresponding Secretary.

The Kansas Gamma chapter was pleased to be able to initiate five new members this spring. Members Erik Klinckman and Linda Myers were awarded the Sister Helen Sullivan scholarships at the Benedictine College Honors Banquet on April 30. We celebrated with a party at a local pizza establishment at the end of the school year to congratulate the seven graduating members.

New Initiates – Christina Henning, Emily James, Caitlin Kelly, Scott Storm, Matthew Weaver.

KY Beta – University of the Cumberlands

Chapter President- Kathryn DeLozier, 24 Current Members, 13 New Members
Charlotte Abel, Vice–President; Dustin Ursrey, Secretary; Deidre Higgins, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary.

On February 13, 2008, the Kentucky Beta chapter held an initiation and a joint banquet with Sigma Pi Sigma, physics honor society, at the Cumberland Inn. Kappa Mu Epsilon inducted thirteen new student members at the banquet, presided over by outgoing president, Kathryn

DeLozier. As an additional feature, senior awards were given by the department at the banquet.

On April 12, several members visited the U.S. Space & Rocket Museum in Huntsville, Alabama. Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 15. He spoke about "How Did You Do That?" (Magic or Math?). On April 16, members also assisted in hosting a regional high school math contest, held annually at University of the Cumberlands. On April 24, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held the annual spring picnic at Briar Creek Park.

LA Delta – University of Louisiana at Monroe

Christine Cumming-Strunk, Corresponding Secretary.

New Initiates – Leanna Darland, Anthony Drummer, Emily Judice, Verlencia Jordan, Jie Lin, Courtney Lucas, Zarna Patel, Mike Tu, Lauren Watson, Sharon Brooks, Susan Burchfield, Faisal Kaleem.

MA Alpha - Assumption College

Chapter President – Kristin Kenney, 5 Current Members, 9 New Members
Ashley Daly, Vice-President; Charles Brusard, Corresponding Secretary.

New Initiates - Tanya M. Breault, Andrew A. Buckley, Elizabeth R. Fortino, Tara M. Fountain, Sandra J. Garney, Julia A. Hazlett, Lauren E. Konicki, Kathryn S. Richard, Neil A. Volungis.

MD Beta – McDaniel College

Dr. Harry Rosenzweig, Corresponding Secretary.

New Initiates - Stephen Hardy, Christopher Alan Martin, Dan M. Thornton.

MD Delta – Frostburg State University

Dr. Mark Hughes, Corresponding Secretary.

New Initiates - Joe Bascelli, Elizabeth Gitelman, Kelly Seaton.

MI Alpha – Albion College

Mark Bollman, Corresponding Secretary.

New Initiates – Mary Bizon, Whitney Patton, Timothy Rambo, Michael Robinson.

MI Epsilon – (Section A) Kettering University

Chapter President – Lynette Fulk, 92 Current Members, 18 New Members
Gayle Ridenour, Vice-President; George Hamilton, Treasurer; Kathleen Moufore, Secretary; Boyan Dimitrov, Corresponding Secretary.

Also two Pizza Party/Movie: on 3rd Tuesday (January 29) at noon with the movie “Decoding Nazi Secrets” (part 1), and on 6th Tuesday (February 19 noon) “Decoding Nazi Secrets” - the second part, took place. A new initiative (Wednesday noon 30 minutes mathematics attractive lectures called “Zero to Infinity” started for students and faculty in a math auditorium. These series of lectures are on the history of Mathematics and numbers' secrets and are watched with great interest.

The Initiation Ceremony for the KME Mathematics Honor Society was marked with a dinner at the Kettering cafeteria with the new members, their families, and faculty. 18 initiates and 26 guests participated. Two successful Kettering alumnus and former KME members of our chapter addressed their special invited speech for the new initiates.

New Initiates – Joshua J. Alphonse, Terry A. Barr, Phillip J. Besoiu, Adrienne M. Billiau, Kaleen E. Canevari, Kelly A. Christensen, Benjamin G. Czinski, Caryn N. Homsher, Tu Le, Daniel R. McPeak, Michelle A. Mitchell, Jeffrey M. Nolen, Rachel A. Rabideau, Bradley A. Sewell, Melissa S. Scholl, Matthew A. Titus, Kenneth E. Weimer, Lakithia R. Williams.

MI Epsilon – (Section B) Kettering University

Chapter President – Kennet, 103 Current Members, New Members
Joey Campbell, Vice-President; Casey Stevenson, Treasurer; Nick Timkavish, Secretary; Boyan Dimitrov, Corresponding Secretary.

There were Pizza Parties and mathematics Movies at noon time (as usual) in some of the largest class rooms. The movies as "Leonardo da Vinci. Renaissance Master", and "Infinite Secrets" presented an amazing story about the lost book of Archimedes (which could accelerate the development of science by hundreds of years) attracted the student' attention.

The Initiation Ceremony for our new KME members was holding on 10th Thursday (December 14) at 6:30 p.m. in the Kettering University Council room. There were 13 initiates and 21 family members attending the ceremony. The Keynote Speaker, Bill Guttrich, now Global Sales Manager at Gasoline Management Systems at Delphi Energy and Chases System, a former Kettering graduate talked about his experience in the use of Mathematics as a student, worker, manager, and in his inter-relationships in the market.

MO Alpha – Missouri State University

Chapter President– Chris Inabnit, 31 Current Members, 12 New Members
Michael McDonald, Vice–President; Nicole Jones, Secretary; Bobbi Gregory, Treasurer; Jorge Rebaza, Corresponding Secretary.

02/19/08 KME Seminar. Speaker: Les Reid, MSU

03/13/08 KME Seminar. Speakers: Aaron Yeager and Brandon Turner, MSU.

04/17/08 KME Seminar. Speakers: Michael McDonald and Immanuel McLaughlin, MSU.

New Initiates – Caleb Bennett, Ashlie Blanzy, Cheree Coffman, Chris Farley, Alex Hutchins, Courtney Kolb, Brandi Mills, Taylor Porter, Steven Taylor, April Treaster, Emily Wilson, Darren Wynne.

MO Beta – University of Central Missouri

Chapter President – Joshua Liberman, 30 Current Members, 10 New Members

Allison Monroe, Vice-President; Tristan Sullins, Secretary; Sandy Davidson, Treasurer; Mindi Douglas, Historian; Dr. Rhonda McKee, Corresponding Secretary.

The Missouri Beta Chapter of KME held 3 regular meetings during the spring 2008 semester. On Feb. 12, Dr. Nicholas Baeth spoke on "Demystifying Merlin's Magic." On March 18, student Brittney Hinds spoke on "An Extension of the Second Derivative Test," and on April 8, the group enjoyed "Math Quiz Bowl," pizza and officer elections. In addition, we held a book sale on Feb. 27-28, volunteered at the UCM Math Relays for high school students and participated in the regional KME convention in Pittsburg, KS. The end-of-semester social activity was a Royals game in KC.

New Initiates – Todd Carlstrom, Brittney Hinds, Phat Hoang, Jason Merten, Delana Nicholson.

MO Epsilon – Central Methodist University

Chapter President – Ian Young, 10 Current Members, 5 New Members

Amy Geurin, Vice-President; Linda O. Lembke, Corresponding Secretary.

We had an initiation dinner in April. At that time we initiated five new members and elected officers for next year. We also distributed graduation cords for those members who were graduating in May 2008.

New Initiates: Blair Bigham-Crosswhite, Amy Guerin, Scott Haefele, Valerie Robinette, Ian Young.

MO Eta –Truman State University

Jason Miller, Corresponding Secretary.

New Initiates – Brian Manning, Chris Owens, Jon Eman, Elle Little, Michael Solomon, Kylie Raithel.

MO Gamma – William Jewell College

Dr. Mayumi Sakata Derendinger, Corresponding Secretary.

New Initiates: Jennifer McKnight, Robert Christopher Sharp, Hayley VanderStel, Carl Ziegler, John Spiegel.

MO Iota - Missouri Southern State University

Chapter President - John Carr

Chip Curtis, Corresponding Secretary.

New Initiates - Deidrick Beckett, Kayleigh Cecil, Michael Harris, Garen Hartley, Andrew Waugh, Chad Whitbeck.

MO Kappa – Drury University

Carol Browning, Corresponding Secretary.

New Initiates – Lindsey Courtney, Blair Ellington, H M Mohasin Uddin Mithu.

MO Lambda – Missouri Western State University

Chapter President – Tysa Updike, 20 Current Members, 9 New Members
Michelle Ritter, Vice-President; Rebecca Shipers, Secretary; Rylan Sampson, Treasurer; Dr. Steve Klassen, Corresponding Secretary.

Nine initiates participated in our KME ceremony in April. Retiring professor and department chairperson Dr. Ken Lee was our guest speaker. With the beginning of the Fall 2008 semester, we welcome Dr. Tingxiu Wang to Missouri Western as our new chairperson.

New Initiates – Julie Allen, Brett Cagg, Elena Castanada, Geoffrey Gould, Chad Klein, Aaron Lewis, Brock Schmutzler, Stephanie Thomsen, Raymond Williams.

MO MU – Harris Stowe State College

J. Behle, Corresponding Secretary.

New Initiates – Natasha Hurd, Gregory Taylor, Julie Farance, Stephanie Grice, Dawn J. Haney, Deborah A. Murphy, Jerome L. Schmidt, Tawana White.

MO Nu – Columbia College

Dr. Nataliya Latushkina, Corresponding Secretary.

New Initiates - Kristin Crane, Andrew S Grote, Garrett Koon, Neal Lines, Magdalene Joy Pride, Wade Vandelicht.

MO Theta – Evangel University

Chapter President – Adrienne Arner
Rebekah Holmes, Vice-President; Don Tosh, Corresponding Secretary.

Meetings were held monthly. In the January meeting, Adrienne Arner was elected President and Rebekah Holmes was elected Vice President, and one faculty and five students were initiated. In April, Dr. Tosh and seven students attended the regional convention in Pittsburg, KS, where our new vice president, Rebekah Holmes, presented a paper. The final meeting of the semester was an ice cream social held at the home of Dr. Tosh.

New Initiates - Adrienne Elizabeth Arner, Kassondra Broihier, Fred A. Fortunato, Chantel Galipeau, Rebekah D. Holmes, Sarah E. Ritchie.

MS Alpha – Mississippi University for Women

Chapter President – Dana Derrick, 13 Current Members, 1 New Member
Michelle L. Hitt, Vice-President; Dana Derrick, Secretary; Michelle L. Hitt , Treasurer; Dr. Shaochen Yang, Corresponding Secretary.

Mississippi Alpha participated in Sonya Kovalexky High School Mathematics Day. Mississippi Alpha also participated in Multiple Sclerosis Walk raising $1096. Dr. Jiu Ding, The University of Southern Mississippi, presented “Dynamical Geometry: From Order to Chaos and Sierpi’nski Pedal Triangles.”

New Initiate – Stefanie Zegowitz.

MS Epsilon – Delta State University

Paula Norris, Corresponding Secretary.

New Initiates - Dr. Leslie Horton, Cynthia N. Jones, Anita Bell Miller, Stephan Roberts, Jacquelyn Ann Stowe, Audra L. Sullivan.

NC Epsilon – North Carolina Wesleyan College

Chapter President – Denise Pope, 15 New Members

Josh Allen ,Vice-President; Josh Jenkins, Secretary; Kimberly Garrett, Treasure; Bill Yankosky , Corresponding Secretary.

The North Carolina Epsilon Chapter of Kappa Mu Epsilon was installed on Monday, March 24, 2008, at a ceremony in the Braswell Administration Building Trustees' Room on the campus of North Carolina Wesleyan College, Rocky Mount, North Carolina. The meeting was conducted by Bill Yankosky. KME President-Elect Ron Wasserstein served as the Installing Officer.

Twelve students and 3 mathematics faculty were inducted into the North Carolina Epsilon Chapter as charter members. The students inducted were Joshua Lee Allen, Ashley Ball, Michaela Rose Case, Kimberly Garrett, Ashley Nikole Hawkins, Joshua Randall Jenkins, Amanda Landi, Ben Lilley, Tiffany McCord, Kathleen Penrod, Susan Denise Pope and Brittany Marie Wright. The mathematics faculty inducted were L. Carol Lawrence, Mulugeta Markos and Gail T. Stafford.

The first officers of North Carolina Epsilon, President Susan Denise Pope, Vice President Joshua Randall Jenkins, Recording Secretary Joshua Lee Allen, Treasurer Kimberly Garrett, and Corresponding Secretary/Faculty Sponsor Bill Yankosky were also installed.

Several guests including family and friends of the inductees along with the Associate Academic Dean of North Carolina Wesleyan College, Jay Stubblefield, attended the Installation Ceremony. The inductees were also recognized during North Carolina Wesleyan College's Honors Convocation on May 3, 2008.

New Initiates - Joshua Lee Allen, Ashley Ball, Michaela Rose Case, Kimberly Garrett, Ashley Nikole Hawkins, Joshua Randall Jenkins, Amanda Landi, Ben Lilley, Tiffany McCord, Kathleen Penrod, Susan Denise Pope, Brittany Marie Wright, L. Carol Lawrence, Mulugeta Markos, Gail T. Stafford.

NE Beta – University of Nebraska at Kearney

Other spring 2008 officer: Dr. Katherine Kime, Corresponding Secretary.

New Initiates – Robert Langan, Carrie Miller, Andrew Olson, Cameron Push, Grant Saltzgaber.

NE Delta – Nebraska Wesleyan University

Melissa Erdmann, Corresponding Secretary.

New Initiates - Tyrone Franklin, Alicia Halter, Thao Nguyen, Heather Schmidt.

NH Alpha – Keene State College

Vincent J. Ferlini, Corresponding Secretary.

New Initiates – Michael Barry, Amanda Benware, Christine Griswold, Nicole Lewis, Jennifer McDougall, Kristin Rennie, Alisha Stevens, Kimberly Thompson, Christina Honeycutt, Kristin Tolman, Patrick Rodden.

NJ Beta – Montclair State University

John G. Stevens, Corresponding Secretary.

New Initiates - Pablo Apablaza, Timothy Buli, Nicole Drag, Turrell Jones, Jin Park, Lauren Truncale, Timothy Valik, Alexandra Woroniecka.

NJ Gamma – Monmouth University

Chapter President – Erin Humphries, 116 Current Members, 17 New Members

Meaghan Joyce, Vice-President; Carolyn Morris, Secretary; Jennifer Roman, Treasurer; Judy Toubin, Corresponding Secretary.

Because most of the senior members of KME had graduated and the new membership was small, the New Jersey Gamma chapter was not able to do as much this year as in past years. However, the chapter was able to sponsor two colloquium speakers for the Math Department. On November 14, Michael Leibrock, Commerzbank spoke on The Merging of Financial Mathematics and Wall Street; Trends and Opportunities all Math Students Should Know. On April 9, Dr. Lynn Bodner, a Monmouth University Professor, spoke on Symmetries of Islamic Patterns.

The annual induction ceremony was held on April 18 in Wilson Hall Auditorium. The guest speaker was Dr. Bruce Normandie, Associate Professor for Curriculum and Instruction in the School of Education. After the ceremony, family and friends enjoyed refreshments with the 17 new inductees.

During exam week, KME had a candy sale in The Math Learning Center. It proved to be very successful.

New Initiates – Areej M. Abed, Kara A. Danbrowney, Crystal Lynn Dannecker, William V. Feldmann II, Gabriella Anne Furmato, Nicole M. Gagliano, Michelle N. Hacker, Michael Karney, Sarah A. Keppel, Renee Koblan, Brianne Lauren McDonough, Sarah Frances Miller, Colby Dennis Mueller, Michael J. Pereira, Kati Romangnoli, Christopher Santorelli, Joseph A. York.

NY Eta - Niagara University

Chapter President – Meagan Reeb, 43 Current Members, 9 New Members

Jamie Rahr, Vice-President; Adam Sokol, Secretary; Richard Cramer-Benjamin, Corresponding Secretary.

We had a very active year, organizing several fund raisers, including a concert. We had two faculty talks and attended three local conferences.

NY Kappa – Pace University

Lisa Fastenberg, Corresponding Secretary.

New Initiates – Miguel Acobo, Akihiko Kurimoto.

NY Lambda – C.W. Post Campus of Long Island University

Other spring 2008 officers: Andrew Rockett, Corresponding Secretary.

Thirteen students were initiated into the New York Lambda Chapter by Jennifer L. Flynn and Zhen Chen during our annual banquet at the Greenvale Town House restaurant on the evening of April 27th, bring the Chapter membership to 301. Our evening concluded with the announcement by Dr. Katherine Hill-Miller, Dean of the College of Arts and Sciences, of the 2007-2008 department awards: the Claire F. Adler Award to Richard F. Mazanek, the Lena Sharney Memorial Award to Salvatore Cardillo and Tara J. Koebel, and the Hubert B. Huntley Memorial Award to Kaitlin Egan; the Dean Schmidt Graduate Scholarship Award to Robert Tuosto; and the presentation by Dr. James V. Peters of three MAA students memberships.

NY Mu – St. Thomas Aquinas College

Dr. Marie Postner, Corresponding Secretary.

New Initiates – Jennifer Akguc, Chelsey Ires-Cohen, Cuong Luc, Jill Sheridan.

NY Nu – Hartwick College

Chapter President – Dustin Jones, 18 Current Members, 13 New Members
Amelia Berchtold, Vice-President; Elizabeth Salmeron, Secretary; Tamdin Sherpa, Treasurer; Ron Brzenk, Corresponding Secretary.

New Initiates – Daniel Buehrens, Cassie Dresser, Austen Groener, Melanie Hart, Rachel Keller, Helena Khazdozian, Kaitlyn King, Stephanie Miller, Daniel Parisian, Elizabeth Salmeron, Matthew Shoudy, James Walsh, Kaitlin Woskoff.

NY Omicron – St. Joseph's College

Chapter President – Heather Kramer, Current Members, New Members
Nicole Hatzispirou, Vice–President; Kristine Vaccaro, Secretary; Brian Callen, Treasurer; Elana Epstein, Corresponding Secretary.

In Spring 2008 our chapter held an induction where we inducted 25 new members. We also got new chapter officers. We held some fundraisers, including bagel sales and bake sales. All of our members volunteered to tutor at our Saturday morning math clinic for local high school students.

New Initiates - Matthew Caputo, Eileen Conner, Vasil Skenderi, William Vojir, Mary Ellen Becker, George Bernius, Brian Callen, Vincent Cosentino, Gina DeTomasso, Jessica Durant, Gina Fumai, Nicole Giordano, Nicole Hatzispirou, David Jacobsen, Leigh Johnson, Heather Kramer, Jessica Lombardo, Heather O'Conner, Louis Petersen, Larry Plompen, Krystina Pylyp, Jessica Ragazzi, Nicolas Sciallo, Kristine Vaccaro, Joseph Poma.

NY Pi – Mount Saint Mary College

Dr. Lee Fothergill, Corresponding Secretary.

New Initiates – Elizabeth D'Angelo, Kelly Bergen, Melissa Antinoro, Erin Knight, Kyle Reiss, Christina Nardi.

OH Alpha – Bowling Green State University

Dr. Maria Rizzo, Corresponding Secretary.

New Initiates – Nathan A. Boss, Amber J. Brewer, Christina R. Cary, Sean A. Crisafi, Kyle R. Frank, Bridget A. Fritsch, Steven D. Groh, Michael A. Hewitt, Jeana R. Hilvers, Sydney C. Jones, Candice T. Loehrke, Kacey D. Mayer, Matthew K. Mullen, Gerald L. Nahrebecki Jr., Erin M. Noss, Jonathan H. Oliver, Kathryn F. Osborne, Monica A. Schneiderman, Kyle D. Schwieterman, Sarah R. Stout, Jacob J. Worley, Kyle M. Zwyer.

OH Epsilon – Marietta College

Chapter President – Kelsie McCartney, 25 Current Members, 13 New Members

Megan Brothers, Vice-President; Dr. John C. Tynan, Corresponding Secretary.

OH Eta – Ohio Northern University

Donald Hunt, Corresponding Secretary.

New Initiates - Bailey M. Blake, Axel Brandt, Matthew Fagert, Michael Thomas Gabrieli, James Christopher Gallagher, Joshua A. Gargac, John Holodnak, Michael P. Johnson, Kyle D. Maurer, Janet Ondrake, Christopher Senesi, Josh Stoffel, Justin Stone, Lauren Sutherland, Christopher M. Vukelich, William R. Wolfgang Jr., James Wollaeger.

OH Gamma– Baldwin-Wallace College

Chapter President – Gregory Skupski, 28 Current Members, 20 New Members

Bryan Haslett, Vice-President; Mallory Underwood, Secretary; Maria Stopak, Treasurer; Dr. David Calvis, Corresponding Secretary.

New Initiates - Meghan Black, Soha Hassan, Erin Hegarty, Amanda Lasko, Leslie Meinhardt, Cassandra Sears, Kathleen Sebastian, Rachel Sundberg, Mallory Underwood, Antoinette Ward, Jonothan Wilson.

OH Zeta – Muskingum College

Dr. Richard Daquila, Corresponding Secretary.

New Initiates – Kayla Bebout, Eric Brumbaugh, Logan Driggs, Amy Miller, Lacy Schafer.

OK Alpha – Northeastern State University

Chapter President – Caleb Knowlton, 66 Current Members, 16 New Members

Ramona Medlin, Vice–President; Felicia Lotchleas, Secretary; Amanda Barker, Treasurer; Dr. Joan E. Bell, Corresponding Secretary.

Our spring initiation brought 16 new members into our chapter. During our January meeting, we worked on a problem from a math journal and submitted the solution for publication. The speaker at our February meeting was Bill Kay. He became a member of the Oklahoma Alpha chapter in 1955. He spoke of his degree in math in relation to his career in petroleum engineering. He also shared with us the activities of KME in the 1950's, including some memories of some of his math teachers, including W. C. Carpenter and L. P. Woods. We celebrated "Pi Day," which fell during our spring break, during our April meeting with the movie "The Great Pi versus e Debate." It was hilarious! We then enjoyed homemade pie brought by Dr. Carment, Mrs. Megee, Mrs. Hladick, Mr. Sisk, and Dr. Bell. Our annual Ice Cream Social was in May, as was our last event, a presentation by Dr. Mark Buckles on "To Infinity and Beyond."

New Initiates – Angela C. Arnold, Cody J. Boggs, Branda L. Briscoe, Cristina N. Cooper, Kyle W. Elam, Jennifer L. Johnson, Aaron J. LaBounty, Katie M. Needham, Martha Y. Parrott, Katherine L. Rabe, Michael J. Rex, Ryan P. Ridley, Brian L. Riley, Joan E. Bell, Robin L. Tilley, Rustyn J. VanDeventer, Jory S. Wade.

OK Epsilon – Oklahoma Christian University

Ray Hamlett, Corresponding Secretary.

New Initiates – Lannea Blackerby, Travis Bow, Lara Killingsworth, Spencer Goad, Carly Marshall, Laura Oestmann, Tyler Stevens, Alex Tulikumwenayo.

OK Gamma – Southwest Oklahoma State University

Bill Sticka, Corresponding Secretary.

New Initiates – Debra Boone, Neil Funwie, Caitlin Harwell, Amy Janzen, Adam McCown.

PA Eta – Grove City College

Dale L. McIntyre; Corresponding Secretary.

New initiates – Jonathan Baisch, Tyler Constable, James McNamara, Jordan Ritchie, Andrew Shesman, Michelle Bowser, Andrew Falcone, Susannah Johnson, Shaun Mills, Greg Mottet, Trevor Partridge, Lauren Powers, Lucas Waddell, April Whiting, Lindsay Willett, Lauren Woodring, Lauren Zientek.

PA Gamma – Waynesburg College

James R. Bush, Corresponding Secretary.

New Initiates – Matthew Deveaney, Kerri Holsopple, James Johnson, Michael Kaminski, Samantha Klink, Jennifer May, Nicholas Miller, Andrew Naymick, Lawrence Newbauer, Megan Orndoff, John Paiani, Laina Seemiller.

PAIota-ShippensburgUniversity

Dr. Paul Taylor, Corresponding Secretary.

New Initiates - Jessica Armstrong, Kelly Dirks, Brooks Emerick, Michael Freed, Joshua Harrington.

PA Lambda – Bloomsburg University of Pennsylvina

New Initiates – Daniel Bartleson, Reese Bethel, Cody Brady, Brian Caiazzo, Jesse Childress, Kyle DeFelice, Cara Holman, Alexander Paoletti, Anthony Trubiano, Beth Wulff.

PA Mu – Saint Francis University

Chapter President – Jonathan Miller, 42 Current Members, 14 New Members

Tim Gaborek, Vice – President; David Kirby, Secretary; Joe Rosmus, Treasurer; Dr. Peter Skoner, Corresponding Secretary.

Induction ceremonies were held on January 31, 2008 along with a dinner in the Raymond Hall Conference Room. Inducted were students Nicholas Baro, Ashley Bellmont, Jessica Boyer, Tiffany Bradford, Heather Harteis, Kurt Hoffman, Ben Long, Abi May, Rikki Myers, Kaylyn Oshaben, Aaron Osysko, Ellie Pecharka, Anne Stock, and Paul Strenko.

The Eighth Math Day was held on Wednesday, March 12, 2008. A total of 117 high school students and 11 teachers from 9 area high schools attended. Each student had the chance during the day to attend two presentations, participate in a Who Wants to Look Like a Millionaire (In a New Tee Shirt) quiz contest, and complete other mathematics challenges. The presentations included, for example, Geocaching: The Amazing GPS Race, Sacred Geometry, Struck by Lightening or Win the $300 Million Powerball: Which is more likely?

Saint Francis University students, faculty, and staff conducted four separate Math Quiz Bowls during three different sessions as one of three different activities at Try Math A Lot, a mathematics competition for middle school students held on Wednesday, April 30, 2008 on the campus of the University of Pittsburgh at Johnstown. The event was attended by 330 sixth and seventh grade students from 25 schools.

The Mathematics Department held the Annual Celebration of Pi Day on Friday, March 14, 2008 by serving pies for anyone on campus from 10:00 A.M. to 2:00 P.M. in the physics lab.

New Initiates - Nicholas Baro, Ashley Bellmont, Jessica Boyer, Tiffany Bradford, Heather Harteis, Kurt Hoffman, Ben Long, Abigale May, Rikki Myers, Kaylyn Oshaben, Aaron Osysko, Ellie Pecharka, Anne Stock, Paul Strenko.

PA Pi – Slippery Rock University

Chapter President – Emily Hendrickson, 49 Current Members

Michelle Komo, Vice-President; Tyler Druschel, Secretary; Dr. Elise Grabner, Corresponding Secretary.

PA Rho – Thiel College

Chapter President – Angela Crone, 9 Current Members, 11 New Members

Kaitlyn Scherer, Vice-President; Michael Ryan, Secretary; Cassandra Beck, Treasurer; Max Shellenbarger, Corresponding Secretary.

The RHO chapter inducted 11 new members during spring semester, 3 faculty and 8 students. We also held a fund raiser which resulted in a donation of $125 to the Greenville Good Sheppard Center.

New Initiates – Syrell Rodriguez Carreras, Jennifer Curry, Andrew Grover, Amanda Hawkins, Michael Johnson, Patrick Mitch, Elise Moore, Haley Pickard, Devin Todd, Laura Wise, Jie Wu.

PA Theta – Susquehanna University

Lisa Orloff Clark, Corresponding Secretary.

New Initiates - Robert T Davis, Patrick M Donegan, Sarah B Hawk, Molly M Knapsack, Catarina B Manney, Casey Oliver, Sarah K Pfeiffer, Nicholas E Stepanik, Emily Teller.

PA Xi – Cedar Crest College

Patrick M Ratchford, Corresponding Secretary.

New Initiates – Vanessa Coderre, Noelle Grube, Diana Holder, Hilda Lai, Ashley Rancourt, Melanie Smith, Ewelina Witko.

SC Delta – Erskine College

3 Current Members, 0 New Members
Kokou Abalo, Corresponding Secretary.

KME was inactive this year. The larger Math Club (of which KME is a subset) was active and was led by Art Gorka. It is expected that both organizations will be active next year.

SC Gamma – Winthrop University

Chapter President – Josh Jones, 7 Current Members, 5 New Members
Lauren Cairco , Vice – President; Kristen Huete, Secretary; Ashley Batson, Treasurer; Dr. Trent Kull, Corresponding Secretary.

New Initiates – Ashley Virginia Batson, Stephanie Ann Eckstein, Soo Kyung Han, Trent Christian Kull, Megan Ann Patterson.

SD Alpha – Northern State University

Mike Melko, Corresponding Secretary.

New Initiates – Molly Graves.

TN Alpha – Tennessee Technological University

Andrew J. Hetzel, Corresponding Secretary.

New Initiates – Benjamin Eckart, Barry Elliott, Amy Forgey, Matthew Hall, Robert Porter, Matthew Putnam.

TN Beta – East Tennessee State University

Lyndell Kerley, Corresponding Secretary.

New Initiates – Kelly Jefferson, Daniel Phillips, Andrew Smith, Amanda Phebus.

TN Delta – Carson-Newman College

Chapter President – Alex Cate, 8 Current Members, 6 New Members
Brittany Hall, Vice-President; Brian A. McLaughlin, Secretary; B. A. Starnes, Treasure; B. A. Starnes, Corresponding Secretary.

Tenn. Delta held a game night, lacrosse game, bowling, and statistical journal night. At Cherokee Lake we held our initiation and a picnic.

New Initiates - Lucas Morton, Andrew Stubblefield, Gretchen Hill, Andrew Hansen, Sarah Tweed, Joshua Sprewell.

TN Gamma – Union University

Chapter President – Matthew Dawson. 13 Current Members, 10 New Members

Will Trautman, Vice–President; Robbyn Reynolds, Secretary; Joshua Brooks, Treasurer; Bryan Dawson, Corresponding Secretary.

The TN Gamma chapter held two events this spring, despite the challenges of recovering from an EF-4 tornado that destroyed most of our campus housing and damaged academic buildings on February 5. Although many were injured, no lives were lost, and for that we are exceedingly grateful. We held our annual initiation banquet at the Old Country Store on April 22; ten students were initiated. Alumnus Amanda Cary was the speaker. The second event was our end-of-year picnic May 9, at which graduating seniors were honored.

New Initiates - Sarah A. Conway, Joey Easterling, Caroline J. Hughey, Bradley Kiddie, Kristen Alise Kirk, David Moore, Dylan Riekeman, Matthew Frederick Sengstock, Will Sipes, Kent Avery Willis.

TX Alpha – Texas Tech University

Dr. Magdalena Toda, Corresponding Secretary.

New Initiates – Ashley Farrenkothen, Cory Coltharp, David Pace, Nicholas Rohr, Zachary Powell, William Thomas, Kelli Friar, Lawrence Warren, Nina Attel, Kyle Welden, Janell Appelhans, Brittney Parks, Justin Frerich, Andrew Hardin, Justin Riley McCall, Kenneth Rogers III, Benjamin Welch, Eric Lunsford, Randi Mellon, Darci Barney, Ruslan Filyukov, Jason Kongsataya, Tracy Cowin, Armias DeBella, Jeffery Thomason, Christopher Marquez, Jemelyn Palentinos, Rodolfo Palomo Jr., Kimberly Shipley, Marvin Koerth, Kiowa Sibley-Cutforth, Colin Lauer, Ashley Ray, Sean Trotter, John Kallal, Levi Speer, Joelle Fuhrmann, Susan Caudill, Danielle Felty.

TX Kappa – University of Mary Hardin-Baylor

Chapter President – Brittany Mumme, 10 Current Members, 4 New Members

Daphne Kahlig, Vice-President; Joel Munoz, Secretary; Andrew Brenek , Treasure; Dr. Peter H. Chen, Corresponding Secretary.

New Initiates - Chelsea Caspell, Jennifer Pinkert, Ashley Shafer, Lauren Vander Laan.

TX Lambda – Trinity University

Diane Saphire, Corresponding Secretary.

New Initiates – Matthew Brandt, Mary Beth Browning, Kristin Golmon, Megan Holtz, Matthew Kieke, Alexandra Leamy, Michelle Olivier, Kristin Poole, Jeffrey Stine, Sawan Vaidya.

TX Mu – Schreiner University

Chapter President – Ashley Moore, 14 Current Members, 9 New Members

Mark Freund, Vice-President; Amy Vickers, Secretary; Timothy Tucker, Treasurer; William Sliva, Corresponding Secretary.

New Initiates - Leigh Ann Browm, Robert Brad Baker, Stephen Patrick Franklin, Mark Anthony Freund, Anna Marie Oran, Joseph Michael Parker, Tanya Lynn Jimenez.

VA Gamma – Liberty University

Glyn Wooldridge, Corresponding Secretary.

New Initiates – Kelsey Adams, Ashley Allen, Kaitlin Brill, Caleb Cohoon, Paul Dakum, Erica Fritz, Anthony Jarman, Jennifer Pittman, Jennifer Presson, Nathan Putney, Amy Whicker, Robert Young, Jr.

WI Gamma – University of Wisconsin-Eau Claire

Dr. Simei Tong, Corresponding Secretary.

New Initiates – Gwen Belden, Lindsay Brunshidle, Stephen Connelly, Jared Farmer, Alyssa Haugen, Shaun Koktav, Hatem Kouraichi, Amaris Lieske, Brian Lockwood, Austin Lynch, Bryan Nestingen, Mari Orendorff, Joel Peppler, Chris Peterson, Amanda Rodning, Hillary Rugroden, Maggi Jo Varsho.

Kappa Mu Epsilon National Officers

Don Tosh *President*
Department of Science and Technology
Evangel University
1111 N. Glenstone Avenue
Springfield, MO 65802
toshd@evangel.edu

Ron Wasserstein *President-Elect*
American Statistical Association
732 N Washington Street
Alexandria, VA 22314-1943
ron@amstat.org

Rhonda McKee *Secretary*
Department of Mathematics
University of Central Missouri
Warrensburg, MO 64093-5045
mckee@ucmo.edu

Cynthia Woodburn *Treasurer*
Department of Mathematics
Pittsburg State University
Pittsburg, KS 66762-7502
cwoodbur@pittstate.edu

Connie Schrock *Historian*
Department of Mathematics
Emporia State University
Emporia, KS 66801-5087
schrockc@emporia.edu

Kevin Reed *Webmaster*
Department of Science and Technology
Evangel University
1111 N. Glenstone Avenue
Springfield, MO 65802

KME National Website:
http://www.kappamuepsilon.org/

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960

MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Domincan University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986

TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph’s College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Villa Julie College, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007