

THE PENTAGON

A Mathematics Magazine for Students

Volume 69 Number 1

Fall 2009

Contents

<i>Subscription Renewals</i>	3
<i>Kappa Mu Epsilon National Officers</i>	4
Palindrome Probabilities <i>Anthony Fraticelli</i>	5
A Tasty Combination: Multivariable Calculus and Differential Forms <i>Edray Herber Goins and Talitha M. Washington</i>	11
<i>The Problem Corner</i>	29
<i>Report of the 37th Biennial Convention</i>	40
<i>Kappa Mu Epsilon News</i>	59
<i>Active Chapters of Kappa Mu Epsilon</i>	78

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions must include both an electronic copy and a hard copy. The electronic copy should be included as an attachment to an e-mail sent to the editor. Either a TeX file or Word document is acceptable. It is the responsibility of the author to ensure that any images are in a camera-ready form that can be scaled to the page size of *The Pentagon*. The typewritten copy should be double-spaced with wide margins on white paper. Standard notational conventions should be respected. Graphs, tables, or other materials taken from copyrighted works MUST be accompanied by an appropriate release form from the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

Individual domestic subscriptions: \$5.00 for two issues (one year) or \$10.00 for four issues (two years); individual foreign subscriptions: \$7.00 (US) for two issues (one year). Library rate: \$10.00 (US) for two issues (one year) or \$20.00 (US) for four issues (two years). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced. Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

Editor:

Chip Curtis
 Department of Mathematics
 Missouri Southern State University
 3950 E Newman Road
 Joplin, MO 64801-1595
 curtis-c@mssu.edu

Business Manager:

Don Tosh
 Department of Science and Technology
 Evangel University
 1111 N. Glenstone Ave.
 Springfield, MO 65802-2191
 toshd@evangel.edu

Associate Editors:

The Problem Corner: Pat Costello
 Department of Mathematics and Statistics
 Eastern Kentucky University
 521 Lancaster Avenue
 Richmond, KY 40475-3102
 pat.costello@eku.edu

Kappa Mu Epsilon News: Peter Skoner
 Department of Mathematics
 Saint Francis University
 Loretto, PA 15940
 pskoner@francis.edu

Subscription Renewals and Change of Address

Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is Volume 69 Number 1, if the code 69-1 appears on your label then this is your last issue!

Early renewals save us the cost of mailing out renewal notices. To renew, please send your check - just \$10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subscriptions) - together with your name and address and a copy of your old address label to:

Don Tosh, *The Pentagon* Business manager
Department of Science and Technology
Evangel University
1111 N. Glenstone Ave.
Springfield, MO 65802-2191 USA

Please renew promptly to avoid gaps in your journal collection. **If you have a change of address**, please don't forget *The Pentagon*! Send your change of address to the business manager at the above address. Failure to do so may cost KME additional postage fees!

Back Issues

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for \$5.00 per copy. Please send inquiries to:

Don Tosh, *The Pentagon* Business manager
Department of Science and Technology
Evangel University
1111 N. Glenstone Ave.
Springfield, MO 65802-2191 USA

Kappa Mu Epsilon National Officers

Ron Wasserstein

President

American Statistical Association
732 N Washington Street
Alexandria, VA 22314-1943
ron@amstat.org

Rhonda McKee

President-Elect

Department of Mathematics
University of Central Missouri
Warrensburg, MO 64093-5045
mckee@ucmo.edu

Mark Hamner

Secretary

Department of Mathematics and Computer Science
Texas Woman's University
Denton, TX 76204
mhamner@twu.edu

Cynthia Woodburn

Treasurer

Department of Mathematics
Pittsburg State University
Pittsburg, KS 66762-7502
cwoodbur@pittstate.edu

Peter Skoner

Historian

Department of Mathematics
Saint Francis University
Loretto, PA 15940
pskoner@francis.edu

Kevin Reed

Webmaster

Department of Science and Technology
Evangel University
1111 N. Glenstone Avenue
Springfield, MO 65802

KME National Website:

<http://www.kappamuepsilon.org/>

Palindrome Probabilities

Anthony Fraticelli, *student*

MO Theta

Evangel University
Springfield, MO 62803

Presented at the 2009 National Convention and awarded "top four" status by the Awards Committee.

1. Introduction

A palindrome is a string of at least two characters which reads the same forwards as backwards. We consider palindromes of digits, for example 27372. We answer the question: If we start writing digits randomly, what is the probability that at some point the digits we have written so far will form a palindrome? It can be shown easily by enumeration that the probability is greater than 0.19. And by considering the probability that a string of length n is a palindrome and adding these together for $n > 1$ we can show that an upper bound for this probability is $2/9 = 0.222\dots$. Enumerating the cases did not lead to a recognizable pattern, and so to find the exact value we need to prove the following theorem: Given a palindrome s of length n , if there is a proper initial substring of s which is also a palindrome, then there is an initial substring of s of length less than $n/2 + 1$ which is a palindrome. This result leads to a recursive formula giving the probability in our original question as 0.20889 (to 5 decimal places). We then extend the result to arbitrary bases. In particular, the base 26 result gives the asymptotic probability of a random string of letters eventually forming a "word" palindrome as 0.07834.

2. An Estimate

The question we answer is the following: If an infinite string of digits is randomly generated, what is the probability that for some $n > 1$ the first n digits form a palindrome? We define a palindrome as a string of at least two digits which reads the same forwards as backwards, for example 30103. We will call this probability p^* .

It is easy to find the probability that a string of length n forms a palindrome. Define

L_n = the set of all infinite strings for which the first n digits form palindromes.

Then

$$L_2 = \{00\dots, 11\dots, 33\dots, 44\dots, 55\dots, 66\dots, 77\dots, 88\dots, 99\dots\}.$$

Letting $P(E)$ stand for the probability of event E , the probability of generating a string with the first two digits forming a palindrome is $P(L_2) = 0.1$, since the probability that the second digit will match the first is $1/10$. Similarly $P(L_3) = 0.1$, since the second digit can be any digit and the probability that the third digit matches the first is $1/10$.

The first few values of $P(L_n)$ can easily be seen to be

n	2	3	4	5	6	7
$P(L_n)$	0.1	0.1	0.01	0.01	0.001	0.001

In general, $P(L_n) = (0.1)^{n/2}$ when n is even and $P(L_n) = (0.1)^{(n-1)/2}$ when n is odd. If we use the notation

$\lfloor x \rfloor$ = the floor of x = the greatest integer part of x ,

then we get the general formula

$$P(L_n) = (0.1)^{\lfloor n/2 \rfloor}.$$

Adding these probabilities for $n \geq 2$ gives an upper bound for p^* which can be seen to be

$$\begin{aligned} p^* &\leq 0.1 + 0.1 + 0.01 + 0.01 + 0.001 + 0.001 + \dots \\ &= 0.2 + 0.02 + 0.002 + 0.0002 + \dots \\ &= 0.2 \cdot \sum_{k=0}^{\infty} (1/10)^k \\ &= 0.2 \cdot \frac{1}{1 - \frac{1}{10}} = \frac{2}{9} = 0.222\dots \end{aligned}$$

This is only an upper bound since, for example, L_3 includes the string 222... , which contains a 3 digit palindrome, which should not be included since the string 22... would have already satisfied the condition for p^* and so no more digits would have been examined.

We define an *initial substring* of a string s to be a substring of s which starts with the first digits of s . For example 357 is an initial substring of 35790... , while 579 is not an initial substring. We define the set F_n for $n \geq 2$ as the set of all infinite strings s for which the first n digits of s form a palindrome but no shorter initial substring of s forms a palindrome.

Then, for example $333... \in L_3$ but $333... \notin F_3$. To find the probability of F_3 , we need strings whose second digit differs from the first but the third digit matches the first. The probability of this is

$$\begin{aligned} P(F_3) &= (\text{probability the second digit differs}) \\ &\quad \times (\text{probability the third digit matches}) \\ &= (0.9)(0.1) \\ &= 0.09. \end{aligned}$$

Thus a lower bound for p^* is $P(F_2) + P(F_3) = 0.1 + 0.09 = 0.19$. In fact, $p^* = \sum_{k=2}^{\infty} P(F_k)$. Enumerating F_2 , F_3 , and F_4 is easy using probability trees. Counting F_5 through F_{12} was reasonably straightforward. Counting all possible members of F_k for $k \geq 13$ proved to be quite challenging, and the hope we had of recognizing some pattern faded. We did get what we considered to be a fairly accurate estimate of p^* of 0.2089.

3. Initial Palindromes

The breakthrough came when we tried to determine under what conditions a palindrome could contain a proper initial substring that was also a palindrome. This led to the following theorem.

Theorem 1 If a string s of length n is a palindrome and has a proper initial string s_1 which is also a palindrome, then there is a proper initial substring s_2 of s which is also a palindrome and has length less than or equal to $\lfloor (n+1)/2 \rfloor$.

To facilitate the proof we need to introduce some more notation. If s is a string, we define the *reflection* of s to be s in reverse order and we will write it as \bar{s} . For example, if $s = 270$, then $\bar{s} = \overline{270} = 072$. Note that a string s is a palindrome if and only if $s = \bar{s}$. We define a *central substring* of a string s to be a substring which is formed by removing

the same number of digits from each end of s . For example, the central substring 345 of 1234567 is formed by deleting two digits from each end. Note that every central substring of a palindrome is also a palindrome. We also use the usual string concatenation symbol $\&$ to concatenate strings. So, for example, if $s_1 = 137$ and $s_2 = 405$, then $s_1 \& s_2 = 137405$. With this notation in place we can prove Theorem 1.

Proof of Theorem 1. Assume s is a string of length n ($n \geq 4$) which is a palindrome. Assume s_1 is an initial substring of s of length k which is also a palindrome. If $k \leq \lfloor (n+1)/2 \rfloor$, we are finished, so assume $k > \lfloor (n+1)/2 \rfloor$. Then $s = s_1 \& s_2$ for some string s_2 of length $n-k$, where $n-k < k$. Since s is a palindrome, the reflection of s_2 forms an initial substring of s_1 . Thus we can write $s_1 = \overline{s_2} \& s_3$ for some string s_3 of length $2k-n$, making $s = \overline{s_2} \& s_3 \& s_2$. Since s_3 is a central substring of a palindrome, it must also be a palindrome, so $s_3 = \overline{s_3}$. Since $s_1 = \overline{s_2} \& s_3$ and is a palindrome, it is equal to its reflection $\overline{\overline{s_2} \& s_3}$, placing s_3 (which equals $\overline{s_3}$ and is a palindrome) as an initial substring of s . Since s_1 is a proper substring of s , k must be less than n , so the length of s_3 must be at least 2 and no more than $n-2$. If the length of s_3 is greater than $\lfloor (n+1)/2 \rfloor$, then s_1 is a palindrome of length less than n with a proper initial substring s_3 which is also a palindrome. The above argument can be repeated to yield an initial substring s_4 which will be at least 2 digits long but at least 2 shorter than s_3 . Eventually this leads to an initial substring of s which is a palindrome and has the required length. This completes the proof. ■

We will call initial substrings which form palindromes *initial palindromes*. We need to introduce one more set. For $n \geq 2$, we define the set A_n as

$$A_n = \text{the set of all infinite strings which have an initial palindrome of length at most } n.$$

For example 12215..., 11234..., and 13431... are all elements of A_5 . This new notation allows us to state a corollary to Theorem 1.

Corollary 2 $L_n \cap A_{n-1} = L_n \cap A_{\lfloor (n+1)/2 \rfloor}$.

Proof of Corollary: Clearly if $s \in L_n \cap A_{\lfloor (n+1)/2 \rfloor}$, then $s \in L_n \cap A_{n-1}$ since $\{A_n\}$ is an increasing sequence of nested sets and $\lfloor (n+1)/2 \rfloor < n-1$ for $n \geq 4$. If $s \in L_n \cap A_{n-1}$, then the first n digits of s form an initial

palindrome, and there is a shorter initial palindrome of s as well. Theorem 1 lets us conclude that there must be an initial palindrome of length less than or equal to $\lfloor (n+1)/2 \rfloor$, and so $s \in L_n \cap A_{\lfloor (n+1)/2 \rfloor}$ as well. ■

This result is important since the intersection $L_n \cap A_{\lfloor (n+1)/2 \rfloor}$ is between independent events, which makes finding the probability easier.

4. A Recursive Formula

The probability p^* of eventually seeing an initial palindrome is simply $\lim_{n \rightarrow \infty} P(A_n)$. We wish to find a formula for $P(A_n)$. While not an explicit formula, the following theorem gives us a recursive result.

Theorem 3 The probability of writing an initial palindrome within the first n digits in base 10 is

$$P(A_n) = P(A_{n-1}) + \frac{1}{10^{\lfloor n/2 \rfloor}} (1 - P(A_{\lfloor (n+1)/2 \rfloor})).$$

Proof of Theorem 2: If an initial substring of length n contains an initial palindrome, then either it first formed the palindrome at the n th digit or on a shorter initial substring. These events are disjoint so their probabilities add:

$$P(A_n) = P(F_n) + P(A_{n-1}).$$

If $s \in F_n$, then $s \in L_n$ but $s \notin A_{n-1}$. If $s \notin A_{n-1}$, then $s \notin A_k$ for any $k < n-1$. In particular, $s \notin A_{\lfloor (n+1)/2 \rfloor}$. Thus the first $\lfloor (n+1)/2 \rfloor$ digits do not contain an initial palindrome ($s \notin A_{\lfloor (n+1)/2 \rfloor}$) and the next $\lfloor n/2 \rfloor$ digits are the reflection of the first $\lfloor n/2 \rfloor$ digits ($s \in L_n$). Since digits are drawn randomly, the choice of the first $\lfloor (n+1)/2 \rfloor$ digits does not affect the probability of selection for the next $\lfloor n/2 \rfloor$ digits, so these events are independent. That lets us find the probabilities by simply multiplying, giving

$$P(F_n) = P(L_n) \cdot (1 - P(A_{\lfloor (n+1)/2 \rfloor})) = \frac{1}{10^{\lfloor n/2 \rfloor}} (1 - P(A_{\lfloor (n+1)/2 \rfloor})).$$

Substituting this quantity in the previous equation completes the proof. ■

Since $A_2 = L_2 = F_2$, we have that $P(A_2) = 1/10$. Then the first several values of $P(A_n)$ are:

$$\begin{aligned}
P(A_2) &= \frac{1}{10} = 0.1 \\
P(A_3) &= P(A_2) + \frac{1}{10^{\lfloor 3/2 \rfloor}} (1 - P(A_{\lfloor 4/2 \rfloor})) = 0.19 \\
P(A_4) &= P(A_3) + \frac{1}{10^{\lfloor 4/2 \rfloor}} (1 - P(A_{\lfloor 5/2 \rfloor})) = 0.199 \\
P(A_5) &= P(A_4) + \frac{1}{10^{\lfloor 5/2 \rfloor}} (1 - P(A_{\lfloor 6/2 \rfloor})) = 0.2071 \\
P(A_6) &= P(A_5) + \frac{1}{10^{\lfloor 6/2 \rfloor}} (1 - P(A_{\lfloor 7/2 \rfloor})) = 0.20791 \\
P(A_7) &= P(A_6) + \frac{1}{10^{\lfloor 7/2 \rfloor}} (1 - P(A_{\lfloor 8/2 \rfloor})) = 0.208711
\end{aligned}$$

These values correspond to the hand calculations we found for the enumeration approach. Using a simple program to generate the values gives an asymptotic value for p^* of 0.208888 to six decimals.

5. Extension to Other Bases

These arguments could be easily extended to bases other than 10. If we denote the new base as b , then the sets L_n , F_n , and A_n are defined in the same way. We have $P(L_2) = P(F_2) = P(A_2) = 1/b$, and $P(L_n)$ becomes

$$P(L_n) = (1/b)^{\lfloor n/2 \rfloor}.$$

Theorem 1 and the Corollary still hold and Theorem 2 becomes

Theorem 4 The probability of writing an initial palindrome within the first n digits in base b is

$$P(A_n) = P(A_{n-1}) + \frac{1}{b^{\lfloor n/2 \rfloor}} (1 - P(A_{\lfloor (n+1)/2 \rfloor})).$$

In base 2 this gives $P(A_n) = 1 - (\frac{1}{2})^{n-1}$, so the asymptotic probability of a binary palindrome is 1. Base 26 would correspond to “word” palindromes. The asymptotic probability of eventually writing a word palindrome by randomly writing letters of the alphabet is 0.078343 to six decimal places.

We were hoping to find an explicit formula for the probability of writing palindromes. We were not able to do this. However, the recursive formula we did arrive at is relatively easy to use and was surprisingly easy to extend to other bases.

A Tasty Combination: Multivariable Calculus and Differential Forms

Edray Herber Goins and Talitha M. Washington

Purdue University 150 North University Street West Lafayette, IN
47907-2067

University of Evansville 1800 Lincoln Avenue Evansville, IN 47722

1. Introduction

Differential Calculus is a staple of the college mathematics major's diet. It is relatively easy to explain the Fundamental Theorem of Calculus: Given a differentiable function $f : I \rightarrow \mathbb{R}$ defined on closed interval $I = [a, b]$, there is the identity

$$\int_a^b \frac{d}{dt} [f(t)] dt = f(b) - f(a).$$

Eventually one becomes tired of the same routine, and wishes for a more diverse meal. The college math major may seek to generalize applications of the derivative that involve functions of more than one variable, and thus enjoy a course on Multivariate Calculus. Actually, there is a veritable buffet of ways to differentiate and integrate a function of more than one variable: there is the gradient, curl, divergence, path integrals, surface integrals, and volume integrals. Plus, there are many "Fundamental" Theorems of Multivariate Calculus, such as Stokes' Theorem, Green's Theorem, and Gauss' Theorem.

We serve this article as a culinary guide to differentiating and integrating functions of more than one variable – using differential forms which are the basis for de Rham Cohomology.

2. Gradient, Curl, and Divergence

Let's focus on functions of three variables. First, let's fix a "simply connected" closed subset $D \subseteq \mathbb{R}^3$; this means we can define an integral between two points without concern of a choice of path in D which connects them. (For example, a subset in the form $D = [a, b] \times [c, d] \times [p, q]$ is simply connected.) Recall that the *gradient* of a scalar function $f : D \rightarrow \mathbb{R}$ is the vector-valued function $\nabla f : D \rightarrow \mathbb{R}^3$ defined by

$$\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}.$$

(Here $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$ are the standard basis vectors for 3-dimensional space.) The *curl* of a vector field $\mathbf{F} : D \rightarrow \mathbb{R}^3$, which we write in the form $\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$, is the vector-valued function $\nabla \times \mathbf{F} : D \rightarrow \mathbb{R}^3$ defined by

$$\begin{aligned} \nabla \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} \\ &= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) \mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) \mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \mathbf{k}. \end{aligned}$$

Finally, the *divergence* of a vector field $\mathbf{G} : D \rightarrow \mathbb{R}^3$, which we write in the form $\mathbf{G} = S \mathbf{i} + T \mathbf{j} + U \mathbf{k}$, is the scalar function $\nabla \cdot \mathbf{G} : D \rightarrow \mathbb{R}$ defined by

$$\nabla \cdot \mathbf{G} = \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial U}{\partial z}.$$

Larson [4] provides more information on Multivariable Calculus. Marsden and Tromba [5] is also an excellent source. Hughes-Hallet et al. [3] provides a novel approach to the subject using concept-based learning.

We would like to answer the following questions:

1. Which $\mathbf{F} : D \rightarrow \mathbb{R}^3$ have a scalar potential $f : D \rightarrow \mathbb{R}$ such that $\mathbf{F} = \nabla f$?
2. Which $\mathbf{G} : D \rightarrow \mathbb{R}^3$ have a vector potential $\mathbf{F} : D \rightarrow \mathbb{R}^3$ such that $\mathbf{G} = \nabla \times \mathbf{F}$?
3. Which $f : D \rightarrow \mathbb{R}$ have a vector potential $\mathbf{G} : D \rightarrow \mathbb{R}^3$ such that $f = \nabla \cdot \mathbf{G}$?

Relating Gradients and Curls. Let's answer the first of our motivating questions. We'll show the following:

$$\mathbf{F} = \nabla f \text{ is a gradient if and only if the curl } \nabla \times \mathbf{F} = \mathbf{0}.$$

First assume that $\mathbf{F} = \nabla f$. That is,

$$\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k} = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}.$$

To find the curl of \mathbf{F} , we have

$$\begin{aligned} \nabla \times \mathbf{F} &= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) \mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) \mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \mathbf{k} \\ &= \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y} \right) \mathbf{i} + \left(\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z} \right) \mathbf{j} \\ &\quad + \left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right) \mathbf{k} \\ &= \mathbf{0}. \end{aligned}$$

Note that we can change the order of differentiation as long as f is continuously twice-differentiable: with this assumption, Clairaut's Theorem states that the mixed partial derivatives are equal. (This theorem and its proof can be found in [6, pp. 885 and A-48].)

Now what if $\nabla \times \mathbf{F} = \mathbf{0}$? Can we find an f such that $\mathbf{F} = \nabla f$? Well, if $\nabla \times \mathbf{F} = \mathbf{0}$, then

$$\left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) = \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) = \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = 0.$$

Now define the scalar function $f = u + v + w$ in terms of the definite integrals

$$\begin{aligned} u(x, y, z) &= \int_{x_0}^x M(\sigma, y, z) d\sigma, \\ v(x, y, z) &= \int_{y_0}^y \left[N(x, \tau, z) - \frac{\partial u}{\partial y}(x, \tau, z) \right] d\tau, \\ w(x, y, z) &= \int_{z_0}^z \left[P(x, y, \zeta) - \frac{\partial u}{\partial z}(x, y, \zeta) - \frac{\partial v}{\partial z}(x, y, \zeta) \right] d\zeta, \end{aligned}$$

where we have fixed a point $x_0 \mathbf{i} + y_0 \mathbf{j} + z_0 \mathbf{k} \in D$. (Here's where we

subtly use the assumption that D is simply connected: the integrals are independent of a choice of path in D which connects $x_0 \mathbf{i} + y_0 \mathbf{j} + z_0 \mathbf{k}$ and $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$.) Upon changing the order of differentiation and integration, we have the derivatives

$$\begin{aligned}\frac{\partial v}{\partial x} &= \int_{y_0}^y \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] d\tau = 0 \\ \frac{\partial w}{\partial x} &= \int_{z_0}^z \left[\left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z} \right) - \frac{\partial v}{\partial x} \right] d\zeta = 0, \\ \frac{\partial w}{\partial y} &= \int_{z_0}^z \left[\frac{\partial P}{\partial y} - \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right) \right] d\zeta = \int_{z_0}^z \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) d\zeta = 0.\end{aligned}$$

(We can only do this if M , N , and P are continuously differentiable functions; this also follows from Clairaut's Theorem.) It follows that $\mathbf{F} = \nabla f$:

$$\begin{aligned}\nabla f &= \left[\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial x}} + \cancel{\frac{\partial w}{\partial x}} \right] \mathbf{i} + \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial y}} \right] \mathbf{j} \\ &\quad + \left[\frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} + \frac{\partial w}{\partial z} \right] \mathbf{k} \\ &= M \mathbf{i} + N \mathbf{j} + P \mathbf{k}.\end{aligned}$$

We conclude that $\mathbf{F} : D \rightarrow \mathbb{R}^3$ has a scalar potential $f : D \rightarrow \mathbb{R}$ such that $\mathbf{F} = \nabla f$ if and only if the curl $\nabla \times \mathbf{F} = \mathbf{0}$.

Example. Let's see this result in action. Consider the scalar function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

We compute the gradient using the Chain Rule:

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{1}{2} (x^2 + y^2 + z^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \\ \frac{\partial f}{\partial y} &= \frac{1}{2} (x^2 + y^2 + z^2)^{-1/2} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \\ \frac{\partial f}{\partial z} &= \frac{1}{2} (x^2 + y^2 + z^2)^{-1/2} \cdot 2z = \frac{z}{\sqrt{x^2 + y^2 + z^2}}.\end{aligned}$$

This gives the vector field

$$\mathbf{F} = \nabla f = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}.$$

(See Figure 1 for a plot. To keep with the gastronomical motif of this article, perhaps we should call this direction field a "Prickly Pear"?)

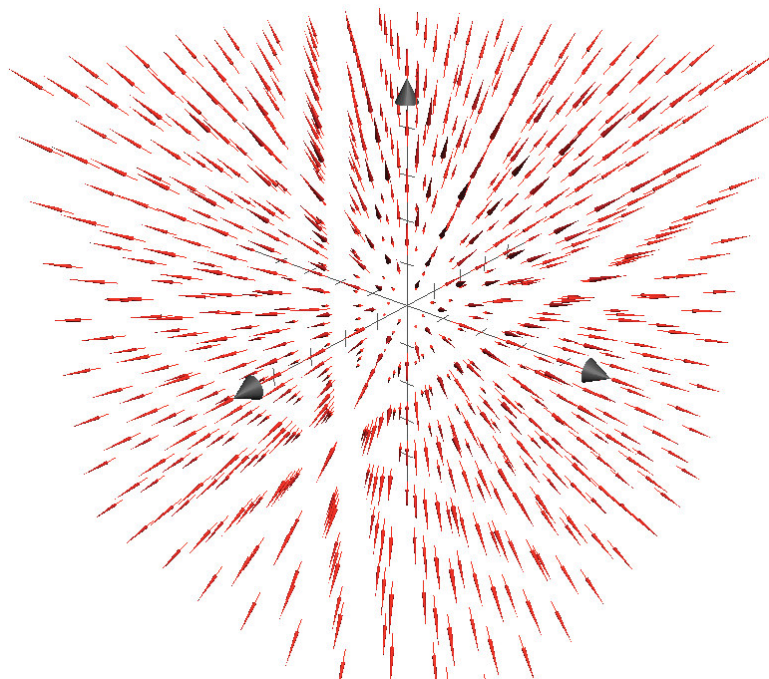


Figure 1: Plot of $\mathbf{F} = \nabla f$ for $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$

Recall that the curl of this vector field is

$$\nabla \times \mathbf{F} = \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y} \right) \mathbf{i} + \left(\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z} \right) \mathbf{j} + \left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right) \mathbf{k}.$$

We compute the mixed partial derivatives as follows:

$$\begin{aligned} \frac{\partial^2 f}{\partial y \partial z} &= \frac{\partial}{\partial y} \left[\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right] = -\frac{1}{2} z (x^2 + y^2 + z^2)^{-3/2} \cdot 2y \\ &= -\frac{yz}{(x^2 + y^2 + z^2)^{3/2}} \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 f}{\partial z \partial y} &= \frac{\partial}{\partial z} \left[\frac{y}{\sqrt{x^2 + y^2 + z^2}} \right] = -\frac{1}{2} y (x^2 + y^2 + z^2)^{-3/2} \cdot 2z \\ &= -\frac{yz}{(x^2 + y^2 + z^2)^{3/2}} \end{aligned}$$

Note that the other mixed partial derivatives give rise to a similar function.

Thus, $\nabla \times \mathbf{F} = \mathbf{0}$.

Relating Curls and Divergence. Let's return to the second of our motivating questions. We'll show the following:

$\mathbf{G} = \nabla \times \mathbf{F}$ is a curl if and only if the divergence $\nabla \cdot \mathbf{G} = 0$.

First assume that $\mathbf{G} = \nabla \times \mathbf{F}$ for some $\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$. That is,

$$S \mathbf{i} + T \mathbf{j} + U \mathbf{k} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) \mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) \mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \mathbf{k}.$$

We compute the divergence as

$$\begin{aligned} \nabla \cdot \mathbf{G} &= \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial U}{\partial z} \\ &= \left(\frac{\partial^2 P}{\partial x \partial y} - \frac{\partial^2 N}{\partial x \partial z} \right) + \left(\frac{\partial^2 M}{\partial y \partial z} - \frac{\partial^2 P}{\partial y \partial x} \right) + \left(\frac{\partial^2 N}{\partial z \partial x} - \frac{\partial^2 M}{\partial z \partial y} \right) \\ &= 0. \end{aligned}$$

(Here, we assume that M , N , and P are continuously twice-differentiable so that we can interchange the order of differentiation.)

Now what if $\nabla \cdot \mathbf{G} = 0$? In this case, we write

$$\begin{aligned} \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial U}{\partial z} &= 0 \\ \implies U(x, y, z) &= U(x, y, z_0) - \int_{z_0}^z \left[\frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} \right] d\zeta \end{aligned}$$

for any fixed point $x_0 \mathbf{i} + y_0 \mathbf{j} + z_0 \mathbf{k} \in D$. Define the function $\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$ in terms of the definite integrals

$$M(x, y, z) = \int_{z_0}^z T(x, y, \zeta) d\zeta - \int_{y_0}^y U(x, \tau, z_0) d\tau$$

$$N(x, y, z) = - \int_{z_0}^z S(x, y, \zeta) d\zeta$$

$$P(x, y, z) = 0.$$

Upon changing the order of differentiation and integration, we have the derivatives

$$\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} = S(x, y, z)$$

$$\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} = T(x, y, z)$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = U(x, y, z_0) - \int_{z_0}^z \left[\frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} \right] d\zeta = U(x, y, z).$$

It follows that $\nabla \times \mathbf{F} = \mathbf{G}$.

Example. Consider the vector function

$$\mathbf{G} = \nabla f = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$$

defined in terms of

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

(Remember the “Prickly Pear”?) We will show that there is *no* vector potential \mathbf{F} such that $\mathbf{G} = \nabla \times \mathbf{F}$. The idea is to show that the divergence of \mathbf{G} is nonzero. To this end, we compute the higher-order partial derivatives using the Quotient Rule:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left[\frac{x}{\sqrt{x^2 + y^2 + z^2}} \right] = \frac{y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left[\frac{y}{\sqrt{x^2 + y^2 + z^2}} \right] = \frac{x^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z} \left[\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right] = \frac{x^2 + y^2}{(x^2 + y^2 + z^2)^{3/2}}$$

(Check as an exercise!) Hence we find the divergence

$$\nabla \cdot \mathbf{G} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{2x^2 + 2y^2 + 2z^2}{(x^2 + y^2 + z^2)^{3/2}} = \frac{2}{f}.$$

Since $\nabla \cdot \mathbf{G} \neq 0$, there cannot exist a function \mathbf{F} such that $\mathbf{G} = \nabla \times \mathbf{F}$.

3. Differential Forms

Naturally, the three motivating questions in the introduction are rather naive questions to ask – although a bit difficult to answer! – so let’s make them a little more interesting. We’ll rephrase the definitions above in a way using differential k -forms; this will make integration more natural.

- A 0 -form is a continuously differentiable function $f : D \rightarrow \mathbb{R}$. These are the functions we know and love from the one-variable Differential Calculus.

- A 1-form is an arc length differential in the form $\eta = M dx + N dy + P dz$ for some continuously differentiable vector field $\mathbf{F} : D \rightarrow \mathbb{R}^3$ in the form $\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$. Note that we can express this in the form $\eta = \mathbf{F} \cdot d\mathbf{r}$ using the dot product. We'll use this to define path integrals later.
- A 2-form is an area differential in the form $\omega = S dy dz + T dx dz + U dx dy$ for some continuously differentiable vector field $\mathbf{G} : D \rightarrow \mathbb{R}^3$ in the form $\mathbf{G} = S \mathbf{i} + T \mathbf{j} + U \mathbf{k}$. Note that we can express this in the form $\omega = \mathbf{G} \cdot d\mathbf{A}$. As the notation suggests, we'll use this to define surface integrals.
- 3-form is a volume differential in the form $\nu = f dx dy dz$ for some continuously differentiable function $f : D \rightarrow \mathbb{R}$. Note that we can express this in the form $\nu = f dV$. As the notation suggests, we'll use this to define volume integrals.

We'll denote by $\Omega^k(D)$ the collection of k -forms on D for $k = 0, 1, 2,$ and 3 . Boothby [2] provides a rigorous treatment of differential forms.

We claim that each $\Omega^k(D)$ is a linear vector space. This means that the linear combination of two k -forms is another k -form. Consider, for example, 1-forms and 2-forms. Given scalars α and β , we have the identities

$$\begin{aligned} \alpha(M_1 dx + N_1 dy + P_1 dz) + \beta(M_2 dx + N_2 dy + P_2 dz) \\ = (\alpha M_1 + \beta M_2) dx + (\alpha N_1 + \beta N_2) dy + (\alpha P_1 + \beta P_2) dz \end{aligned}$$

$$\begin{aligned} \alpha(S_1 dy dz + T_1 dx dz + U_1 dx dy) + \beta(S_2 dy dz + T_2 dx dz + U_2 dx dy) \\ = (\alpha S_1 + \beta S_2) dy dz + (\alpha T_1 + \beta T_2) dx dz + (\alpha U_1 + \beta U_2) dx dy. \end{aligned}$$

Thus $\Omega^k(D)$ is a linear vector space for $k = 1, 2$. The argument for $k = 0, 3$ is similar.

Maps Between Differential Forms. How are these linear vector spaces related? Well, there are linear maps between them! Consider the following “differential” map $d : \Omega^0(D) \rightarrow \Omega^1(D)$ from 0-forms to 1-forms:

$$\begin{array}{c}
 f \\
 \downarrow \\
 df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = \nabla f \cdot d\mathbf{r}.
 \end{array}$$

This is linear because $d(\alpha f_1 + \beta f_2) = \alpha df_1 + \beta df_2$. Here, we write $d\mathbf{r} = dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k}$ as the arc length differential – expressed as a vector. Hence we find the answer to our first motivating question:

The vector – valued function $\mathbf{F} = \nabla f$ has a scalar potential f if and only if the 1 – form $\mathbf{F} \cdot d\mathbf{r} = df$ is the differential of some 0 – form f .

Similarly, consider the following “differential” map $d : \Omega^1(D) \rightarrow \Omega^2(D)$ from 1-forms to 2-forms:

$$\begin{array}{c}
 \eta = M dx + N dy + P dz = \mathbf{F} \cdot d\mathbf{r} \\
 \downarrow \\
 d\eta = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} \right) dy dz + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} \right) dz dx \\
 + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy \\
 = (\nabla \times \mathbf{F}) \cdot d\mathbf{A}.
 \end{array}$$

This is linear because $d(\alpha \eta_1 + \beta \eta_2) = \alpha d\eta_1 + \beta d\eta_2$. Here, we write $d\mathbf{A} = dy dz \mathbf{i} + dz dx \mathbf{j} + dx dy \mathbf{k}$ as the area differential – also expressed as a vector. Hence we find the answer to our second motivating question:

The vector – valued function $\mathbf{G} = \nabla \times \mathbf{F}$ has a vector potential \mathbf{F} if and only if the 2 – form $\mathbf{G} \cdot d\mathbf{A} = d\eta$ is the differential of some 1 – form $\eta = \mathbf{F} \cdot d\mathbf{r}$.

Recall that $\mathbf{F} = \nabla f$ is a gradient if and only if the curl $\nabla \times \mathbf{F} = \mathbf{0}$. In other words, given a 1-form $\eta = M dx + N dy + P dz = \mathbf{F} \cdot d\mathbf{r}$, we have $\eta = df$ as the differential of a 0-form if and only if the 2-form $d\eta = 0$. This is yet another answer to our first question!

Finally, consider the following “differential” map $d : \Omega^2(D) \rightarrow \Omega^3(D)$ from 2-forms to 3-forms:

$$\omega = S \, dy \, dz + T \, dx \, dz + U \, dx \, dy = \mathbf{G} \cdot d\mathbf{A}$$

↓

$$d\omega = \left(\frac{\partial S}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial U}{\partial z} \right) dx \, dy \, dz = (\nabla \cdot \mathbf{G}) \, dV.$$

Again, this is linear because $d(\alpha \omega_1 + \beta \omega_2) = \alpha d\omega_1 + \beta d\omega_2$. Remember that $dV = dx \, dy \, dz$ is the volume differential. Hence we find the answer to the last of our three motivating questions:

The vector – valued function $f = \nabla \cdot \mathbf{G}$ has a vector potential \mathbf{G} if and only if the 3 – form $f \, dV = d\omega$ is the differential of some 2 – form $\omega = \mathbf{G} \cdot d\mathbf{A}$.

Recall that $\mathbf{G} = \nabla \times \mathbf{F}$ is a curl if and only if the divergence $\nabla \cdot \mathbf{G} = 0$. In other words, given a 2-form $\omega = S \, dy \, dz + T \, dx \, dz + U \, dx \, dy = \mathbf{G} \cdot d\mathbf{A}$, we have $\omega = d\eta$ as the differential of a 1-form if and only if the 3-form $d\omega = 0$. This is yet another answer to our second question!

We’ll summarize this via the following diagram of maps:

$$\{0\} \longrightarrow \Omega^0(D) \xrightarrow{\text{gradient}} \Omega^1(D) \xrightarrow{\text{curl}} \Omega^2(D) \xrightarrow{\text{divergence}} \Omega^3(D) \longrightarrow \{0\}.$$

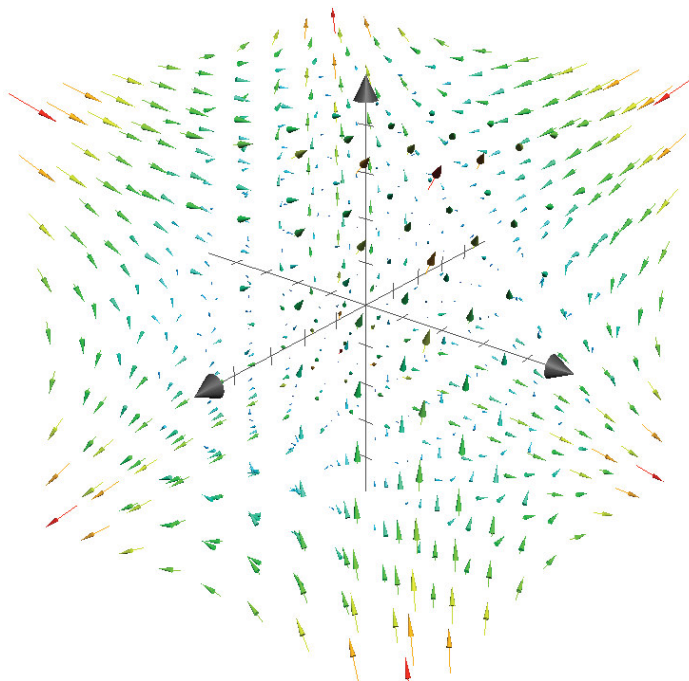
Example. Let’s consider the differential 1-form

$$\eta = yz \, dx + xz \, dy + xy \, dz.$$

This is in the form $M \, dx + N \, dy + P \, dz$ for $M = yz$, $N = xz$, and $P = xy$. (See Figure 2 for a plot of $\mathbf{F} = M \mathbf{i} + N \mathbf{j} + P \mathbf{k}$.) Figure 2: We’ll compute the differential $d\eta$. We have the partial derivatives

$$\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z} = x - x = 0, \quad \frac{\partial M}{\partial z} - \frac{\partial P}{\partial x} = y - y = 0, \quad \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = z - z = 0.$$

Hence $d\eta = 0$.

Figure 2: Plot of $\mathbf{F} = yz \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$

4. Integration

We know now how the various forms of differentiation are all related, but what about integration? First, we review what we mean by integration. As $D \subseteq \mathbb{R}^3$ is a subset of three-dimensional space, we can define integration for one variable, two variables, or even three variables.

Fundamental Theorem of Calculus. In order to define an integral for one variable, let $\mathbf{r} : I \rightarrow D$ be a continuously differentiable map, which we write in the form $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, defined on a closed interval $I = [a, b]$. We say that the image $C \subseteq D$ is a *path*. Given vector field $\mathbf{F} : D \rightarrow \mathbb{R}^3$, the composition gives the 1-form $\eta = \mathbf{F}(\mathbf{r}(t)) \cdot d\mathbf{r}$, so naturally a *path integral* is

$$\int_C \eta = \int_a^b \left[M(\mathbf{r}(t)) \frac{dx}{dt} + N(\mathbf{r}(t)) \frac{dy}{dt} + P(\mathbf{r}(t)) \frac{dz}{dt} \right] dt.$$

Recall that $\mathbf{F} = \nabla f$ has a scalar potential f if and only if $\eta = df$ is the differential of a 0-form f . In this case, the path integral simplifies to

$$\int_a^b \frac{d}{dt} [f(\mathbf{r}(t))] dt = \int_C df = f(\mathbf{r}) \Big|_{\partial C} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

Hence the integral is independent of the path C , and it is only dependent on the endpoints $\partial C = \{\mathbf{r}(a), \mathbf{r}(b)\}$. This is just the *Fundamental Theorem of Calculus*.

Example. Let $D = \mathbb{R}^3$ be all of three-dimensional space, and let C denote the circle of radius r in the plane. Note that C is the set of points $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ such that $x^2 + y^2 = r^2$ and $z = 0$. We may think of this as being the image of the map $\mathbf{r} : [0, 2\pi] \rightarrow \mathbb{R}^3$ which sends $t \mapsto r \cos t \mathbf{i} + r \sin t \mathbf{j}$. The circle is just an example of a path!

Now let's consider a vector field $\mathbf{F} = -(y/2)\mathbf{i} + (x/2)\mathbf{j}$. (See Figure 3 for a plot.) We'll explain why there does *not* exist a scalar function f such that $\mathbf{F} = \nabla f$. The idea is to suppose that f does indeed exist and then compute the integral of \mathbf{F} around the path C . Let's consider the following differential 1-form:

$$\begin{aligned} \eta = \mathbf{F}(\mathbf{r}(t)) \cdot d\mathbf{r} &= \left(-\frac{r \cos t}{2} \mathbf{i} + \frac{r \sin t}{2} \mathbf{j} \right) \cdot (-r \cos t dt \mathbf{i} + r \sin t dt \mathbf{j}) \\ &= \frac{r^2 \cos^2 t dt + r^2 \sin^2 t dt}{2} = \frac{r^2}{2} dt. \end{aligned}$$

If $\mathbf{F} = \nabla f$, then $\int_C \eta = f(\mathbf{r}(2\pi)) - f(\mathbf{r}(0)) = 0$. But actually,

$$\int_C \eta = \int_0^{2\pi} \frac{r^2}{2} dt = \pi r^2.$$

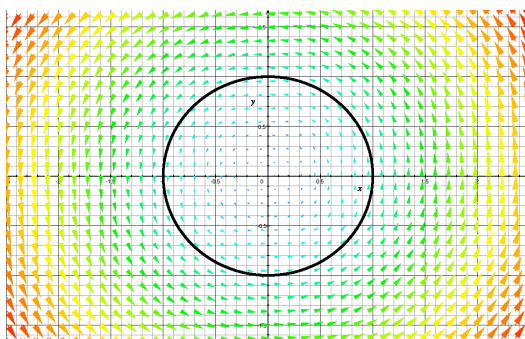


Figure 3: Plot of $\mathbf{F} = -(y/2)\mathbf{i} + (x/2)\mathbf{j}$

Hence there is no function f such that $\mathbf{F} = \nabla f$. (Of course, we could have seen this sooner by computing the curl $\nabla \times \mathbf{F} = \mathbf{k}$ and realizing it's a nonzero vector.)

Stokes' Theorem and Green's Theorem. In order to define an integral for two variables, let $\mathbf{r} : R \rightarrow D$ be a continuously differentiable map, which we write in the form $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, that is defined on an closed region $R = [a, b] \times [c, d]$. We say that the image $S \subseteq D$ is a *surface*, and assume that its boundary $C = \partial S$ is a curve as before. Given vector field $\mathbf{G} : D \rightarrow \mathbb{R}^3$, the composition gives the 2-form $\omega = \mathbf{G}(\mathbf{r}(u, v)) \cdot d\mathbf{A}$, so naturally *surface integral* is defined as

$$\iint_S \omega = \int_a^b \int_c^d \begin{vmatrix} S(\mathbf{r}(u, v)) & T(\mathbf{r}(u, v)) & U(\mathbf{r}(u, v)) \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix} du dv.$$

Recall that $\mathbf{G} = \nabla \times \mathbf{F}$ has a vector potential \mathbf{F} if and only if $\omega = d\eta$ is the differential of a 1-form η . In this case, the surface integral simplifies to

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \iint_S d\eta = \int_{\partial S} \eta = \int_C \mathbf{F} \cdot d\mathbf{r}.$$

Hence the integral is independent of the surface S , and it is only dependent on the boundary $C = \partial S$. This is known as *Stokes' Theorem*.

Let's consider a special case, where $\mathbf{r} : R \rightarrow \mathbb{R}^2$ actually maps into the plane. Then $S \subseteq \mathbb{R}^2$, so write $\mathbf{F} = M \mathbf{i} + N \mathbf{j}$. Stokes' Theorem in the plane reduces to the statement

$$\iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy = \iint_S d\eta = \int_{\partial R} \eta = \int_{\partial S} [M dx + N dy].$$

This is known as *Green's Theorem*. Similarly, for the *orthogonal* vector field $\mathbf{F}_\perp = -N \mathbf{i} + M \mathbf{j}$, the *Divergence Theorem in the Plane* is the expression

$$\iint_S \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy = \int_{\partial S} [-N dx + M dy].$$

Example. Let $D = \mathbb{R}^3$ be all of three-dimensional space, and let S denote the disk of radius r in the plane. The latter is just the set of points $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ such that $x^2 + y^2 \leq r^2$ and $z = 0$. We may think of this as being the image of the map $\mathbf{r} : [0, r] \times [0, 2\pi] \rightarrow \mathbb{R}^3$ which sends $(\rho, \theta) \mapsto \rho \cos \theta \mathbf{i} + \rho \sin \theta \mathbf{j}$. Note that the boundary $C = \partial S$ is simply the circle $x^2 + y^2 = r^2$, which we considered before.

Let's return to the vector field $\mathbf{F} = -(y/2)\mathbf{i} + (x/2)\mathbf{j}$. We'll consider the differential 2-form $\omega = (\nabla \times \mathbf{F}) \cdot d\mathbf{A} = \mathbf{k} \cdot d\mathbf{A} = dx dy$. Stokes' Theorem (or really Green's Theorem, since it's in the plane) states that

$$\iint_S \omega = \int_C \mathbf{F} \cdot d\mathbf{r} = \pi r^2.$$

(Recall the previous example.) Let's compute the integral on the left-hand side in a different way. This differential 2-form can also be expressed in *polar coordinates* using the determinant

$$\begin{aligned} \omega &= \begin{vmatrix} S(\mathbf{r}(\rho, \theta)) & T(\mathbf{r}(\rho, \theta)) & U(\mathbf{r}(\rho, \theta)) \\ \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix} d\rho d\theta \\ &= \begin{vmatrix} 0 & 0 & 1 \\ \cos \theta & \sin \theta & 0 \\ -\rho \sin \theta & \rho \cos \theta & 0 \end{vmatrix} d\rho d\theta \\ &= \rho d\rho d\theta. \end{aligned}$$

Hence we have the integral

$$\iint_S dx dy = \iint_S \omega = \int_0^r \int_0^{2\pi} \rho d\rho d\theta = \left[\int_0^r \rho d\rho \right] \left[\int_0^{2\pi} d\theta \right] = \pi r^2.$$

Of course, this is just the area of the disk S .

Gauss' Theorem and the Divergence Theorem. In order to define an integral for three variables, let $\mathbf{r} : B \rightarrow \mathbb{R}^3$ be a continuously differentiable map, which we write in the form $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, defined on an closed region $B = [a, b] \times [c, d] \times [p, q]$. We say that the image $D \subseteq \mathbb{R}^3$ is a *region*, and assume that its boundary $S = \partial D$ is a surface as before. Given a scalar function $f : D \rightarrow \mathbb{R}$, the composition gives the differential 3-form $\nu = f(\mathbf{r}(u, v, w)) dV$, so naturally define a *volume integral* as

$$\iiint_D \nu = \int_a^b \int_c^d \int_p^q f(\mathbf{r}(u, v, w)) \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial w} & \frac{\partial y}{\partial w} & \frac{\partial z}{\partial w} \end{vmatrix} du dv dw.$$

(As with surface integrals, we've expressed the integrand above using a 3×3 determinant. Most texts refer to this as the *Jacobian* of the

transformation $\mathbf{r} : B \rightarrow D$.) Recall that $f = \nabla \cdot \mathbf{G}$ has a vector potential \mathbf{G} if and only if $\nu = d\omega$ is a differential of a 2-form ω . In this case, the volume integral simplifies to

$$\iiint_D (\nabla \cdot \mathbf{G}) dV = \iiint_D d\omega = \iint_{\partial D} \omega = \iint_S \mathbf{G} \cdot d\mathbf{A}.$$

Hence the integral is independent of the region V , and it is only dependent on the boundary $S = \partial D$. This is known as *Gauss' Theorem*, or the *Divergence Theorem*.

Example. Let D denote the solid sphere of radius r in three-dimensional space. This is just the set of points $x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ such that $x^2 + y^2 + z^2 \leq r^2$. We may think of this as being the image of the map $\mathbf{r} : [0, r] \times [0, \pi] \times [0, 2\pi] \rightarrow \mathbb{R}^3$ which sends

$$(\rho, \phi, \theta) \mapsto \rho \sin \phi \cos \theta \mathbf{i} + \rho \sin \phi \sin \theta \mathbf{j} + \rho \cos \phi \mathbf{k}.$$

Note that the boundary $S = \partial D$ is simply the sphere $x^2 + y^2 + z^2 = r^2$.

Let's return to the vector field

$$\mathbf{G} = \nabla g = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$$

defined in terms of

$$g(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

(The "Prickly Pear" again!) We will compute integrals and verify the Divergence Theorem. We may express the volume element $dV = dx dy dz$ in *spherical coordinates* using the determinant

$$\begin{aligned} dV &= \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix} d\rho d\phi d\theta \\ &= \begin{vmatrix} \cos \theta \sin \phi & \sin \theta \sin \phi & \cos \phi \\ \rho \cos \theta \cos \phi & \rho \sin \theta \cos \phi & -\rho \sin \phi \\ -\rho \sin \theta \sin \phi & \rho \cos \theta \sin \phi & 0 \end{vmatrix} d\rho d\phi d\theta \\ &= \rho^2 \sin \phi d\rho d\phi d\theta. \end{aligned}$$

Hence we have the differential 3-form

$$\nu = \nabla \cdot \mathbf{G} (\mathbf{r}(\rho, \phi, \theta)) dV = \frac{2}{\rho} \cdot (\rho^2 \sin \phi d\rho d\phi d\theta) = 2 \rho \sin \phi d\rho d\phi d\theta.$$

(Recall that $\nabla \cdot \mathbf{G} = 2/g$.) This gives the integral

$$\begin{aligned} \iiint_D \nu &= \int_0^r \int_0^\pi \int_0^{2\pi} 2\rho \sin \phi \, d\rho \, d\phi \, d\theta \\ &= \left[\int_0^r 2\rho \, d\rho \right] \left[\int_0^\pi \sin \phi \, d\phi \right] \left[\int_0^{2\pi} d\theta \right] \\ &= 4\pi r^2. \end{aligned}$$

On the other hand, we may express the area element as the determinant

$$\begin{aligned} d\mathbf{A} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix} d\phi \, d\theta \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \rho \cos \theta \cos \phi & \rho \sin \theta \cos \phi & -\rho \sin \phi \\ -\rho \sin \theta \sin \phi & \rho \cos \theta \sin \phi & 0 \end{vmatrix} d\phi \, d\theta \\ &= \rho \sin \phi (\rho \sin \phi \cos \theta \mathbf{i} + \rho \sin \phi \sin \theta \mathbf{j} + \rho \cos \phi \mathbf{k}) \, d\phi \, d\theta. \end{aligned}$$

Hence we have the differential 2-form

$$\omega = \mathbf{G}(\mathbf{r}(\rho, \phi, \theta)) \cdot d\mathbf{A} = \frac{\mathbf{r}}{\rho} \cdot (\rho \sin \phi \mathbf{r} \, d\theta \, d\phi) = \rho^2 \sin \phi \, d\phi \, d\theta.$$

This gives the integral

$$\iint_{\partial D} \omega = \int_0^\pi \int_0^{2\pi} r^2 \sin \phi \, d\theta \, d\phi = r^2 \left[\int_0^\pi \sin \phi \, d\phi \right] \left[\int_0^{2\pi} d\theta \right] = 4\pi r^2.$$

(Remember that $\rho = r$ along the boundary $S = \partial D$.) This indeed verifies that

$$\iiint_D (\nabla \cdot \mathbf{G}) \, dV = \iint_S \mathbf{G} \cdot d\mathbf{A}.$$

Arfken and Weber [1] provide a plethora of formulas for integrating in different coordinate systems other than spherical.

5. The Moral of the Story

In this paper we have shown that all of the theorems – those for differentiation and integration – can be expressed using differential forms. Here’s the idea in a nutshell for *any* region D . We consider a series of linear vector spaces $\Omega^k(D)$, where we have “differential” maps

$$\dots \longrightarrow \Omega^{k-1}(D) \xrightarrow{d} \Omega^k(D) \xrightarrow{d} \Omega^{k+1}(D) \longrightarrow \dots$$

We would like to know the answer to the following question involving differentiation:

Which k – forms $\omega \in \Omega^{k+1}(D)$ are in the form $\omega = d\eta$ for some $(k - 1)$ – form $\eta \in \Omega^k(D)$?

The *partial* answer should be:

*If $\omega = d\eta$ for some $(k - 1)$ – form η ,
then $d\omega = 0$ as a $(k + 1)$ – form.*

A complete answer involves computing something called *de Rham Cohomology*. (The dessert after the Main Course?) For instance, we have computed de Rham Cohomology in this article for certain subsets of three-dimensional space. We expect to generalize the integration formulas above by saying something like

$$\int_D \omega = \int_D d\eta = \int_{\partial D} \eta$$

so that the integral would be independent of the region D , and would be only dependent on the boundary ∂D . This is the *Generalized Stokes’ Theorem*. Surprisingly, this entire theory can be worked out for “many” sets D . But don’t take our words for it – just a friendly Differential Geometer!

References

- [1] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists*, Harcourt/Academic Press, Burlington, MA, 5th ed., 2001.
- [2] W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, volume 120 of *Pure and Applied Mathematics*, Academic Press Inc., Orlando, FL, 2nd ed., 1986.
- [3] D. Hughes-Hallett, A. M. Gleason, W. G. McCallum, D. E. Flath, P. F. Lock, T. W. Tucker, D. O. Lomen, D. Lovelock, D. Mumford, B. G. Osgood, D. Quinney, K. Rhea, and J. Tecosky-Feldman, *Calculus: Single and Multivariable*, Wiley-Interscience [John Wiley & Sons], 4th ed., Dec. 2004.
- [4] R. Larson, R. P. Hostetler, and B. H. Edwards, *Calculus: Early Transcendental Functions*, Houghton Mifflin, 4th ed., 2007.
- [5] J. E. Marsden and A. J. Tromba, *Vector Calculus*, W. H. Freeman, 5th ed., 2004.
- [6] J. Stewart, *Calculus: Early Transcendentals*, volume 6E of *Calculus*, Brooks Cole, 2008.

The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before August 1, 2010. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall, 2010 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051)

NEW PROBLEMS 649-658

Problem 649. *Proposed by Tuan Le, Fairmont High School, Anaheim, CA.*

Suppose x, y, z are positive real numbers such that $xyz \geq 10 + 6\sqrt{3}$. Prove that

$$\frac{y}{x + y^3 + z^2} + \frac{z}{x^2 + y + z^3} + \frac{x}{x^3 + y^2 + z} \leq \frac{1}{2}.$$

Problem 650. *Proposed by Tuan Le, Fairmont High School, Anaheim, CA.*

Suppose a, b, c are positive real numbers. Prove that

$$\frac{a^2b + b^2c + c^2a}{3(a^3 + b^3 + c^3)} + \frac{a^2 + b^2 + c^2}{ab + bc + ca} \geq \frac{4}{3}.$$

Problem 651. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Find all triplets (x, y, z) of real numbers for which

$$4x - y^2 - 2\sqrt{4y - z^2 - 2\sqrt{4z - x^2 - 2\sqrt{41x + 43y + 44z}}}$$

is a positive integer, and determine the values.

Problem 652. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Let a, b, c be the lengths of the sides of triangle ABC . Prove that

$$\left(\frac{\sqrt{bc}}{a}\right) \sin A + \left(\frac{\sqrt{ca}}{b}\right) \sin B + \left(\frac{\sqrt{ab}}{c}\right) \sin C \leq \frac{3\sqrt{3}}{2}.$$

Problem 653. Proposed by Mohammad K. Azarian, University of Evansville, Evansville, IN.

Find the general solution of the recurrence relation

$$\sum_{k=0}^{2010} (2010)^{2011-k} x_{n-k} = 0, n \geq 2010.$$

Problem 654. Proposed by Mohammad K. Azarian, University of Evansville, Evansville, IN.

Find all real roots of the exponential equation

$$e^{9x} - 2e^{8x} + e^{7x} - 29e^{6x} + 87e^{5x} - 87e^{4x} + 29e^{3x} - e^{2x} + 2e^x - 1 = 0.$$

Problem 655. Proposed by Ovidiu Furdui, Campia Turah, Cluj, Romania.

Let $f(n)$ be the function defined by $f(n) = \frac{\ln 2}{2^{k-1}}$ if $2^{k-1} \leq n < 2^k$. Prove that

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - f(n)\right) = \gamma,$$

where γ denotes the Euler-Mascheroni constant.

Problem 656. *Proposed by Ovidiu Furdui, Campia Turah, Cluj, Romania.*

Let $k \geq 2$ be a natural number. Find the sum

$$\sum_{n_1, \dots, n_k \geq 1} (-1)^{n_1 + \dots + n_k} (\zeta(n_1 + \dots + n_k) - 1),$$

where ζ denotes the Riemann zeta function.

Problem 657. *Proposed by Panagiote Ligouras, Leonardo Da Vinci High School, Noci, Italy.*

Let a, b, c be positive real numbers. Prove that

$$\frac{5c^2 + 11ab}{(a+b)^2} + \frac{5a^2 + 11bc}{(b+c)^2} + \frac{5b^2 + 11ca}{(c+a)^2} \geq 12.$$

Problem 658. *Proposed by Panagiote Ligouras, Leonardo Da Vinci High School, Noci, Italy.*

Let a, b, c be the sides, m_a, m_b, m_c the medians, h_a, h_b, h_c the heights, l_a, l_b, l_c the angle bisectors and R the circumradius (radius of the circle inside which the triangle can be inscribed) of triangle ABC . Prove that

$$\frac{l_a^2}{h_a} \sqrt{\frac{m_a^2 - h_a l_a}{l_a^2 - h_a^2}} + \frac{l_b^2}{h_b} \sqrt{\frac{m_b^2 - h_b l_b}{l_b^2 - h_b^2}} + \frac{l_c^2}{h_c} \sqrt{\frac{m_c^2 - h_c l_c}{l_c^2 - h_c^2}} \leq 6R.$$

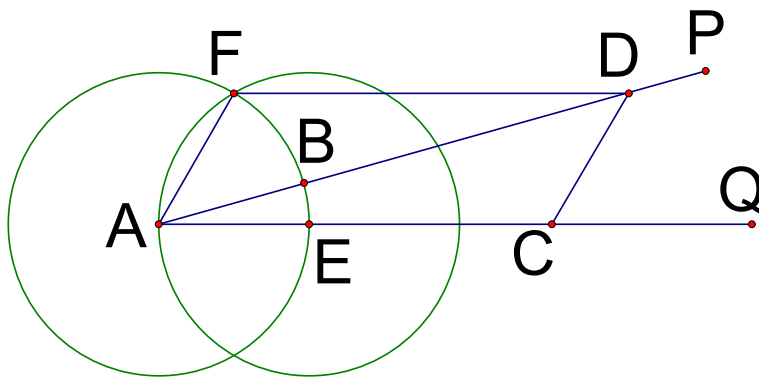
SOLUTIONS 632-640

Problem 632. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

Let two rays meet at point A , and let P be a point on one ray and Q a point on the other ray. Let B be a point between A and P . Suppose the angle measure of $\angle PAQ$ is less than 60° . Show how to construct, with only compass and straightedge, points D on AP and C on AQ such that $CD = AB$ and DC makes an angle of 60° with AQ .

Solution *by the proposers.*

Draw a circle with center A , radius AB to determine a point E on AQ with $AB = AE$. Draw a circle with center E , radius AB to determine a point F such that AFE is an equilateral triangle (with F on the same side of AQ as B). Draw FD parallel to AQ with D on AP and DC parallel to AF with C on AQ . Since $AFDC$ is a parallelogram, it is straightforward to show that $CD = AB$ and that CD makes an angle of 60° with AQ . The following diagram shows the construction.



Also solved by the Dead Poet Society at Berry College, Mount Berry, GA.

Problem 633. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, IL.*

The integers beginning with 2008 and without spaces between them are written down:

200820092010201120122013...

Then commas are placed to form an infinite sequence of 5-digit arrangements:

20082, 00920, 10201, 12012, 20132, ...

Prove or disprove: Every 5-digit arrangement appears infinitely many times in this sequence.

Solution *by the proposers.*

Let $xyztu$ be a 5-digit arrangement. Since all of the numbers are written down, eventually the original list of numbers will contain

$xyztuAxyztuBxyztuCxyztuDxyztu,$

where $A, B, C,$ and D are some arbitrary integers. In fact, the list will contain an infinite number of such 29-digit arrangements. When the commas are placed, the 29-digit arrangement will be partitioned into five 5-digit arrangements, plus some digits left over. By considering the various possibilities for the location of the comma in the first 5-digit arrangement, $xyztu,$ we see that the list contains one of the following

$,xyztu, Axyzt, uBxyz, tuCxy, ztuDx, yztu$
 $x, yztuA, xyztu, Bxyzt, uCxyz, tuDxy, ztu$
 $xy, ztuAx, yztuB, xyztu, Cxyzt, uDxyz, tu$
 $xyz, tuAxy, ztuBx, yztuC, xyztu, Dxyzt, u$
 $xyzt, uAxyz, tuBxy, ztuCx, yztuD, xyztu$

In particular, the list contains the 5-digit sequence $xyztu$ an infinite number of times.

Problem 634. *Proposed by Ovidiu Furdui, University of Toledo, Toledo, OH.*

Find the limit

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{n}{\frac{1}{2} + \frac{2}{3} + \cdots + \frac{n}{n+1}} \right)^n.$$

Solution by the proposer.

The limit equals $e^{\gamma-1}$. We have

$$\begin{aligned} \frac{1}{2} + \frac{2}{3} + \cdots + \frac{n}{n+1} &= n - \left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n+1} \right) \\ &= n + 1 - \gamma_{n+1} - \ln(n+1), \end{aligned}$$

where

$$\gamma_{n+1} = 1 + \frac{1}{2} + \cdots + \frac{1}{n+1} - \ln(n+1).$$

The original terms above become

$$x_n = \frac{1}{n} \left(\frac{1}{1 - \frac{\gamma_{n+1} + \ln(n+1) - 1}{n}} \right)^n = \frac{1}{n} \left(\frac{1}{1 - a_n} \right)^n,$$

where

$$a_n = \frac{\gamma_{n+1} + \ln(n+1) - 1}{n}.$$

We note that

$$\begin{aligned} \ln x_n &= -n \ln(1 - a_n) - \ln n \\ &= n \left(a_n + \frac{a_n^2}{2} + \cdots \right) - \ln n \\ &= n \left(a_n + O \left(\left(\frac{\ln n}{n} \right)^2 \right) \right) - \ln n \\ &= na_n - \ln n + n \cdot O \left(\left(\frac{\ln n}{n} \right)^2 \right) \\ &= \gamma_{n+1} - 1 + \ln \left(\frac{n+1}{n} \right) + n \cdot O \left(\left(\frac{\ln n}{n} \right)^2 \right). \end{aligned}$$

Letting $n \rightarrow \infty$, we get the desired result since $n \cdot O \left(\left(\frac{\ln n}{n} \right)^2 \right) \rightarrow 0$.

Problem 635. Proposed by Ovidiu Furdui, University of Toledo, Toledo, OH.

Let $k \geq 2$ be a natural number. Find the sum

$$\sum_{n_1, \dots, n_k \geq 1} (\zeta(n_1 + \cdots + n_k) - 1),$$

where ζ denotes the Riemann zeta function.

Solution by the proposer.

The series equals $\zeta(k)$. We have

$$\begin{aligned} \sum_{n_1, \dots, n_k \geq 1} (\zeta(n_1 + \dots + n_k) - 1) &= \sum_{n_1, \dots, n_k \geq 1} \left(\sum_{p=2}^{\infty} \frac{1}{p^{n_1 + \dots + n_k}} \right) \\ &= \sum_{p=2}^{\infty} \left(\sum_{n_1=1}^{\infty} \frac{1}{p^{n_1}} \right) \cdots \left(\sum_{n_k=1}^{\infty} \frac{1}{p^{n_k}} \right). \end{aligned}$$

Since these sums are geometric series with sum $\frac{1}{p-1}$, we get that

$$\sum_{n_1, \dots, n_k \geq 1} (\zeta(n_1 + \dots + n_k) - 1) = \sum_{p=2}^{\infty} \frac{1}{(p-1)^k} = \zeta(k).$$

Problem 636. Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, MO.

Let p be a fixed prime. Find the dimensions of all rectangles with integral side lengths and whose areas are numerically equal to p times their semiperimeters.

Solution by the proposers.

Let x and y be the dimensions of the rectangle. One needs to solve the Diophantine equation $xy = p(x + y)$. Notice that $x \neq p$, for if $x = p$, then $py = p(x + y)$, so $x = 0$. Solving the equation for y gives $y = p + \frac{p^2}{x-p}$. Since y must be a positive integer, $x - p$ must be a divisor of p^2 . Since p is a prime, $x - p \in \{-p^2, -p, -1, 1, p, p^2\}$. Since x must be a positive integer, $x \in \{p-1, p+1, 2p, p+p^2\}$. Thus $y = p - p^2, p^2 + p, 2p$, or $p + 1$, respectively. Since the first of the y -values will not be positive, the possible pairs (x, y) are $(p+1, p^2 + p)$, $(2p, 2p)$, and $(p^2 + p, p+1)$.

Problem 637. Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.

Let $x, y, z \in [1, \infty)$. Prove that

$$\frac{x}{x^2 + yz} + \frac{y}{y^2 + xz} + \frac{z}{z^2 + xy} \leq \frac{3}{2}.$$

Solution by the Dead Poet Society at Berry College, Mount Berry, GA.

Pick $x, y, z \in [1, \infty)$. Then we have $\frac{x}{x^2+yz} \leq \frac{1}{2}$ if and only if $\frac{x^2+yz}{x} \geq$

2. Notice that since $y, z \geq 1$ we have $\frac{x^2+yz}{x} \geq \frac{x^2+1}{x}$. Now notice that

$$\frac{x^2+1}{x} \geq 2 \Leftrightarrow x^2+1 \geq 2x \Leftrightarrow x^2-2x+1 \geq 0 \Leftrightarrow (x-1)^2 \geq 0,$$

which of course is true. Hence, by transitivity, we have that $\frac{x}{x^2+yz} \leq \frac{1}{2}$. But the three terms on the left-hand side of the desired inequality are symmetric in $x, y,$ and z . Therefore each term is less than or equal to $\frac{1}{2}$. Therefore,

$$\frac{x}{x^2+yz} + \frac{y}{y^2+xz} + \frac{z}{z^2+xy} \leq \frac{3}{2}.$$

Also solved by the proposer and the Oklahoma Alpha Chapter, Northeastern State University, Tahlequah, OK.

Problem 638. Proposed by Jose Luis Diaz-Barrero, Universitat Politcnica de Catalunya, Barcelona, Spain.

Let a be a positive integer. Find the least common multiple of the number $A = a^n(a+1)^{n+1} + a$ and $B = a^{n+1}(a+1)^n + a - 1$, where n is any natural number.

Solution by the proposer.

Let us denote the GCD and LCM of A and B by (A, B) and $[A, B]$, respectively. If $d = (A, B)$, then

$$d \mid a^n(a+1)^{n+1} + a \text{ and } d \mid a^{n+1}(a+1)^n + a - 1.$$

Hence, we have that

$$d \mid a \left[a^n(a+1)^{n+1} + a \right] \text{ and } d \mid (a+1) \left[a^{n+1}(a+1)^n + a - 1 \right],$$

or equivalently

$$d \mid a^{n+1}(a+1)^{n+1} + a^2 \text{ and } d \mid a^{n+1}(a+1)^{n+1} + a - 1.$$

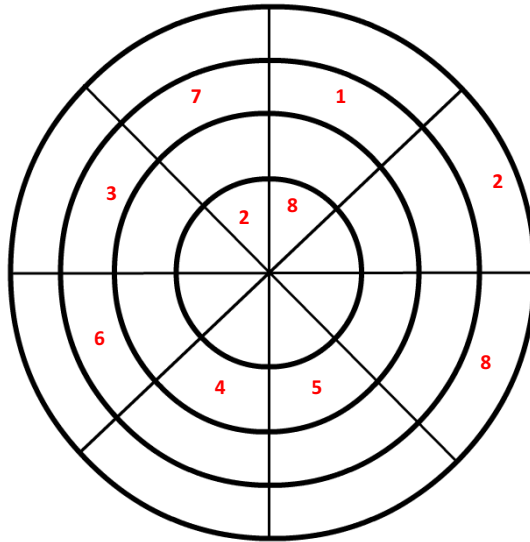
Thus d divides their difference. That is, $d \mid 1$. Therefore, $(A, B) = 1$, and taking into account that $(A, B)[A, B] = AB$, as is well known, we get

$$\begin{aligned} [A, B] &= \left[a^n(a+1)^{n+1} + a \right] \left[a^{n+1}(a+1)^n + a - 1 \right] \\ &= a^{2n+1}(a+1)^{2n+1} + a^n(a+1)^n(2a^2-1) + a(a-1), \end{aligned}$$

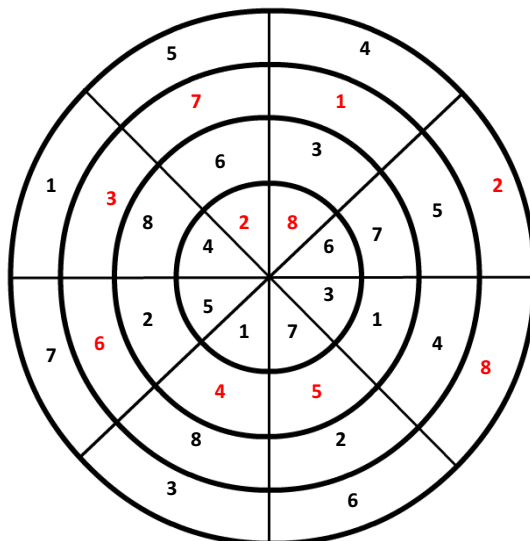
and we are done.

Problem 639. *Proposed by Peter M. Higgins and Caroline Higgins (authors of the book Circular Sudoku), Essex University, England.*

The following is a Circular Sudoku puzzle. Each of the numbers 1-8 must appear once in every ring and once in every pair of touching slices. Fill in the missing values of the puzzle.



Solution *by the proposers.*



Also solved by the Dead Poet Society at Berry College, Mount Berry,

GA; Sister Marcella Wallowicz, Holy Family University, Philadelphia, PA; Oklahoma Alpha Chapter, Northeastern State University, Tahlequah, OK; Joshua Liberman and Craig Edwards, Central Missouri State University, Warrensburg, MO; Timothy Groh, John Gruenewald, Hunter Hayes, Paige Hebard, Michael Henry, Lucas Hoots, Lisa Notier, Morgan Smith, and Brian Sneed, Centre College, Danville, KY; Eddie Bailey, Amanda Bowling, Donald Brewer, Jeremy Britt, Matt Columbus, Jenny Fischesser, Sarah Hale, Hunter, Nuckols, and Joe Snyder, Eastern Kentucky University, Richmond, KY.

Problem 640. *Proposed by the editor.*

The sequence 19, 199, 1999, ... starts off with three primes; most of the numbers in the sequence, however, are composites, and there are lots of divisors of the numbers in the sequence. Prove the following:

1. The prime 19 divides infinitely many of the numbers in the sequence.
2. The composite number 551 divides infinitely many of the numbers in the sequence.
3. The composite number 323 does not divide any of the numbers in the sequence.

Solution *by the proposer.*

The numbers in the sequence are of the form $2 \cdot 10^n - 1$, where the exponent n creates the value having $n + 1$ digits.

1. All of the numbers in the sequence having $18m + 2$ digits will be divisible by 19. To see this, note that

$$10^{18m+1} = 10 (10^{18})^m \equiv 10 \cdot 1^m \pmod{19}$$

by Fermat's Little Theorem. Thus $10^{18m+1} \equiv 10 \pmod{19}$ for all positive integers m . This means that $2 \cdot 10^{18m+1} - 1 \equiv 0 \pmod{19}$ or 19 divides all values in the sequence having $18m + 2$ digits. Note further that 1 (mod 18) is the only exponent on 10 in the last congruence that gives 0.

2. All of the numbers in the sequence having $252m + 74$ digits will be divisible by 551. To see this, we start by observing that

$$\begin{aligned} 10^{28m+17} &= 10^{17} (10^{28})^m \equiv 15 \cdot (10^{28})^m \pmod{29} \\ &\equiv 15 \cdot 1^m \pmod{29} \end{aligned}$$

by Fermat's Little Theorem. Thus 29 divides all values in the sequence having $28m + 18$ digits. Since $551 = 19 \cdot 29$, we need to see whether

any of the values from part 1 will have $28n + 18$ digits for some n . This will happen if the system of linear congruences $m \equiv 1 \pmod{18}$ and $m \equiv 17 \pmod{28}$ has any solutions. This system has solution $m \equiv 73 \pmod{252}$. Therefore 551 divides all values in the sequence having $252m + 74$ digits.

3. Since $323 = 17 \cdot 19$, we would need a subsequence of the values in part 1 that are divisible by 17 in order for the values to be divisible by 323. When we consider powers of 10 $\pmod{17}$, we see that

$$10^{16m+6} = 10^6 (10^{16})^m \equiv 10^6 \cdot 1^m \pmod{17}$$

by Fermat's Little Theorem. Thus $2 \cdot 10^{16m+6} - 1 \equiv 0 \pmod{17}$ or 17 divides all values in the sequence having $16m + 7$ digits. Moreover, these are the only values in the sequence that are divisible by 17. Since these values have an odd number of digits and the values divisible by 19 have an even number of digits, there are no values divisible by both 17 and 19.

Report of the 37th Biennial Convention

Kappa Mu Epsilon
March 26-28, 2009
Doubletree Hotel
Philadelphia, PA

Thursday, March 26, 2009

On Thursday evening, March 26, a mixer was held at the Lucky Strike Bowling Alley near the hotel. Participants were able to pick up registration packets at this mixer. At 8:00 p.m., the National Council, the Regional Directors and the Pentagon Editor and Business Manager held their biennial business meeting in the Chamber Board Room at the Doubletree Hotel.

Friday, March 27, 2009

First General Session and Business Meeting

Friday's activities began with breakfast and continued registration in the lobby outside the Ormandy East Ballroom at the Doubletree Hotel. At 8:30 a.m. in the Ormandy East Ballroom, the first general session began, with KME President Don Tosh presiding. After President Tosh welcomed the participants, Rhonda McKee, KME Secretary called the roll. Twenty-one faculty members, 46 students and 2 visitors were in attendance, for a total of 69 participants from 16 chapters. The following new chapters, installed during the 2007-2009 biennium were recognized: Oklahoma Epsilon at Oklahoma Christian University, Hawaii Alpha at Hawaii Pacific University and North Carolina Epsilon at North Carolina Wesleyan College.

Chip Curtis of Missouri Iota, chair of the Nominating Committee, reported for the committee. Rhonda McKee of Missouri Beta and Vincent Dimiceli of Oklahoma Delta were nominated for the office of National President Elect. Peter Skoner of Pennsylvania Mu, Mark Hamner of Texas Gamma and Christopher Barat of Maryland Epsilon were nominated for the office of National Historian. The nominees were introduced to the delegates, and additional nominations were requested from the floor. There being none, nominations were closed. Ballots, to be cast during the business meeting on Saturday morning, were distributed to the delegates from each chapter.

First Paper Session

Ron Wasserstein, KME President Elect presided over the paper presentations. The following papers were presented during the morning session:

- "Prime Number Formulas", by Ashley Moore, Texas Mu, Schreiner University
- "The Mathematics in Music", by Jenna Haines, Gina Palino and Stephanie Salvator, New York Omicron, St. Joseph's College
- "The Self-Duality of Some Complete Bipartite Graphs", by Andrea Chaney, Georgia Alpha, University of West Georgia
- "Tortoise and the Hare", by Brad Baker, Texas Mu, Schreiner University
- "A Neuron Model", by Katie Schniebs and Rae Lynn McFarlin, Texas Gamma, Texas Woman's University
- "The Peg Game", by Christine Potter, Kansas Delta, Washburn University
- "The Lady Philosopher: Hypatia of Alexandria", by Holden Kraus, Kansas Alpha, Pittsburg State University
- "Global Warming: A Statistical Analysis of 'An Inconvenient Truth'", by Kristine Vaccaro and Louis Petersen, New York Omicron, St. Joseph's College

At 11:45 a.m., a group photograph was taken on the roof of the Hotel. The photo session was followed by lunch in the Ormandy West Ballroom. The Auditing Committee and the Resolutions Committee met during the lunch period.

Second Paper Session

The paper presentations resumed at 1:30 p.m. The following papers were presented in the afternoon session:

- "Dancing with the Crops: Using Mathematic Modeling to Maximize Earnings for a Family Farm", by Sarah Butler, Kansas Delta, Washburn University
- "Modeling of Folding in a-Conotoxins from Molecular Dynamics and Ab Initio Methods", by Aaron Osysko, Pennsylvania Mu, St. Francis University
- "Phi Patterns in Nature and Beyond", by Leigh Johnson, Theresa Samp-

son and Heather O'Connor, New York Omicron, St. Joseph's College

- "Palindrome Probabilities", by Anthony Fraticelli, Missouri Theta, Evangel University

After the paper session, refreshments were served and the group viewed the video *Flatland: The Movie*. Don Tosh led a post-movie discussion. At 3:45 p.m., the student section met in the East Ballroom, while the faculty section met in the Orchestra Room. The afternoon activities concluded at 4:30 p.m.

Banquet

The convention banquet was held in the Ormandy West Ballroom at 7:00 p.m. Ron Wasserstein, KME President Elect served as emcee. Following dinner, Melanie Matchett Wood of Princeton University gave the keynote address. Her topic was *The Chemistry of Primes*.

Saturday, March 28, 2009

Saturday's activities began with breakfast and the remaining paper presentations.

Third Paper Session

The following papers were presented on Saturday morning:

- "Sans Hypotenuses or 3.04×10^{62} ", by George Bernius, Jennifer Vosilla and Dallas Jones, New York Omicron, St. Joseph's College
- "Beyond the Fibonacci Sequence", by Whitney Turley, Kansas Beta, Emporia State University
- "A Brief Glimpse into the History of Fractals", by Kaitlyn Snyder, Pennsylvania Mu, St. Francis University
- "Best Phone", by Tim McVey, Kansas Alpha, Pittsburg State University

Second General Session and Business Meeting

At 9:50 a.m. the convention broke for refreshments and the Awards Committee met in the Chamber Board Room. After the break, the second general session began with President Don Tosh presiding. The following national officers made reports:

- Charles Curtis, Editor, *The Pentagon*
- Connie Schrock, Historian

- Cynthia Woodburn, Treasurer
- Rhonda McKee, Secretary
- Ron Wasserstein, President-Elect
- Don Tosh, President

Following the national officer reports were reports from the section meetings, the Auditing Committee and the Resolutions Committee. All reports are attached.

Ballots were collected from the delegates from each chapter. Rhonda McKee was elected President Elect and Peter Skoner was elected Historian. These new officers were installed by President Don Tosh. In addition, Ron Wasserstein was installed as President. Ron then presented plaques to out-going president Don Tosh and out-going historian, Connie Schrock, in appreciation for their service to Kappa Mu Epsilon.

National Treasurer Cynthia Woodburn presented checks for travel allowances to each chapter present. The final activity of the convention was the report of the Awards Committee and the presentation of awards. The presenters of the top four papers were: (in alphabetical order)

- Andrea Chaney - GA Alpha
- Anthony Fraticelli - MO Theta
- Leigh Johnson - NY Omicron
- Christine Potter - KS Delta

These four students were awarded \$100 checks. A \$50 award was given for each of the other presentations.

Rhonda McKee
National Secretary

Report of the National President

I was installed as president at the national convention in Kerrville, TX, in April, 2005. In the last four years I have attempted to represent Kappa Mu Epsilon, and improve the level of service we provide to our members.

The main responsibility of the president is to be the contact person for schools wishing to form new chapters. Requests for information about forming new chapters come at various times, and the president has to be ready to provide information and advice in response to these requests. Our website is helping in this regard. In fact, during this biennium I have been

contacted by several schools which made the initial contact based on information gathered from our website. There is every indication that this will become the prevalent contact methodology in the future.

The three chapters that have been installed during the past biennium are:

- Oklahoma Epsilon at Oklahoma Christian University on Friday, April 20, 2007. I was the installing officer.
- Hawaii Alpha at Hawaii Pacific University in Honolulu on October 22, 2007. I was the installing officer.
- North Carolina Epsilon at North Carolina Wesleyan College in Rocky Mount on March 24, 2008. Ron Wasserstein was the installing officer.

Additionally, next week I will be traveling to Redding, California, to install California Zeta at Simpson University on April 4, 2009. Two weeks later, New York Rho will be installed at Molloy College on April 21 in New York City by Andrew Rockett, a previous Pentagon editor and long-time corresponding secretary.

In February, 2008, I attended the Association of Collegiate Honor Societies meeting in St. Louis, MO. KME has been a member of ACHS for many years and the president represents KME at this on alternate years. At this meeting I mainly focused on the new federal regulations regarding non-profit corporations and the resolution we had to pass in the national council meeting to bring our organization into compliance.

I have maintained the position of business manager of *The Pentagon* for the past two and a half years. After considerable time and effort, the database is now current and I am working on trimming out unnecessary addresses. The Fall 2008 issue was sent out on schedule and I anticipate being able to maintain that pattern. I am willing to continue in this role after I step down as president. I have enjoyed working with Charles Curtis, *The Pentagon* editor, who is doing an excellent job. I am thankful for all the time and effort he puts into the publication. However, if anyone is interested in replacing me as the business manager I'm sure the national council would be happy to entertain your offer. I would be happy to entertain your offer.

During the past biennium we have established and maintained contact with a sister organization, Pi Mu Epsilon. I am happy to welcome Eve Torrence to our national convention. Eve is the president of Pi Mu Epsilon and we are hoping to continue developing the relationship and interaction with her society. We have invited PME officers to attend our council meetings, and some of our officers have attended theirs as well. We believe that

through cooperation both organizations will benefit.

The most gratifying aspect of being involved with KME is developing relationships with the quality people in this organization. National council meetings are invariably pleasant and much is accomplished because everyone has a common goal – the furtherance of KME in particular and mathematics in general. I was very pleased to see the level of undergraduate research that has been demonstrated at this meeting. I believe that Kappa Mu Epsilon has a bright future because of the people at the core of the organization. I am grateful for the help I have received in my role as president and I hope that the level of support will continue for the new officers. I especially want to thank the incoming president, Ron Wasserstein, for the amount of work he did to facilitate this meeting. He helped make my workload considerably lighter. I have enjoyed working with Connie Schrock and would like to personally thank her for her contribution as historian over the past eight years. One of the perks of staying on as business manager is that I will be able to maintain many of the relationships that I have formed over the past twelve years on the national council.

Don Tosh
National President

Report of the National President-Elect

This is my final report as KME President-Elect. I find it difficult to believe that four years have already passed since I was elected at the 2005 Convention at Schreiner University in Kerrville, Texas. It has been an extraordinary privilege and honor to serve in this capacity, and the chance to work with dedicated and talented volunteers with deep commitment to this organization and to the success of mathematics students has been inspirational. I take this opportunity, then, to express my thanks to all those who are so committed to Kappa Mu Epsilon, but particularly to the National Council members with whom I've been fortunate to work: President Don Tosh, Secretary Rhonda McKee, Treasurer Cynthia Woodburn, Past Treasurer John Kubicek, Historian Connie Schrock, and Webmaster Kevin Reed.

The primary focus of this report is on conventions, for the benefit of future presidents-elect, since that has been my primary responsibility. It has been a lot of work, but it has also been a delight. During my term as President-Elect, I have been able to attend two national and two regional conventions, and by listening to students talk about their research, I've been reminded why these activities are so important.

2009 Convention

Kappa Mu Epsilon's 37th Biennial Convention began this evening, March 26, 2009 with a mixer at Lucky Strike Lanes in downtown Philadelphia, PA, and continues through Saturday, March 28. Our host is the Doubletree Hotel in Philadelphia. There are 16 chapters in attendance from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas). Sixteen students will present papers over the course of the next two days. Seventy-five people are registered for the convention.

By way of comparison, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in 2007 in Springfield, Missouri.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). There were 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10 states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee)

The following chapters have participated in the last five conventions:

- California Epsilon (2005)
- Colorado Delta (2001)
- Georgia Alpha (2009)
- Iowa Alpha (2001, 2003)
- Iowa Gamma (2001)
- Kansas Alpha (2001, 2005, 2007, 2009)
- Kansas Beta (2001, 2003, 2005, 2007, 2009)
- Kansas Gamma (2001, 2003, 2007)
- Kansas Delta (2001, 2003, 2005, 2007, 2009)
- Kansas Epsilon (2001)
- Kentucky Alpha (2001)
- Maryland Beta (2009)
- Maryland Epsilon (2009)
- Michigan Beta (2003, 2005, 2009)
- Missouri Alpha (2001, 2003, 2007)
- Missouri Beta (2001, 2003, 2005, 2007, 2009)
- Missouri Iota (2001, 2003, 2005, 2007, 2009)

Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Theta (2001, 2003, 2005, 2007, 2009)
New York Eta (2001, 2003, 2005, 2007)
New York Lambda (2003, 2005)
New York Omicron (2005, 2009)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009)
Pennsylvania Lambda (2003, 2009)
Pennsylvania Mu (2005)
Pennsylvania Theta (2001)
Tennessee Gamma (2001, 2003, 2005, 2007)
Texas Gamma (2003, 2005, 2009)
Texas Mu (2003, 2005, 2009)

Thus, in the 2000's, 32 different chapters have participated. Six have participated in all five conventions. Four have not participated since 2001. A chapter in Georgia and two in Maryland participated for the first time in recent years at this 2009 Convention. However, five chapters that had participated in at least three other conventions in this decade did not come this year.

A significant change for the 2009 convention is that it was not hosted by a school, but arranged by the President-Elect. As you recall from my previous report, this action was taken for several reasons: (1) No host invitations were received. (2) Attendance at the convention has fallen off. (3) The Council perceived a need to attract to the convention more schools from outside Region Four. (4) Hosting the convention at a "destination" might attract more chapters to participate.

In hindsight, it might have been wise for the President-Elect to have reached out to one or more of the Philadelphia KME schools and asked for their assistance. As it is, neither of the two schools in the metropolitan area who indicated they might attend actually ended up sending students. (More hindsight later in the report.)

A second change was in the process for submitting papers. Students are only responsible for having submitted by the deadline an abstract and a certifying letter from a faculty advisor. A full written paper is no longer required. This change was based on observation of the Pi Mu Epsilon conventions, and was made to facilitate more student paper submissions. This change seems to have helped.

We also reached out to national mathematics and statistics associations

for support for the conference. We received \$500 each from the AMS and the ASA. The MAA offered in-kind help if we needed it, and I did not get a response from SIAM. Economic difficulties are causing strains on the abilities of these associations to support small conferences.

We are grateful that Melanie Matchett Wood accepted our invitation to present the keynote presentation at the banquet on Friday, March 27. Melanie is completing her Ph.D. at Princeton University, and will soon be a post-doc at Stanford. She was the first female to represent the US in the International Mathematics Olympiad, twice competing and twice winning a silver medal. Melanie is already one of the rising stars in mathematics. She won the Morgan Prize for Outstanding Research by An Undergraduate. I heard her make an outstanding keynote presentation this past summer at the US Math Olympiad Awards Ceremony at the US Dept of State in Washington, DC. Melanie will be interesting and inspiring to our students and faculty.

For future conferences, a few lessons learned (from my perspective, anyway):

- Having the conference in a desirable location is important.
- Too many consecutive conferences in the Midwest has set us back a bit, in my view. Moving the conference around is key to participation, as some chapters seem to be unable to travel great distances.
- While we couldn't have predicted a recession when we planned the conference originally, I still should have been more conservative in predicting the number of students who would participate. Overestimating the number (by a lot) has made this convention much more expensive than it needed to be.
- We can have future conventions at hotels, but
 - We'll have to estimate more conservatively.
 - We need to have better data. (Because the conventions were organized by local chapters, there wasn't a ready source of data on past attendance. We should include total headcount in P-E reports for future meetings.)
 - We'll have to increase revenue from elsewhere, or charge attendees more, or both.

The KME 2009 Convention will be seriously in the red, and I take full responsibility for that. Fortunately, we have a sufficient reserve. However, I will work hard to restore the balance over the coming four years.

Regional Conventions

Less than 1/3 of our chapters have participated at least once in a national convention in the last 10 years. To me, this stresses the importance of regional conventions. Perhaps we should put our heads together to figure out how to emphasize this importance, and how we might help chapters to try to host a convention.

Relationship with Pi Mu Epsilon

One of the most pleasing aspects of my term as P-E has been the development of a good relationship between KME and PME. Thanks to the support of the KME National Council, and due in no small part to the efforts of PME Secretary Leo Schneider, we have been able to establish regular, meaningful communication between the two organizations. I attended PME's Council meeting in August of 2007, and Rhonda McKee attended in August 2008. PME President David Sutherland attended our Council meeting in November 2008, and PME President-Elect Eve Torrence is attending this convention.

The result of these exchanges has been tangibly beneficial to both organizations. I hope to continue this relationship, and plan to attend MathFest in August of this year.

Ron Wasserstein
National President Elect

Report of the National Secretary

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. In the fall of each year, I send out supplies to each chapter. The supplies include information brochures and membership cards. I also take minutes of all business meetings of the organization and all meetings of the national council. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

I am now in my eighth year as KME national secretary, and have enjoyed watching how various processes within the organization have changed over the years. For example, when I began, most of our correspondence was by land mail, as not everyone had email. Now, of course, nearly all of our correspondence is done by email.

The initiation report form has changed immensely over the last 10-12 years. We've gone from tri-fold cards on which information was hand-printed, to a word processing document that was mailed to me, to a word processing document that was emailed, and now to a spreadsheet that can be emailed or filled out on a web form. Kevin Reed, our web master and I have been working on this form recently, and, I think, have a final version in place (for now).

During this, the 37th Biennium of Kappa Mu Epsilon, 2,444 new members were initiated in 121 chapters. Twenty-three active chapters did not report an initiation in this biennium. There are 144 active chapters of KME and 34 inactive chapters for a total of 178 chapters. The total membership in KME at the end of the 37th biennium was 75,527.

Rhonda McKee
National Secretary

Report of the National Historian

It has been a pleasure to serve as the National Historian for the past two terms (8 years). I have most enjoyed working with the other officers and I am impressed with the level of dedication I see from all of the officers. I will miss this group of people and the time spent discussing all the issues that affect KME and our students.

One of the things I have learned is that it is important to send out a second call for information. When the call for information is sent out at the end of the semester many faculty are too busy to respond. This year I made a last request right at the deadline and I heard from more than 10 additional schools.

The historian has a four drawer file cabinet full of information as well as the material for an addition two drawers. This information will need to be transferred to the location of the new historian. Another consideration will be to immediately make sure that the historian is receiving multiple copies of the Pentagon.

Thank you for this opportunity to serve in this role.

Connie S. Schrock
National Historian

Report of the National Treasurer

37th Biennium (March 21, 2007 – March 10, 2009)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows end-of-biennium assets of \$67,002.51. The Cash Flow Report shows that we had an asset loss of \$1,647.78 during the biennium. A National Council goal to maintain an asset base of at least \$40,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (March 20, 2007)		\$68,650.29
Current Assets		
Kansas Teachers Community Credit Union		
Checking	35,889.16	
Share Account	1,113.35	
CD15229	10,000.00	
CD15261	10,000.00	
CD15288	10,000.00	
Total Current Assets		\$67,002.51

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	48,880.00	
Installation fees received	150.00	
Interest income	1,476.08	
Overpayment in	100.00	
Gifts & misc. income	1,000.00	
Total Biennium Receipts		\$51,606.08
Expenditures		
Association of College Honor Soc	1,621.68	
Administrative expenses	3,390.70	
National Convention expenses	4,162.30	
Regional Convention expenses	500.00	
Council Meetings travel	2,438.02	
Certificates, jewelry & shipping	10,851.07	
Installation expenses	100.00	
Overpayment returns	100.00	
Pentagon expenses	29,184.65	
Miscellaneous expenses	905.44	
Total Biennium Expenses		\$53,253.86
Biennium Cash Flow		-\$ 1,647.78

The cash flow last biennium (05-07) was +\$7,555.79. This biennium we initiated 261 fewer members so our initiation income dropped by \$5220 from last biennium. With lower interest rates, our interest income also dropped. Pentagon expenses more than doubled since the last biennium. However, national convention expenses were low due to the centrally located site and initiation expenses were significantly reduced this biennium. We also received generous gifts of \$500 each from the American Mathematical Society and American Statistical Association to help with convention expenses for the 2009 national convention. We have maintained our goal of maintaining assets of at least \$40,000. The financial condition of Kappa Mu Epsilon is sound.

I want to thank John Kubicek for the very organized records he kept as National Treasurer and his willingness to provide help to me as I took over the position. Also, sincere thanks to the dedicated, talented, hard-working professionals of the National Council. Without their help, support, and commitment to keep expenses low, the Treasurer's job would be much more difficult. In addition, thanks to the work of the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon.

Cynthia Woodburn
National Treasurer

Report of the Webmaster

Basic website completed

- Electronic submitting of initiates
- Organization of web pages
- Updating the form mailer to filter the recent junk emails
- Corrections to the form mailer
- Additional chapters entered into the database
- History moved to "documents" area of website

Updates

- Cleaned up some typos and updated school names
- User registration and login for Council and Corresponding secretaries
- Revamped Initiation report form (Excel & web entry)

- National council members can login to:
 - post and delete news
 - email registered users
- National Council members have a form, too.
- Posted The Pentagon from Spring 2006 to Fall 2007.
- Reorganized the “Forms” and “Documents” pages.

Future installments to the website

- Make the site W3C compatible – standardized way of writing of web pages.
- Make sure that after W3C standards are met, that the website works in “Safari”.
- Add the Problem Corner.
- Continue reorganization and addition of functionality.

Kevin Reed
Webmaster

Report of the Pentagon Business Manager

I took over as business manager of The Pentagon in December, 2006. My first issue was the Fall, 2006 issue (Volume 66, Number 1). This bien-nium begins with Volume 66-2. The biggest job is still entering the names of new initiates into the database. There are approximately 1200 - 1500 names and addresses that have to be entered each year. Since initiation reports have changed to being almost exclusively electronically submitted, this is now a case of cutting and pasting rather than retyping names. This has saved a lot of time and effort, and I appreciate the efforts of Rhonda McKee for encouraging that transition. The database I received was not entirely accurate so there was a lot of time spent verifying names and addresses. Also, during the process of learning Excel I made some data manipulation errors, compounding the subscription list problems. Student initiates receive a two-year subscription and so in the last two years virtually all the errors have been resolved. However, because of the errors I extended a great many subscriptions. That led to an influx in the number of issues printed and mailed, leading to a peak circulation for the 67-1 issue of

5000 copies. We have since dropped the circulation to a more typical level just under 3000 which is where it should remain. That should keep the cost of printing, postage, and handling around \$4500 per issue. The subscription income we receive from libraries and individuals roughly covers the cost of miscellaneous supplies, postage and foreign subscriptions. We actually lose money on magazines sent out of the country. The council has authorized me to trim the list of free subscriptions and students with foreign addresses will have their subscriptions sent to a US address. That should also help reduce costs.

The council has also authorized putting back issues on the website in their entirety. That should help reduce requests for back issues.

It has been a pleasure working with Chip Curtis, and I think we are getting the magazine once again to a place of reliability and prominence.

We started the biennium with a balance of \$303.02 and ended with a balance of \$790.06. I should keep the financial records according to biennium, but funding is graciously processed through my university's books which is maintained on an annual basis.

Don Tosh
Business Manager, The Pentagon

Report of the Pentagon Editor

The Pentagon, introduced in 1941, is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the central theme of *The Pentagon*. Continuing a tradition, papers given "Top 4 Status" by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. Many other student papers presented at the KME National and regional conventions are included as well. The key to having one's paper published in *The Pentagon* is submission of all of the necessary materials. For additional information about having an article published in *The Pentagon*, please see the sheet entitled "Information for Authors".

All new initiates receive a two-year subscription to *The Pentagon* and can continue their subscriptions for \$5.00 per year. Having a current address is obviously vital in insuring that you receive your copy. Please check the address on your most recent issue and e-mail corrections to the Business Manager if necessary at toshd@evangel.edu. Dr. Don Tosh has greatly streamlined the process of getting *The Pentagon* printed and getting it where it is supposed to go. Thank you, Don.

Manuscripts received by *The Pentagon* other than those presented at our conventions are refereed by faculty volunteers. Over the past several

months, dozens of referees have volunteered. The service of these individuals is invaluable.

In addition to the articles, each issue of *The Pentagon* contains the Problem Corner, to which you are encouraged to submit problems and solutions to Dr. Pat Costello, Problem Corner Editor. Dr. Costello's energy and enthusiasm, together with your submissions, make sure the Problem Corner is a quality component of *The Pentagon*. Thank you, Pat.

Another regular feature of *The Pentagon* is the Chapter News. The National Historian is responsible for assembling this and the reports of new chapter installations from the individual chapter reports. Dr. Connie Schrock has set a high standard of efficiency and accuracy in the production of the Chapter News. Thank you, Connie.

Thank you as well to the regional directors for contributing reports of the regional conventions and forwarding student papers from the conventions, and to the National Committee for its assistance with the many details crucial to the success of *The Pentagon*.

Charles Curtis
Pentagon Editor

Audit Committee Report

Audit Committee Members

- Elizabeth Mauch, PA Lambda, Bloomsburg University, faculty, chair
- Scott Sykes, GA Alpha, University of West Georgia, faculty
- Vince Dimiceli, OK Delta, Oral Roberts University, faculty
- Pedro Muino, PA Mu, St. Francis University, faculty
- Anne Stock, PA Mu, St. Francis University, faculty

Audit Process

1. Prior to the national convention, Treasurer Cynthia Woodburn mailed biennium financial summary data to Elizabeth Mauch, the committee chair, to facilitate verification of asset account totals prior to the convention. At the convention, Treasurer Woodburn provided the committee with relevant biennium summary and detail documentation for receipt and payment transactions, along with bank and savings account reconciliation documentation.

2. Before the national convention, Kansas Teachers Community Credit Union in Pittsburg, KS was contacted by telephone. The account balances for the Kappa Mu Epsilon checking and savings accounts were verified to correspond exactly to associated totals found on Treasurer Woodburn's biennium reports.
3. At the national convention, committee members interviewed President Don Tosh and Secretary Rhonda McKee to determine their impressions of the accuracy and completeness of the recording throughout the biennium. The committee also perused the financial documentation provided by Treasurer Woodburn to the committee and interviewed Treasurer Woodburn.

Recommendations

1. Information forwarded by the Treasurer to the committee chair prior to the national convention provides the opportunity for verification of assets in a careful manner and should be continued in the future.
2. The internal checks built into the regular financial processing between the Treasurer and the President and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.
3. The committee recommends considering an increase in mileage allowance from .35 per mile to .40 per mile, because of rising gas prices.
4. Finally, the Audit committee recommends the acceptance of the financial records and reports of Kappa Mu Epsilon for the 2007-2009 biennium as presented by Treasurer Cynthia Woodburn.

Commendations

1. The committee commends Cynthia Woodburn on a smooth transition as new treasurer after John Kubicek.
2. We further commend Cynthia for her valuable input to this Audit Committee.
3. The committee commends the national President, Secretary and Treasurer for the manner in which they communicate and cooperate to maintain the internal checks which preserve the integrity of the office of

Treasurer

4. The committee commends the work of the previous audit committees and gratefully received reports provided by the Treasurer.

Report of the Resolutions Committee

Resolutions Committee Members

- Meg Huddleston, TX Mu, Schreiner University, chair
- Rebekah Holmes, Missouri Theta, Evangel University
- Elana Epstein, New York Omicron, St. Joseph's College
- Gaspar Porta, Kansas Delta, Washburn University
- Amanda McCullough, Kansas Delta, Washburn University
- Phyllis Conner, Kansas Beta, Emporia State University
- Sarah Lott, Kansas Beta, Emporia State University.

Resolutions

The committee proposed the following resolutions. "Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved:

1. that this Thirty-seventh Biennial National Convention express its gratitude to (a) Don Tosh, who is completing 12 years as an officer: 4 as historian, 4 as president-elect, and 4 as president; (b) Rhonda McKee, for her 8 years of faithful service as national secretary; (c) Connie Schrock, for her 8 years of dedicated service as national historian; (d) Kevin Reed, for his invaluable work as webmaster; (e) Charles Curtis, for his dedication to the Pentagon; (f) Jason Merten for volunteering to serve as event photographer; and (g) Ron Wasserstein for his efforts in guiding Kappa Mu Epsilon as its president-elect; and
2. that this Convention acknowledge the participation of the students and faculty who served on the Auditing, Awards, Nominating, Paper Selection and Resolutions committees, which are so essential for the success of the meeting."

"Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of a deeper understand-

ing of its beauty, be it further resolved:

1. that this Convention give special commendations to the dedication and hard work of the student presenters who spent numerous hours in researching and preparing their papers and presentations,
2. that this Convention express thanks to Rhonda McKee for bringing “Flatland: the Movie” and to the University of Central Missouri for paying for the license so that it could be shown,
3. that Melanie Matchett Wood, Princeton University, be commended for completing her PhD this spring and moving on to post-doctoral work at Stanford this fall, and for her keynote address “The Chemistry of Primes,” presented at the Friday night banquet, and
4. that this Convention express its gratitude to the American Mathematical Society and the American Statistical Association for their support.”

“Finally, whereas the Doubletree Hotel has provided this Convention with gracious hospitality, be it resolved that this Thirty-seventh Biennial Convention express its heartfelt appreciation to the staff and especially to Ms. Karen Lech for the thorough arrangements they have planned and carried out so successfully.”

Respectfully submitted,
Meg Huddleston, Chairperson

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of September 2009

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive
313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

Installation Report

California Zeta
Simpson University, Redding, CA

The installation of the California Zeta Chapter of Kappa Mu Epsilon was held in the Owen Student Services Center of Simpson University on Saturday, April 4, 2009, at three o'clock in the afternoon. KME National President Emeritus Don Tosh was the installing officer. Dr. Larry McKinney, Simpson University President, Dr. Stanley Clark, University Provost, and Dr. Isaiah Lankham, faculty sponsor and corresponding secretary, each played a part in the ceremony. After Dr. Lankham finished describing KME's crest, the organization was declared to be the California Zeta Chapter of Kappa Mu Epsilon and the chapter's charter was presented to the corresponding secretary. The faculty charter members of California Zeta are: Isaiah Lankham, Harold Lund, and Mel Shuster. The student charter members of California Zeta are: Pearl Baez, Katherine Garringer, Caleb Goerzen, Rebekah Hoffman, Joshua Lin, Thomas Parker, Chalynn Passmore, and Nguyen Tran. Each initiate was invited to sign the California Zeta Chapter Roll, and was presented with a membership certificate and a KME pin. The officers of the California Zeta chapter were also installed during the ceremony. They are: Dr. Isaiah Lankham, faculty sponsor and corresponding secretary; Thomas Parker, president, and Nguyen

Tran, vice president. Each officer was charged with the responsibilities of his new office, and each chose to accept those responsibilities. A time of congratulations and fellowship with delicious refreshments was enjoyed by all following the installation ceremony.

New York Rho
Molloy College, Rockville Centre

The installation of the New York Rho Chapter of Kappa Mu Epsilon was held on Tuesday afternoon, 21 April 2009, in the Multi-Purpose Room of Molloy College in Rockville Centre, New York, and twenty charter members were initiated. Dr. Andrew M. Rockett, a former editor of *The Pentagon*, was the installing officer, and Dr. Anthony Tolvo, Dean of the Division of Natural Sciences, Mathematics, Computer Studies and Allied Health Division, attended. Dr. Vincent Pane, chairman of Mathematics and Computer Studies, opened the proceedings and Dr. Manyiu Tse presented the initiation candidates. Officers installed were: Thomas Anderle, president; Michelle Topel, vice-president; Nicole Scandariato, recording secretary; Rebecca Stern, treasurer; Dr. Deborah Upton, faculty sponsor; and Dr. Manyiu Tse, corresponding secretary. Refreshments were served afterwards.

North Carolina Zeta
Catawba College, Salisbury, North Carolina

The North Carolina Zeta Chapter of Kappa Mu Epsilon was installed at 4pm on Thursday, September 17, 2009, at a ceremony in Kentner Hall on the campus of Catawba College, Salisbury, North Carolina. The meeting was conducted by Douglas Brown. KME President Ron Wasserstein served as the Installing Officer. The charter members, students Cody Ashby, Scott Campbell, Cynthia Cook, John Hoehman, Stacy Michael and Zachary Owen and faculty members Paul Baker, Jason Hunt, Sharon Sullivan, Cyndi Osterhus, John Zerger and Douglas Brown, were initiated into the chapter. The first officers of North Carolina Zeta, President Scott Campbell, Vice President John Hoehman, Recording Secretary Zachary Owen, Treasurer Cody Ashby, and Corresponding Secretary/Faculty Sponsor Douglas Brown were installed. About 30 people were in attendance. After the formal ceremonies, Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the North Carolina Lottery."

Chapter News

AL Alpha – Athens State University

*Chapter President – Kasey Taylor; 30 Current Members, 13 New Members
Other spring 2009 officers: Rick Martindale, Vice-President; Guy Stafford, Secretary; Dottie Gasbarro, Corresponding Secretary.*

Alabama Alpha Chapter of Kappa Mu Epsilon held initiation of new members on Sunday afternoon, April 5, 2009 in the Chapel of Founders Hall with a reception for initiates and their approximately 50 guests following in the Parlor. Professor Dottie Gasbarro, KME Advisor, presided and Ms. Kasey Taylor, chapter president and Mr. Guy Stafford, chapter secretary conducted the ceremony for the initiates. Dr. Ron Fritze, Dean of College of Arts and Sciences and historian, was guest speaker and spoke about the rich history of mathematics and Algebra of the middle east. Alabama Alpha chapter was chartered in 1935, the first KME chapter in Alabama. With Sunday's initiation, there are 219 members on the roll of Alabama Alpha.

New initiates – Cossandra Paige Bishop, Angela Davis, Amber Lashay Drummund, Rachael Ellison, Suzanne Forrest, Donna Evette Hood, Lacey Elizabeth Keenum, Roger Kilpatric, Carl J. Kuby, Jr., Kenneth Lynn Mullins, Georgette Terry, Samuel Charles Wallace, Trisha Ann Woodley.

AL Eta – University of West Alabama

Hazel Truelove, Corresponding Secretary.

New initiates – Clint Brown, Abdoulaye Ndiaye, Kimberly Posey, Brandi Roe, Stephanie Turner.

AL Gamma – University of Montevallo

John Herron, Corresponding Secretary.

New initiates – Alicia M. Chatham, Ann K. Chester, Corey R. Gros, Veny Liu, Sendrick Smith, II, Megan N. Traweek, Catherine L. Wilson.

AL Zeta – Birmingham Southern College

*Chapter President – Loree Kilbrew; 14 Current Members, 1 New Member
Other spring 2009 officers: Xinyan Yan , Vice-President; Stephanie Barlow, Secretary and Treasurer; Mary Jane Turner, Corresponding Secretary.*

Fall Meeting: Dr. Stephen Curry presented “The Trouble with Randomness is that it is Random”. Dr. Curry at the time was a member of the mathematics faculty of Birmingham Southern College. The event was well attended. Refreshments were enjoyed afterward.

Spring Meeting: Dr. Emily Hynds presented “Mathematical Puzzles and Games”. Dr. Hynds is an Associate Professor at Samford University in Birmingham. Refreshments were enjoyed by all.

CA Delta – Cal State Polytech - Pomona*Patricia Hale, Corresponding Secretary.*

New initiates – Damian Allen, Brandi Bailes, Daniel Bortis, Jared Call, Sonia Chauhan, Rafael Delgado, Lawrence Fraser, Rana Haber, Gregory Hewitt, Adeline Kornelus, Suzanne Lavertu, Jonathan Norton, Natalie Mateo, April Morton, Christopher Phillips, Amanda Rich, Christopher Sanchez, Vikram Savani, Karim Tartoussia, Brian Vega, Ke Wu.

CA Zeta – Simpson University*Chapter President – Thomas Parker**Other spring 2009 officers: Nguyen Tran, Vice President, Isaiah Lankham, Corresponding Secretary.*

New initiates – Pearl Baez, Katie Garringer, Caleb Goerzen, Rebekah Hoffman, Isaiah Lankham, Joshua Lin, Harold Lund, Thomas Parker, Chalynn Passmore, Mel Shuster, Nguyen Tran.

CT Beta – Eastern Connecticut State University*Christian L. Yankov, Corresponding Secretary.*

New initiates – Jenae Beauchamp, Nicole Beauchamp, Bryce Casavant, Maureen Cop, Mary Costello, Amy Couture, Margaret Deptula, Heather Desmond, Katherine Dudzinski, Alexandra Garry, Scott Johnson, William Lathrop, Christine Nadelman, Scott Seaman, Ashley Smith, Ryan Tomchek, Jessica Wilson.

GA Alpha – University of West Georgia*Scott R. Sykes, Corresponding Secretary.*

New initiates – Amanda Cook, Aderinre Dada, Erika Harvey, Holly Meeks, Natalie Wan, Tara Westervelt, Tyler Young.

GA Beta – Georgia College and State University

163 Current Members, 14 New Members

Dr. Rodica Cazacu, Corresponding Secretary; and Dr. Jason Huffman, Faculty Sponsor.

HI Alpha – Hawaii Pacific University*Andrea Ciletti, Corresponding Secretary.*

New initiates – Sarah Davis, Ryan K. Tenn, Justin Reeves, Danielle Mooshol, Imee Caraang.

IA Alpha – University of Northern Iowa*Chapter President – Kellen Miller; 29 Current Members, 9 New Members**Other spring 2009 officers: Darcy Thomas, Vice-President; Megan Klein, Secretary; Beth Kolsrud, Treasurer; Mark D. Ecker, Corresponding Secretary.*

Our first Spring KME meeting was held on March 24, 2009 at Professor Mark Ecker's residence where student member Reanna Collins presented her paper on "Possible Factors Related to the Achievement of a Bachelor's Degree". Student member Darcy Thomas addressed the spring

initiation banquet with "Analysis of US Divorce Rates." Our banquet was held at Godfather's Pizza in Cedar Falls on April 28, 2009 where nine new members were initiated.

New initiates – Michelle Breen, Nicole Hancock, Samantha Jaeger, Nicholas Jensen, Andrew Meyer, Tristram Nebelsick, David Rygh, Adam Wauters, Jaime Zeigler.

IA Delta – Wartburg College

Chapter President – Jen Czachura; 36 Current Members, 14 New Members

Other spring 2009 officers: Stacy Berns, Vice-President; Ashley Schulteis, Secretary; Blake

Haugen, Treasurer; Dr. Brian Birgen, Corresponding Secretary.

In March, fourteen new students were welcomed at our annual banquet and initiation ceremony.

New initiates – David Carlson, Kylee Frush, Jacob Hinrichsen, Masa Maruyama, Joshua Osbeck, Adam Pedersen, Jared Robb, Eric Ruggles, Aditya Salimputra, Sage Schissel, Ashley Schulteis, Kelsey Steffens, Hyerim Stuhr, Shuhei Yamamoto.

IA Gamma – Morningside College

Eric Canning, Corresponding Secretary.

New initiates – Garrett Allen, Chiao Ying Chang, Emily Christen, Jennifer McWashington, Jessica Rix, Margo Stanforth, Riley Swedberg, Kelsey Uherka.

IL Eta – Western Illinois University

Chapter President – Jacob Brown; 9 Current Members, 3 New Members

Other spring 2009 officers: Sarah Cramsey, Vice President, Carla Webb, Secretary, Timothy Gross, Treasurer, Boris Petracovici, Corresponding Secretary.

We started an Undergraduate Math Colloquium; four of our faculty gave the following talks: Combinations and The Binomial Theorem; Some Interesting Happenings in the Pascal's Triangle; and Jack and the Beanstalk 2.0: Space Tethers and Elevators. The club organized the yearly Calculator Seminar (to help incoming freshmen learn how to use the TI-83/84 calculator), and the yearly tutoring session for the math part of the Basic Skills Test required for the Illinois teaching certification.

New initiates – Lindsay Henderson, Shufang Liu, Danielle Taulbee.

IL Iota – Lewis University

Margaret Juraco, Corresponding Secretary.

New initiates – Douglas Anderson, Katrine Binaku, Sylvia Ciezak, Kim Donlan, Jason Domico, Kristy Gaynor, Jose Guerrero, Sarah Ingram, Joshua Kaluzny, Jessica Kos, Mike Lerman, Kaitlin Merrion, Michael Patrick, Bryan Pierard, Raymond Pellicore, Brian Robinson, Simon Rock, Allison K. Schmitz, Katherine Skurski, Joseph Stephens, Keith Taddei, Paul Dagens, Dnielle Toranzo, Vincent Schade, Rhonda Walsh.

IL Theta – Benedictine University

Dr. Thomas Wangler, Corresponding Secretary.

New initiates – Hunan Chaudhry, Jared Gustafson, Michael Whitley.

IL Zeta – Dominican University

Chapter President – Nancy Gullo; 20 Current Members, 5 New Members

Other spring 2009 officers: Monika Vidmar, Vice President; Angelina Myers, Secretary; Phillip Lenzini, Treasurer; Aliza Steurer, Corresponding Secretary.

An initiation ceremony was held to induct five new members. A Dominican University math professor and KME member, Don Marzen gave the induction talk. As for the rest of the semester, we held our annual Pi day pie-giveaway activity and participated in the ACCA Calculus Competition.

New initiates – Andre Burrell, Yoshio Piediscalzi, Kim Plesnicar, Karl Rickert, Natalie Waksanski.

IN Alpha – Manchester College

Stan Beery, Corresponding Secretary.

New initiates – Raheel Ahmad, Karla Conrad, Abel Mengistu, Tsega Mengistu, Jessamy Rogers.

IN Delta – University of Evansville

Chapter President – Mary Craighead; 32 Current Members, 20 New Members

Other spring 2009 officers: Carrie Schindler, Vice President; Annette DeWold, Secretary; Dr. Adam Salminen, Corresponding Secretary.

In the Spring semester of 2009, the Indiana Delta Chapter hosted the annual University of Evansville Mathematics competition. KME members also provided free tutoring for UE students to prepare for final exams. Several members of KME also competed in the Indiana College Mathematics Competition.

New initiates – Jace E. Bittner, Mary Craighead, Bryan Crist, Annette C. DeWolf, Jordan Haycock, Jamin W. Heldt, Jacob M. Kahle, Darrel J. Matouba, Ashley M. Nelson, Kien Trung Pham, Mark A. Riehle, Joshua L. Scherschel, Carrie Schindler, Robert Seitzinger, Ji Heun Tak, Brian L. Varner, Lauren E. Wahl, Chelsea Wargel, Joshua Woody, Guy E. Wyant.

KS Beta – Emporia State University

Chapter President – Heather Julian

Other Spring 2009 Officers: Melissa Swager, Vice President; Ryan Wilson, Treasurer; Yuchen Chen, Secretary; Connie Schrock, Corresponding Secretary.

Our chapter traveled to the KME Biannual meeting in Philadelphia and to a mathematics Colorado Springs conference. One of the fundraisers

students enjoyed was our penny wars with the faculty members who received over a specific amount receiving pies in the face. Pi Day was celebrated on March 14.

KS Epsilon – Fort Hays State University

Jeffrey Sadler, Corresponding Secretary.

New initiates – Kory Koehn, Antoinette Leiker, Troy Morash, Marcella Rhoads, Kylie Simpson, Michelle Webb.

KS Gamma – Benedictine College

Chapter President – Matthew Weaver; 14 Current Members, 6 New Members

Other Spring 2009 Officers: Caitlin Kelly, Vice President; Christina Henning, Secretary; Christina Henning, Treasurer; Dr. Eric West, Corresponding Secretary.

In Spring 2009 our chapter initiated 6 new members, and met to elect new officers. At the spring Honors Banquet, the Sister Helen Sullivan scholarship in mathematics was awarded to chapter members Caitlin Kelly, Emily Marsolek, and Matthew Weaver. We ended the year with a dinner to honor graduating seniors.

KY Alpha – Eastern Kentucky University

Chapter President – Joshua Sparks; 15 Current Members, 17 New Members

Other spring 2009 officers: Brittney Walker, Vice President; Alex Henegar, Secretary; Katherine Carter, Treasurer; Pat Costello, Corresponding Secretary.

The first meeting was February 12th where we worked problems posed by Ken Dutch and decided to view lectures by Arthur Benjamin from the series entitled Joy of Mathematics. We viewed the Joy of Numbers on 2/27, the Joy of Primes on 3/20, the Joy of Counting on 4/3, and the Joy of Fibonacci Numbers on 4/17. Initiation of new members was April 30, 2009. The President, Josh Sparks, gave a talk on Encrypted Chess.

New initiates – Sarah K. Beauchamp, Justin R. Bray, Johnathan R. Cooper, Valencia S. Darnell-Ortt, Aaron R. Decker, Paul S. Duncan, Charles J. Fahringer, Jennifer L. Fischesser, Sarah L. Hale, Meghan L. Hill, Jonathan M. Jones, Adison D. Merida, Naomi L. Meyer, Luke T. Pinkston, Katara N. Roaden, William D. Spears, Natalie L. Venable.

KY Beta – University of the Cumberland

Chapter President – Teresa Shaffer; 29 Current Members, 10 New Members

Other Spring 2009 Officers: Dustin Ursrey, Vice President; Joshua Ward, Secretary; Andrzej Lenard, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary.

On February 27, 2009, the Kentucky Beta chapter held an initiation and a banquet at the Cumberland Inn. Kappa Mu Epsilon inducted ten new student members at the banquet, presided over by outgoing president, Teresa Shaffer. As an additional feature, senior awards were given by the department at the banquet. Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 16. He spoke about "Rabbits, Art, and Pineapple." On April 17, members also assisted in hosting a regional high school math contest, held annually at the University of the Cumberlands. On April 28, the entire department, including the Math and Physics Club, Sigma Pi Sigma (Physics Honors Society), and the Kentucky Beta Chapter, held a picnic at Briar Creek Park to celebrate the end of the semester.

LA Delta – University of Louisiana at Monroe

David Hare, Corresponding Secretary.

New initiates – Dina Abdelhalim, Brooke Allison, Brooke Ardoin, Kaynala Baker, Sarah Barrett, Brody Bourque, Jake Borque, Margaret Broussard, Tony Brown, Paul Caliste, Renisha Converse, Rebecca Davezac, Miranda Dejean, Youssef Dib, Kacie Dore, Jessica Earnest, Nick Fenner, Kristin Frankhouser, Alika George, Jeremy Hattaway, Charles Head, Destinee Hebert, Atheia Holland, Gary Jackson, Mallory Kelly, Lara Keys, Sara Ledoux, Krystal Lee, Tiffany Legendre, Cally Newsom, An Nguyen, Caitlyn Olivier, Tiffany Riley, Mariam Saadat, Heather Simmons, Ashlee Sisson, Abel Stevenson, Ruthie Stone, Jessica Talley, Anna Zaunbrecher.

LA Gamma – Northwestern State University

Leigh Myers, Corresponding Secretary.

New initiates – Lindsay Browning, Brittany Domangue, Alex Ferguson, Allison Hardy, Anton Kodochygov, Jacob Matherne, Amanda Vines.

MA Alpha - Assumption College

*Chapter President – Sandra Garney; 7 Current Members, 9 New Members
Other spring 2009 officers: Tara Fountain, Vice-President; Kathryn Richard, Corresponding Secretary, Charles Brusard.*

New initiates – Kate Basque, Brittany L. Brannelly, Stephanie M. Enos, Kayla L. Fraser, Nichole A. McIntyre, Amanda C. Milliken, Nicholas M. Mordarski, Nha Trang Thi Nguyen, Kerry M. O'Rourke.

MD Beta – McDaniel College

Dr. Harry Rosenzweig, Corresponding Secretary.

New initiates – Gregory Allen, Robert Kelvey.

MD Delta – Frostburg State University

*Chapter President – Lisa Gitelman; 20 Current Members, 5 New Members
Other Spring 2009 Officers: Kelly Seaton, Vice President/Secretary; Joe Bascelli, Treasurer; Mark Hughes, Corresponding Secretary.*

The Maryland Delta Chapter held two successful fundraisers this spring. Our annual bake sale in celebration of Pi Day was well received by students looking for some treats in between classes. Later in the term we had many customers for our candy Easter Egg sale, providing us with pizza money for the upcoming semester! In March, Dr. Mark Hughes presented a lecture on Bernoulli Numbers and Sterling's Formula. March also saw our 2009 induction ceremony where we welcomed five new members to the chapter. New officers were elected during our April meeting. They are: Kelly Seaton (President), Josh Wilson (Vice President), Joe Bascelli (Secretary) and Brad Phillips (Treasurer). We offer best wishes to our outgoing President, Lisa Gitelman, as she moves on to graduate school – a great artist and excellent student.

MD Epsilon – Stevenson University

*Chapter President – Krystal Burns; 18 Current Members.
Other Spring 2009 Officers: Catherine Gerber, Vice President; Liesl Feinour, Secretary; Brandon Cooper, Treasurer; Dr. Christopher E. Barat, Corresponding Secretary.*

The Chapter sponsored its second annual "Pi Day" during March, selling pies to raise funds. On April 23, the Chapter sponsored an "Afternoon of Mathematical Presentations" to commemorate Mathematics Awareness Month. Presentations included "Finding Cantor's Infinities in Poetry" (Ms. Eileen McGraw, Associate Professor of Mathematics and Economics); "A Gentle Introduction to Microarray Analysis" (Timothy Potter, senior and Chapter member); "Some Reflections of the 'New Math' in Popular Culture" (Dr. Christopher E. Barat, Associate Professor of Mathematics and faculty sponsor of the Chapter). Dr. Barat represented the Chapter at the Biennial Convention in Philadelphia.

New initiates – Jennifer Barrick, Sarah Bauer, Marisa Bender, Matthew Bramble, Nicole Caprio, Justine Crutchfield, Stefani D'Addario, Lauren Gordon, Dr. Susan Gorman, Heather Hanley, Chelsea Insley, Kelsey Lien, Brittany Miller, Dr. Hildagarde Sanders, Jennifer Sauers, Julia Wikoff, Kelly Witte.

MI Alpha – Albion College

Mark Bollman, Corresponding Secretary.

New initiates – Yang Chen, Rachel Kamischke, Kathryn Wagner.

MI Epsilon – Kettering University

*Chapter President – Jeff Nolen; 213 Current Members, 24 New Members
Other Spring 2009 Officers: Phillip Besoiu, Vice President; Kathleen Moufore (A Section) and Justin Via (B Section), Secretaries; Jessi Harden and Jamie Taylor (for B section), Treasurers; Boyan Dimitrov, Corresponding Secretary; Ruben Hayrapetyan (Section A, Winter and Summer terms), and Ada Cheng (Section B, Spring and Fall terms, Faculty Sponsors.*

Kettering University has an active KME Society life. The former Science and Mathematics Department was split into 4 separate departments including the Department of Mathematics (Dr Leszek Gawarecki); Department of Physics (Dr Bahram Roughani); Department of Chemistry and Biochemistry (Dr. Stacy Seeley); and Computer Science Department (Dr. John Geske). The former Department Head, Dr. David Green, retired.

The Winter 2009 KME Initiation was on March 20. There were 24 new members initiated. The dinner finished with an amazing wiz mathematical logic competition for parents (helped by students) instead of traditional speech of an invited speaker.

There were Pizza/Movie Parties on January 30, the movie was "Lost at sea. The search for longitude," and on August 20, the movie was "Infinite Secrets." For 6 years, Kettering University has financially supported the Mathematics Olympiads, organized by the mathematics group of enthusiastic faculty. The last Math Olympiad was held on November 22, 2008 at Kettering with about 80 participants.

New initiates – Michael Beaudin, Adam J. Benish, Jeremy Brickner, Robert Brown, Ryan M. Burt, Steven M. Carmain, Matthew D. Carter, Nackia Chamberlain, Elizabeth M. Cook, Chelsey E. Crabtree, Vanessa Croll, Philip A. DeMonaco, Bradley Dunnill, Lucy Engle, Karsten P. Gamble, Weston J. Hages, Phillip V. Hamilton, Jeffrey R. Hanba, Jessi Harden, Derek E. Hazard, John M. Hendrick, Valentin Hernandez, Randi R. Holderbaum, Jacob Howarth, Tareco Johnson, Sumit Kamboj, James D. Kinsey, Jr., Bi Zan Valery Lorou, Chris Luman, Furqan Memon, Christopher Mitchell, Paul F. Reichert, Blake J. Riley, Charity Rogers, Andrew Serrian, Paul Silversmith, Michael Silvestri, Tevita Skeine, Frederick J. Snoy III, Adam J. Terbrack, Leena Thomas, Alexander O. Thomson, Angela M. Tremble, Alexandria N. Wilson, Kathryn M. Zayan.

MO Alpha – University of Central Missouri

Chapter President – Christina Tharp; 32 Current Members, 15 New Members

Other spring 2009 officers: Jacob Swett, Vice President; Ashley Lewis, Secretary; Brett Foster, Treasurer; Jorge Rebaza, Corresponding Secretary.

KME Seminars were held on the following dates with the following speakers: 02/09/09 Eric Shade (Computer Science), MSU; 03/09/09 Songfeng Zheng (Mathematics), MSU; 4/09/09 Kara Gideon and Lisa Tiernan (Mathematics), MSU.

MO Beta – University of Central Missouri

Chapter President – Thomas Gossell; 25 Current Members, 16 New Members

Other spring 2009 officers: Todd Carlstrom, Vice President; Phat Hoang, Secretary; Cynthia Craft, Treasurer; Jason Merten, Historian; Rhonda McKee, Corresponding Secretary; Steve Shattuck and Dale Bachman, Faculty Sponsors.

MO Gamma – William Jewell College

Neil Nicholson, Corresponding Secretary.

New initiates – Riley Anderson, Emma Farris, Jesse Funk, Katie Tongue, Kenneth Whitmore.

MO Iota – Missouri Southern State University

Charles Curtis, Corresponding Secretary; Richard Laird and Grant Lathrom, Faculty Sponsors.

New initiates – David Benham, Nathan Clymer, Ethan Johns, Clayton Lungstrum, Cheng Yang.

MO Kappa – Drury University

Carol Browning, Corresponding Secretary.

New initiates – Stephanie Flynn, Sam Pirtle.

MO Lambda – Missouri Western State University

Steve Klassen, Corresponding Secretary.

New initiates – Holt Ellis, Rachel Howe, Emily Justin, Curtiss Lane, Adelaide Quaney, Ashley Schnoor, Siya Sun.

MO Mu – Harris-Stowe State College

Ann Podleski, Corresponding Secretary.

New initiates – Kalifa Gray, Muziwi Nyamapfene, Travis Parker, Brooke Presley, Rico Smith, Kathleen Youngbauer.

MO Nu – Columbia College

*Chapter President – Magda Pride; 18 Current Members, 13 New Members
Other spring 2009 officers: Tomas Horvath, Vice President; Becca Kunce,
Secretary; Chris Hawkins, Treasurer; Dr. Kenny Felts, Corresponding
Secretary.*

MO Theta – Evangel University

*Chapter President – Jennifer Brocker; 9 Current Members, 5 New
Members
Other spring 2009 officers: Rachel Hughes, Vice-President; Don Tosh,
Corresponding Secretary.*

Meetings were held monthly. In January we installed 13 new members. In March Dr. Tosh and two students attended the national convention in Philadelphia. One of the students, Tony Fraticelli, presented a paper at the convention. His paper was selected as one of the top four at the convention. In April we had our semester social at the home of Don Tosh.

MS Alpha – Mississippi University for Women

*Chapter President – Dana Derrick; 13 Current Members, 1 New Member
Other spring 2009 officers: Stephanie Zegowitz, Vice-President; Stephanie
Zegowitz, Secretary; Dana Derrick, Treasurer; Dr. Shaochen Yang,
Corresponding Secretary.*

The Mississippi Alpha Chapter's initiation ceremony was held on Monday, April 6, 2009. We participated in the Multiple Sclerosis Walk on April 4, 2009 and raised \$1113. Several KME faculty and student members served as moderators, judges, score keepers, timers, and runners for the Mississippi Regional Science Bowl held on campus on February 13, 2009.

New initiate – Brandon Newsome.

MS Epsilon – Delta State University

Paula Norris, Corresponding Secretary.

New initiates – Ricardo Callender, Brittany Eikner, Sara Moore, Jenny Thorn.

NC Zeta – University of Nebraska at Kearney

Doug Brown, Corresponding Secretary.

*New initiates – Cody Ashby, Paul Baker, Doug Brown, Scott Campbell,
Cynthia Cook, John Hoehman, Jason Hunt, Stacey Michael, Cyndi
Osterhus, Zachary Owen, Sharon Sullivan, John Zerger.*

NE Beta – Catawba College

Dr. Katherine Kime, Corresponding Secretary.

New initiates – Riley Howsden, Brian Flannery, Valerie Sis, Jeremy Stromer, Tierra Webb, Aaron Steele.

NE Delta – Nebraska Wesleyan University

Cheryl Miner, Corresponding Secretary.

New initiates – Ana Burgers, Alicia Granger, Mitchell Hain, Brent McKain, Cole Meyer, Nathan Pribnow, Derek Werner.

NJ Beta – Montclair State University

John G. Stevens, Corresponding Secretary.

New initiate – Natlalia Sisti.

NJ Delta – Centenary College

Kathy Turrisi, Corresponding Secretary.

New initiates – Michael L. Ahrens, Lauren Culbert, Ismael Garcia III, Sean Hutchinson, Christine Merriman, Andrew Pancoast, Kristen A. Wirasnik, Penny J. Zitomer.

NM Alpha – University of New Mexico

Pedro F. Embid, Corresponding Secretary.

New initiates – John Clay Ash, Aniseh Bro, Cecil R. Compeau, Joan C. Dziejewski, Nathan E. Foust, Jesse S. Garrett, Nyssa T. Gilkey, Jeremy Guthrie, Kassandra A. Johnston, David Martin, Daniel J. McGraw, Jean Carlo Pech de Moraes, Brandon S. Runnels, Jonathan F. Torres, Lauren C. Wilson, Emily Cate Wisdom, Sze-Wing Wong.

NY Eta – Niagara University

Maritza M. Branker, Corresponding Secretary.

New initiates – Desmond M. Betteto, Rosalie Ferrari, Kimberly Garrison, Lori Virginia Ohanessian, Brittany L. Sumbler.

NY Iota – Wagner College

Zohreh Shahvar, Corresponding Secretary.

New initiates – Derek G. Blausner, Niya Aisha Branham, Michael Burke, Corina L. Casey, Kristen E. Costello, Michael V. Farina, Dina S. Hussam, Alexandra B. Johnston, Kara F. Lapitsky, Kenneth D. Lucante, Amber R. Mottola, Derek C. Rammelkamp, Kartrina Stec, Christina J. Sperrazza.

NY Kappa – Pace University

Lisa Fastenberg, Corresponding Secretary.

New initiates – Adam Crail, Ivanna Cheban, Jacki Munson, Ariel Sanzo, Andrew Williams.

NY Lambda – C.W. Post Campus of Long Island University

Chapter President – Kaitlin Egan

Other Spring 2009 Officers: Ashley Vaughan, Vice-President; Daniel Barone, Secretary; Jennifer Hanly, Treasurer; Dr. Andrew M. Rockett, Corresponding Secretary; and Dr. Geoffrey C. Berresford, Faculty Sponsor.

Twelve students were initiated by the chapter officers during our annual banquet at the Greenvale Town House restaurant on the evening of April 26th, bringing the chapter membership to 313. Dr. Katherine Hill-Miller, Dean of the College of Arts and Sciences, recognized the recipients of the departmental awards for 2008-2009: the Claire F. Adler Award to Peter Marini; the Lena Sharney Memorial Award to Angelica Atkins and Matthew Strong; the Joseph Panzeca Memorial Award to Christopher Levy and Allory Schroeder; and the Hubert B. Huntley Memorial Award to Jennifer Hanly. Michelle DiDomenico spoke on ““The Black-Scholes-Merton Model”” and included remarks on the research and writing of an honors thesis.

New initiates – Elyse Capozza, Kristine D’Antuono, Michelle DiDomenico, Jamie Earley, Jaclyn Heffel, J. Matthew Jordan, Peter Marini, Sonal Patel, David Phillips, Saima Qaseem, Richard Querni, Elliot Zomber.

NY Nu – Hartwick College

Chapter President – Melanie Hart; 22 Current Members, 14 New Members

Other Spring 2009 Officers: Dan Parisian, Vice President; Matthew Shoudy, Secretary; Kaitlin King, Treasurer; Ron Brzenk, Corr. Sec.

New initiates – Stephanie Abajian, Michelle Brault, Cheryl Boergesson, Amanda Cappelli, Prabhdeep Cheema, Annabelle Chu Yan Fui, Katherine Cikatz, Timothy Daigle, John Doolittle, Yi Ding, Paul Giokas, Zachary Hopkins, Grace Hurlbutt, Tracy Kirkman, Robert Klipp, Shawn Kowal, Stephen Lyons, Carol Potochney, Amber Tuchovsky.

OH Alpha – Bowling Green State University

Elmas Irmak, Corresponding Secretary.

New initiates – Yu Jin Chen, Leesa D. Corken, Sarah K. Garner, Luci K. Howard, Jiabin Lu, William M. Majcher, Yilan Xia.

OH Epsilon – Marietta College

Dr. John C. Tynan, Corresponding Secretary.

New initiates – Brandon Baylor, Dean Bendele, Frances Bolden, Jessica Brown, Scott Burnham, Michael Casto, Rebecca Collier, Cody Ray Hopkins, Amy Horvat, Marsha Hupp, Britt Meador, Bradford Moreland, Paul Paslay, Tyler Ragan, Madhuram Shah, Samuel Vincent.

OH Gamma – Baldwin-Wallace College

Dr. David Calvis, Corresponding Secretary.

New initiates – Megan M. Abbey, Kimberly J. Berry, Drew M. Binkley, Alexander S. Brikmanis, Josh P. Clevenger, Katelyn D. Fisher, Harry M. Fuller, Mario S. Giacomazzo, Grant W. Gretta, Daniel M. Hill, Chuck Imm, Kyle D. Ketchesin, Kirsten I. Larson, Chelsea C. Law, Benjamin M. Leo, Rui Li, Gretchen K. Pappas, Doug A. Placko, Adriana Popa, Lauren M. Rider, Kelsey E. Rife, Maria C. Shaniuk, Kevin E. Stoll, Kenneth W. Taylor, Rebecca A. Trinter, Kyle J. Ungur.

OH Zeta – Muskingum College

Dr. Richard Daquila, Corresponding Secretary.

New initiates – Anne Blood, Lindsey Lorenz, Tao Huang, Camelia Petre, Christopher Uchtman, Melissa Ufholz.

PA Alpha – Westminster College

Natacha Fontes-Merz, Corresponding Secretary.

New initiates – Karina-Mikayla Barcus, Jonathan Bodle, Stephen Donnel, Kurtis Gills, Courtney Herdt, Heather McQuiston, Megan Sherry, Aaron Zavora.

PA Eta – Grove City College

Dale L. McIntyre; Corresponding Secretary.

New initiates – Tyler Anderson, Lisa Baldwin, Samantha Clarke, Angelina Collins, Brianna Dillon, Kate Donatelli, Ian Gilbert, Jonathan Guglielmon, Peter Jantsch, Elizabeth Michael, Rebekah Newborn, Andra Reed, Courtney Sicher.

PA Gamma – Waynesburg College

James R. Bush, Corresponding Secretary.

New initiates – Jonathan Bowman, Kristy Gidley, Jennifer Kelly, Christopher Ian Lundholm, David Patton, Kevin Vietmeier, Marc Smith, Emily Patterson.

PA Kappa – Holy Family University

Chapter President – Michael Browning; 6 Current Members, 4 New Members

Other Spring 2009 Officers: Jacqueline Galelli, Vice President; Sabrina Luczyszyn, Secretary; Colleen Siemers, Treasurer; Sister Marcella Louise Wallowicz CSFN, Corresponding Secretary.

During Spring 2009, the honor society and the math club (the π gers) co-sponsored several activities on campus. PI-Day was celebrated over a two day span. On March 12 the annual pie-eating and Pi-recitation contests were held. Participation increased over the previous year. Katie Blumenstock, a math secondary education major, recited 70 digits, 20 more than the previous record set in 2007. On March 13, fifty local high school students participated in the annual high school mathematics competition. Certificates of participations were distributed to each high school student. The top school in each category (Geometry, Algebra, Calculus and Best Over-All) received engraved plaques. Following the competition refreshments (pizza pie and soda) were served. As part of Math Awareness activities, the first annual Sudoku competition was held. An electronic Sudoku game was awarded to the winner. Several bake sales were held during the semester to support the honor society's activities.

PA Lambda – Bloomsburg University of Pennsylvania

Elizabeth Mauch, Corresponding Secretary.

New initiates – Ryan Evans, Scott Kiedeisch, Stacey Lukasik, Xianrui Meng, Adam Stockinger, Emily Young.

PA Mu – Saint Francis University

*Chapter President – Tim Gaborek; 43 Current Members, 13 New Members
Other Spring 2009 Officers: Kurt Hoffman, Vice President; Abigale May, Secretary; Kaitlyn Snyder, Treasurer; Peter Skoner, Corresponding Secretary; Katherine Remillard, Faculty Sponsor.*

The Pennsylvania Mu Chapter of held induction ceremonies on Wednesday, February 18, 2009 in the Christian Hall Conference Room at Saint Francis University. A dinner preceded the actual initiation ceremony for the thirteen new members including Caitlin Basile, Christopher Behre, Rachel Capizzi, Kaitlin Hensal, Darci Jones, Alyshia Lacey, Justin Long, Ashley Lucas, Patrick McManamon, Dr. Pedro Muino, Kurt Sedei, Colin Trout, and Michelle Wetzel. The ceremony was led by KME President Tim Gaborek, Treasurer Kaitlyn Snyder, member Ellie Pecharka, member Fr. Joseph Chancler, and corresponding secretary Dr. Peter Skoner. The Pennsylvania Mu chapter at Saint Francis University was founded in 1979 and the present membership stands at 288. Congratulations to all the new members!

A total of 96 high school students and 9 teachers from 8 area high schools attended the Ninth Annual Math Day sponsored by the Laurel Highlands Mathematics Alliance and held Thursday, March 12, 2009 at Saint Francis University. Each student had the chance during the day to attend two presentations, participate in a Who Wants to Look Like a Millionaire (In a New Tee Shirt) quiz contest, and complete other mathematics challenges.

Ten KME members participated in Try Math A Lot, a day of mathematics challenges for sixth and seventh grade students. It was held at the University of Pittsburgh at Johnstown on Wednesday, April 29, 2009. The Saint Francis University contingent coordinated a mathematics quiz bowl, one of the three activities that the more than 300 students from about 25 schools rotated through.

PA Nu – Ursinus College

Jeffrey Nelson, Corresponding Secretary.

New initiates – Jennifer Bayzick, Leah Hoover, Matthew Hummel, Kristen Much, Elysia Price, Justin Sachs, Matthew Stehman.

PA Pi – Slippery Rock University

Chapter President – Nathan Leidwing; 17 Current Members, 6 New Members

Other Spring 2009 Officers: Samantha Corvino, Vice President; Elise Grabner, Corresponding Secretary; Dr. Richard Marchand and Dr. Rachelle Bouchat, Faculty Sponsors.

PA Rho – Thiel College

Chapter President – David Wierzchowski; 15 Current Members, 12 New Members

Other Spring 2009 Officers: Krista Wissenbach, Vice President; Adam Troup, Secretary; Devin Todd, Treasurer; Max Shellenbarger, Corresponding Secretary; Faculty Sponsor, Dr. Jie Wu.

PA Sigma – Lycoming College

Chapter President – Steven Brown; 22 Current Members, 9 New Members

Other Spring 2009 Officers: Jake Crosetto, Vice President; Meg Bittle, Secretary; Christopher Dahlheimer, Treasurer; Santu de Silva, Corresponding Secretary; Dr. Eileen Peluso, Faculty Sponsor.

Except for the induction of new members and Math Awareness Day (around March 14), no significant activities were conducted.

New initiates – Jessica Bugno, Margaret Bittle, Steven Brown, Jake Crosetto, Christopher Dougherty, Christiana Karr, Professor Geoffrey Knauth, Mark McDonough, Eric Smithgall.

PA Theta – Susquehanna University

Lisa Orloff Clark, Corresponding Secretary.

New initiates – Alicia Decker, Stacy Hartman, Colin Healy, Gregory Trout, Ryan Zaharuk.

PA Xi – Cedar Crest College

Patrick M Ratchford, Corresponding Secretary.

New initiates – Kristin Allard, Roxanne Holowienka, Rachael Roeckel, Rebecca Tutino.

SC Gamma – Winthrop University

Dr. Trent Kull, Corresponding Secretary.

New initiates – Janet Marie Custer, Lindsey Carron Mann.

TN Delta – Carson-Newman College

Chapter President – Gretchen Hill; 308 Current Members, 8 New Members

Other Spring 2009 Officers: Luke Morton, Vice President; Andrew Hansen, Secretary; Kenneth Massey, Treasurer/Corresponding Secretary.

C-N chapter KME faculty, students, alumni, and retirees enjoyed our Spring initiation picnic at Cherokee Dam, followed by the traditional student-faculty lacrosse game. All of our members have nicknames, which were immortalized on a department t-shirt.

TN Gamma – Union University

Bryan Dawson, Corresponding Secretary.

New initiates – Joshua J. Armacost, Peter Boedeker, Rebecca Eaton, Richard Kyle Harris, Kayla M. Hauss, Emilie Huffman, James Jones, Seth Kincaid, Lauren Medforth, Carrie Moore, Mikias M. Seid, Jacob White.

TX Alpha – Texas Tech University

Chapter President – Stephanie Krol; 1,926 (since 1941) Current Members, 24 New Members

Other spring 2009 officers: Ryan Kostohryz, Vice President; Julia Head, Secretary and Lecturer; Department of Mathematics and Statistics Administrative Office, Treasurer; Dr. Magdalena Toda, Corresponding Secretary.

New initiates – Rami Alrayes, Matthew Barnes, Austin Beam, Mark Broadway, Daniel English, Hiron Fernando, Spencer Gill, Kyle Grunwald, William Hunter, Bryan Judd, Ryan Kostohryz, Stephanie Krol, Nick Lehman, Philip Oliver, Garren Opdenhoff, Yaseer Parupia, Katherine Priebe, Natalya Read, Craig Rhyne, Elizabeth Scheers, Scott Schroer, Carlos Siri, Mark Taylor, Colin Watts.

TX Gamma – Texas Woman’s University

Chapter President – Katie Schniebs; 70 Current Members, 6 New Members

Other spring 2009 officers: Dani Silva, Vice President; Abby Peters, Secretary; Rebeka Bennett, Treasurer; Mark Hamner, Corresponding Secretary.

New initiates – Norma Barrington, Meredith Bratton, Meredith Corley, Karina Gutierrez, Margarita Perez, Danielle Silva.

TX Kappa – University of Mary Hardin-Baylor

Chapter President – Kellie Thomas; 15 Current Members, 7 New Members

Other spring 2009 officers: Mattie Billington, Vice President; Christi D’Herde, Secretary;

Dr. Peter H. Chen, Corresponding Secretary; Mr. Maxwell M. Hart, Faculty Sponsor.

New initiates – Neil Bealka, Mattie Billington, Christi D’Herde, Seth Fredrickson, Ashli Lawson, Vanessa Smith, Kellie Thomas.

TX Lambda – Trinity University

Diane Sapphire, Corresponding Secretary.

New initiates – Timothy Amen, Cabral Balreira, Sonia Brady, Christopher Castillo, Robert Connop, Ryan Daileda, Peter Garatoni, Robert Harl, Andrea Katz, Thomas McDonald, Brian Miceli, Peter Olofsson, Laney Redus, Emma Lea Treadway.

TX Mu – Schreiner University

Chapter President – Ashley Moore; 20 Current Members, 7 New Members

Other spring 2009 officers: Amy Vickers, Vice President; Stephen Franklin, Secretary; Leigh Ann Brown, Treasurer; William Sliva, Corresponding Secretary.

New initiates – Denise E. Begley, Rebecca Gail Burnham, James R. Heikkenen, Matthew R. Jepson, Matthew J. Moreno, William A. Rhodes, Steven J. Spiegelhoff.

VA Alpha – Virginia State University

Dr. Vidya S. Bakhshi, Corresponding Secretary.

New initiates – Brad Bailey, Maurice Brown, Denitra Flowers, Reneisha Hill, Alexandria Jordon, David Randall Montgomery, Keyouna Pierce, William A. Starr, Melissa Watts, Mervin Woodlin.

VA Gamma – Liberty University

Dr. Tim Van Voorhis, Corresponding Secretary.

New initiates – Felicia Adams, Andrea Beckles, Dr. James Cook, Joseph Endicott, Joseph Fiori, Jeffrey Guenther, Coleman Hall, Chadwick McClure, Amanda McMillian, Melissa Rider, Rachel Smith, Amande Vander Klay, Justin Wallace.

WI Gamma – University of Wisconsin-Eau Claire

Chapter President – Kaitlyn Hellenbrand; 76 Current Members, 18 New Members

Other spring 2009 officers: Chelsey Drohman, Vice President; Alyssa Frey, Secretary; Amaris Lieske, Treasurer; Simei Tong, Corresponding Secretary.

Three students' research posters from UWEC were displayed at the 2009 Joint Mathematics Meetings in Washington D.C. Mitch Phillipson's poster "Extensions and Deformations of Associative Algebra" received outstanding student poster award. Two students from UWEC presented their research papers at the 2009 MathFest. Kaitlyn Hellenbrand's paper "Polynomial Equations over Matrices" received outstanding student presentation award.

WV Alpha – Bethany College

Dr. Mary Ellen Komorowski, Corresponding Secretary.

New initiates – Jonas Ankrom, Darcy A. Brand, Susannah J. Cartland, Logan W. Kowcheck, Amanda M. Law, ILIA Liskooka, Patrick R. Miller, Jacob D. McConnell, Kelly C. Sofka.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 March 1935
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961

NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987

PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February 2001
GA Delta	Berry College, Mount Berry	21 April 2001
TX Mu	Schreiner University, Kerrville	28 April 2001
NJ Gamma	Monmouth University	21 April 2002
CA Epsilon	California Baptist University, Riverside	21 April 2003
PA Rho	Thiel College, Greenville	13 February 2004
VA Delta	Marymount University, Arlington	26 March 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 February 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 March 2005
SC Epsilon	Francis Marion University, Florence	18 March 2005
PA Sigma	Lycoming College, Williamsport	1 April 2005
MO Nu	Columbia College, Columbia	29 April 2005
MD Epsilon	Stevenson University, Stevenson	3 December 2005
NJ Delta	Centenary College, Hackettstown	1 December 2006
NY Pi	Mount Saint Mary College, Newburgh	20 March 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 April 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 October 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 March 2008
CA Zeta	Simpson University, Redding	4 April 2009
NY Rho	Molloy College, Rockville Center	21 April, 2009