THE PENTAGON

*A Mathematics Magazine for Students*

Volume 70 Number 2 Spring 2011

# Contents




Typeset in Scientific WorkPlace. Printed in the United States of America.

# *Kappa Mu Epsilon National Officers*

Ron Wasserstein *President*

American Statistical Association
732 N Washington Street
Alexandria, VA 22314-1943
ron@amstat.org

Rhonda McKee *President-Elect*

Department of Mathematics
University of Central Missouri
Warrensburg, MO 64093-5045
mckee@ucmo.edu

Mark Hamner *Secretary*

Department of Mathematics and Computer Science
Texas Woman's University
Denton, TX 76204
mhamner@twu.edu

Cynthia Woodburn *Treasurer*

Department of Mathematics
Pittsburg State University
Pittsburg, KS 66762-7502
cwoodbur@pittstate.edu

Peter Skoner *Historian*

Department of Mathematics
Saint Francis University
Loretto, PA 15940
pskoner@francis.edu

Kevin Reed *Webmaster*

Department of Science and Technology
Evangel University
1111 N. Glenstone Avenue
Springfield, MO 65802

KME National Website:

http://www.kappamuepsilon.org/

# *The Peg Game*

Christine Potter, *student*

KS Delta

Washburn University
Topeka, KS 66621

Presented at the 2009 National Convention and awarded "top four" status by the Awards Committee.

## 1. Introduction

Through this project, I explored the well-known triangular peg game in which a player jumps pegs on the board until arriving at the desired outcome of one peg left on the board. I was able to analyze solutions of the game using modular arithmetic and group theory. In addition to exploring this triangular board, I was able to draw interesting conclusions about triangular boards of other sizes as well as linear boards.

## 2. Playing a Peg Game

The peg game is a single-player game. It is played on a board that contains a fixed number of holes. To begin the game, a player starts with one arbitrary hole empty and one peg in each of the remaining positions. To play the game, the player makes moves by jumping a peg over an adjacent peg. During a move, the jumped peg is removed from the board altogether so that the player is left with one less peg total. A formal definition of “move” is given below. The game is won by making whatever moves necessary to obtain exactly one peg left on the board.

The most common form of peg game is a triangular shape with fifteen holes. In this paper, we’ll explore not only this game, but linear games and different sizes of triangular games. The following definition and theorem holds for these forms.

**Definition:** A move, or a jump takes place amongst three consecutive positions, $r$, $s$, and $t$, that are either all in a row or all on a diagonal (for triangular shapes) with two pegs, $P_1$ and $P_2$, in adjacent positions, and the remaining position empty. Without loss of generality, given that $r$ contains peg $P_1$ and $s$ contains $P_2$, a jump consists of the following occurrences:

1. transfer of peg $P_1$ from position $r$ to position $t$, and
2. removal of peg $P_2$ from the board entirely, leaving holes in positions $r$ and $s$.

Note that from this definition we can make the statement that a jump results in one less peg (and thus, one additional empty hole) on the board. Also, consider the following theorem.

**Theorem 1** *Last Peg Theorem (LPT): In order for a peg $P_L$ to be the last peg left in the game, it must make the final jump into position $h$, the winning hole.*

**Proof:** Assume that $P_L$ does not make a jump into position $h$. Then, $P_L$ is already in position $h$. Therefore, there exists a peg other than $P_L$ that must make the final jump of the board, say $P_0$. But, by definition of a jump, there is a transfer of $P_0$ from position $r$ to position $t$. Thus, $P_0$ is still left on the board. This is a contradiction to $P_L$ being the last peg left in the game. Hence, the theorem holds. ■

## 3. The Linear Peg Game

It is helpful now to look at trends and patterns that occur when using a simple linear peg game, and to establish some rules. We start by labeling the positions of the board with n numbers: 1 2 3 4 5 6 7 8 9. . . n. Note that jumps in a linear game differ only slightly from those in the triangular game, the difference being the omission of the diagonal move.

Let us look at linear boards of different sizes. If $n = 1$ or $n = 2$, clearly there are no jumps possible, so these cases are not interesting to us. Now, let $n = 3$. Then two positions must contain a peg and one position must be empty. By definition of a move, we need exactly two consecutive pegs and one empty hole in order to have any moves available. Hence a board with configuration peg-hole-peg will result in a loss. If instead, we have either the configuration peg-peg-hole or hole-peg-peg (See Figure 1 below), we make the only possible jump in either case and we win.


Figure 1

**Theorem 2** *For $n > 3$, $n$ being the number of positions on the board (or length of the board), the linear peg game beginning with a hole in the first position or a hole in the last position has no solution.*

**Proof:** Let $L$ be a linear peg board of length $n > 3$. Without loss of generality, let the first position of $L$ be the beginning hole (for a game in the last position, simply consider the reflection of this game). So we know that the peg in position 3 must jump the peg in position 2 and land in position 1. But after this jump occurs, there are two consecutive holes next to position 1. Hence, no peg will ever be able to make a jump to occupy position 2, and thus the peg in position 1 will not be able to jump another peg. Therefore, by LPT, we lose and Theorem 2 holds. Figure 2 below illustrates this. ■


Figure 2

Let $n = 4$. Theorem 2 tells us that in order to have a chance at winning, we must start with a hole in position 2 or 3. Figure 3 below shows the winning solution. Note also that a hole in position 3 indicates merely a reflection of the situation illustrated below.

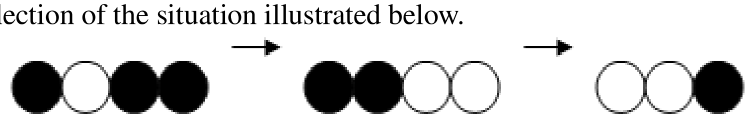


Figure 3

For $n = 5$, we arrive at an interesting result given in Theorem 3.

**Theorem 3** *There is no solution for linear peg game of length $n = 5$.*

**Proof:** Let $n = 5$. Then four positions must contain a peg and one position must be empty. By Theorem 2, we may not begin with a hole in position 1 or 5. If we start with a hole in position 2, the only jump possible leaves a peg in position 5 that is now unreachable. Thus, the hole may not start in position 2 and similarly, by symmetry, the hole cannot start in position 4. This is illustrated in the figure below.


Figure 4

If we start with the hole in position 3, we may move the peg from either position 1 or 5 into position 3. But in either case, we are left with two consecutive holes at an end of the board. In this case, we can view the resulting board as a single hole in an end position adjacent to a board of $n = 4$. Now, by Theorem 2, we have no solution to the 4 position portion of the board. Thus, there is no solution for the linear game of board length $n = 5$. Figures 5 below illustrates this example. ■


Figure 5

## 4. Using Z2 to Label the Board

We wish to label the board in a way that will enable us to eliminate and analyze possible solutions to the game. Using a process as in [3], we used the group $Z_2$. The group $Z_2$ contains the elements $\{0, 1\}$ and uses addition modulo 2 for its binary operation. The addition table for the group is given in Table 1. Note that the only difference with the table and regular addition is that $1 + 1 = 0$ here.

| $(+)$ | $0$ | $1$ |
|---|---|---|
| $0$ | $0$ | $1$ |
| $1$ | $1$ | $0$ |

Table 1

Since the peg game is played by making a sequence of jumps, we begin by analyzing a single jump. Note that each jump involves three consecutive positions on the board. Thus, we examine the ways that we can label three positions using $Z_2$. These possible labelings are illustrated in Table 2.

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

Table 2

Without loss of generality, assume the initial empty hole is represented by the last column in the above chart. Consider the parity of the sum of the positions that contain pegs before and after a jump (we will refer to this as the parity of the board.) Thus, before the jump we sum the first two positions. After the jump, there is only a single peg remaining in the third position. Thus, the parity after the jump will simply be the value of the third position. Table 3 shows the parity before and after a jump using all possible labelings with $Z_2$.

| $Z_2$ labeling of three consecutive positions | | | Parity before jump | Parity after jump |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

Table 3

Notice that in Table 3, the parity before and after a jump is the same with four of the labelings; namely, when the positions in a jump are labeled as 000, 011, 101, and 110. Therefore, we wish to label the board so that each jump has one of these four labelings. This will ensure that parity will stay the same with every jump that occurs.

We can quickly discard the first case, 000, because that is a useless way to label the board since each 0 is indistinguishable. Now, by noting that each of the following three cases contains two 1's and one 0, we can label the board while maintaining the parity of the board by representing each possible jump with two 1's and one 0. The following illustration (Figure 6) uses this labeling. We arbitrarily start with a zero in the top left corner of the triangle. This forces us to label the two adjacent positions with a 1 since the first three positions can be used in a jump. Following a similar technique, the entire board then must be labeled as follows in Figure 6:

```
0   1   1   0   1
  1   0   1   1
    1   1   0
      0   1
        1
```

Figure 6

This labeling provides an interesting insight into the possible solutions of the game, as noted in the following theorem.

**Theorem 4** *Given triangular peg board labeled as in Figure 6, if the empty hole begins at a 1, the last peg in any winning game must be in a position labeled as a 1. Similarly if the empty hole begins at a 0, the last peg in any winning game must be in a position labeled as a 0.*

**Proof:** If we start with an empty hole in any of the positions labeled with a 1, there are then nine holes labeled with a 1 that contain a peg. Since there are an odd number of 1's that contain pegs, the parity of the board before any jumps are made is 1. Since the parity of the board must be maintained with any jump, we know that if the game is won, the ending peg will be in a position labeled with a 1. Similarly, if we start with an empty hole in any one of the positions labeled with a 0, all ten of the positions labeled as 1 contain pegs. Since there are then an even number of 1's that contain pegs, the initial parity of the board is 0. Thus, the ending peg must be in a position labeled as 0. ■

Finally, we wish to ensure that the illustration in Figure 6 accounts for all possible labelings that maintain parity through jumps. We arbitrarily began labeling the board by using 011 in the upper left hand portion of the board as is done in A of Figure 7 below. If we use the other two possibilities of 101 and 110 in the upper left hand portion, we obtain the labelings B and C in Figure 7. Note that as with the labeling for A, once the three positions in the upper left hand corner are chosen, the remaining labels are dictated by the requirement of maintaining parity through jumps. It appears, then, that we have three distinct labelings for the board that will maintain parity through jumps. However, let us note the symmetry that occurs amongstthelabelings. B is a $120°$ - degree clockwise rotation of A, and C is a $240°$ - degree clockwise rotation of A. Thus, our initial labeling is sufficient for all labelings that maintain parity.

```
0 1 1 0 1          1 0 1 1 0          1 1 0 1 1
 1 0 1 1            1 1 0 1            0 1 1 0
  1 1 0              0 1 1              1 0 1
   0 1      A         1 0      B         1 1      C
    1                  1                  0
```

Figure 7

## 5. Labeling the Board with the Klein 4-Group

Now we wish to use another labeling that will distinguish more of the positions of the board. Since the group $Z_2$ was used previously to label the board, we now consider the product $Z_2 \times Z_2$, which itself forms a group [4, p.27]. This group is called the Klein 4-group, denoted $K_4$ [2, p. 132]. For convenience, we can label these elements as $x$, $y$, $z$, 0, where 0 represents the identity element. Using "+" for the binary operation, the table for the Klein 4-group is then:

| $+$ | 0 | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| 0 | 0 | $x$ | $y$ | $z$ |
| $x$ | $x$ | 0 | $z$ | $y$ |
| $y$ | $y$ | $z$ | 0 | $x$ |
| $z$ | $z$ | $y$ | $x$ | 0 |

Table 4

Now, the question of relevance is whether we can use $K_4$ to label the peg game board. Again, the key to labeling the board with the $K_4$ group is to maintain the parity with every jump. As we did with $Z_2$, we could list out the possible combinations for labeling three positions in a jump with the elements of $K_4$ and examine the parity before and after a jump. However, there are 43 ways to do this, so instead, note each element of the group is its own inverse. Also, when we make a jump, we adjust the parity by "subtracting" (i.e. adding the inverse) the value of the positions that had pegs removed and adding the value of the position that the peg landed in. Further, note that $x + y + z = 0$. Thus, labeling the board with an $x$, $y$, and $z$ for each possible jump will maintain the parity. The following, Figure 8, shows this labeling of the board. A similar technique is used in An Application of Elementary Group Theory to Central Solitaire [1].

z y x z y

x z y x

y x z

z y

x

Figure 8

Now, just as we asked of the $Z_2$ labeling, we ask, what does this $K_4$ labeling of the board have to offer us in terms of solutions? Again, the solutions are narrowed down by this labeling, and in fact, even more so than in the $Z_2$ labeling of the board. This time, if we start with no peg in a $z$ position, the parity of the board is $z$. Thus, we know that we must end with a peg in an $z$ position. This result is not surprising since the $z$'s in Figure 8 are in the same position as the 0's in Figure 6. However, it is also true that if we begin with no peg in a $y$ position, our last peg must end in a $y$ position, and if we begin with no peg in a $x$ position, our last peg must end in an $x$ position. The $y$'s and $x$'s in Figure 8 are in the same positions as the 1's in Figure 6. Thus, the Klein 4-group distinguishes the 1's in $Z_2$ with $x$'s and $y$'s. And thus, Theorem 5 naturally follows.

**Theorem 5** *Given triangular peg board labeled with $K_4$ so that parity is maintained with jumps (as in Figure 8 for a board with five rows), if the empty hole begins at an $z$, the last peg in any winning game must be in a position $z$, and similarly for $y$ and $x$.*

One should note that these findings hold true with the example of the winning game shown in the appendix. In the example, the beginning hole starts in a position labeled as an $x$, and therefore, the peg left at the end of the game resides in a hole labeled with an $x$.

## 6. Other Sizes of Triangular Peg Games

So far, we have analyzed triangular boards with 5 rows. Now we will take a look at other sizes of triangular boards. Throughout this discussion, let $N$ represent the number of rows on the triangular board.

Let $N = 3$. Figure 9 illustrates this picture with a $K_4$ labeling.

y x z

z y

x

Figure 9

Theorem 5 tells us that if we start with a hole at $x$, the last peg must be in an $x$. Similarly, this is true with $y$ and $z$. By definition of a move, however, we know that in order to have any moves we must begin with the empty hole at a corner of the triangle. Also, by LPT (Last Peg Theorem), we know that we can only end at the endpoints of this triangle as well. By exhaustion, we see that there are exactly three forced moves to this game.

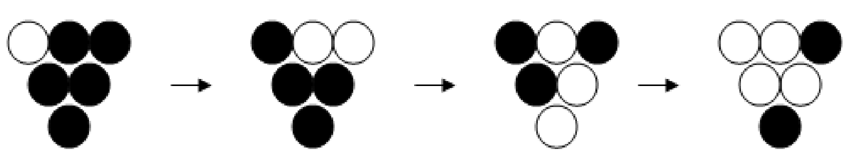


Figure 10

But, these moves do not give us a win. Hence, we have:

**Theorem 6** *There is no solution to the triangular board size $N = 3$.*

Let $N = 4$. Figure 11 illustrates this picture, again with a $K_4$ labeling.

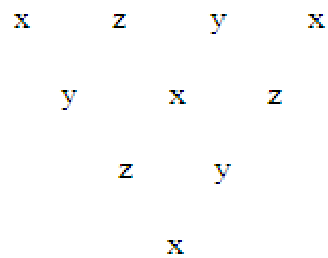



Figure 11

Here, Theorem 5 is sufficient to tell us that there is no solution to this board in some cases. If we start with a hole at $x$, then there are pegs in the three other $x$'s and there are pegs in each of the three $y$'s and $z$'s. Since $x + y + z = 0$, the beginning parity of the game is 0. This means that in order to win the game, the winning peg must land in a position labeled with a 0. But there are no 0's on the board so it is impossible to win. But, a more general claim may be made.

**Theorem 7** *If the number of rows of the triangular board is $N = 3m+1$ and if the board is labeled using $K_4$ so that the upper left corner of the board is an $x$, there is no solution if the beginning hole is in a position labeled with an $x$.*

**Proof:** Note that the total number of positions on a board with $N$ rows is $\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$. We first show that if $N = 3m + 1$, then there exists an integer $j$ such that $\frac{N(N+1)}{2} = 3j + 1$. Since $N = 3m + 1$, we have $\frac{N(N+1)}{2} = \frac{(3m+1)(3m+2)}{2} = \frac{9m^2 + 9m + 2}{2} = \frac{9m(m+1)}{2} + \frac{2}{2}$.
Note that $m(m + 1)$ is even, so $\frac{m(m+1)}{2}$ is an integer. Let

$$j = 3\left(\frac{m(m+1)}{2}\right).$$

Then $\dfrac{9m\,(m+1)}{2} = 9k = 3\,(3k) = 3j$. Therefore, $\dfrac{N\,(N+1)}{2} = 3j+1$. Now note that by the way the board is labeled, if we begin with an $x$ in the upper left position, there must be $j$ number of $y$'s, $j$ number of $z$'s, and $(j+1)$ number of $x$'s on the board. If the beginning hole is in an $x$, the beginning parity is 0. But this means the final peg must land in a position labeled 0 and there are no positions labeled as such. ■

## 7. Summary

It should be noted that most of the analysis in this paper is not new. However, in my explorations of the game, I purposely did not look at the existing literature so that I could try to formulate my own conclusions through a process of self-discovery. Further, I was unable to find any literature on the linear game or on the triangular boards of other sizes; therefore, I believe my results in these areas are new.

The interested reader should consult Bialostocki's article found in [1] for an exploration of a cross-shaped peg game. Also, the Appendix gives one possible solution to the standard five-row triangular peg game.

## 8. Appendix

It is important to note here that we start this example with a hole in a position labeled $x$ in Figure 8 and end with a peg in a position labeled $x$ in Figure 8.

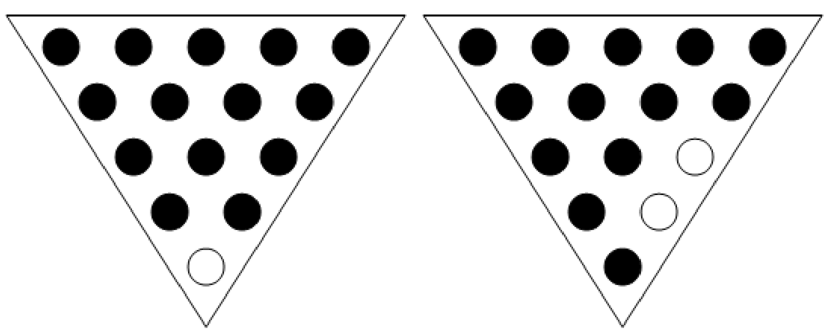


Figure 12

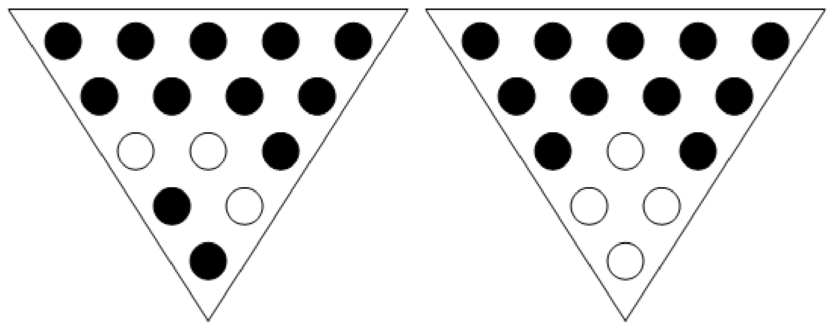


Figure 13

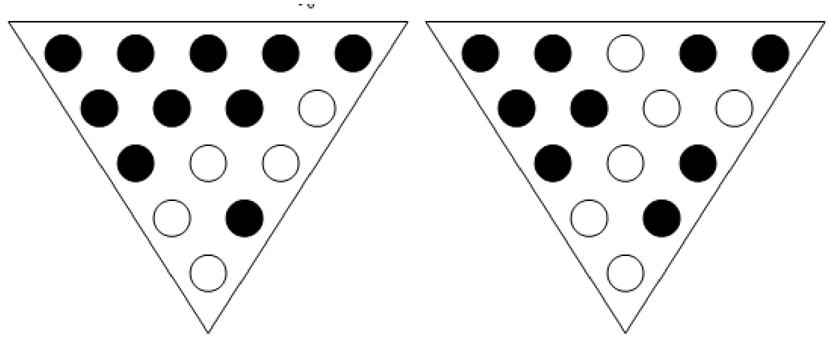


Figure 14

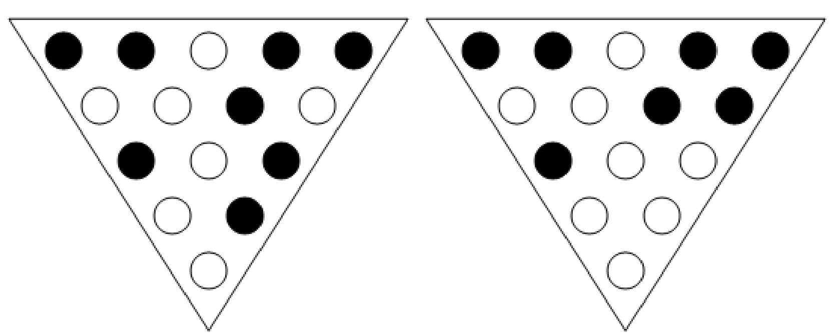


Figure 15

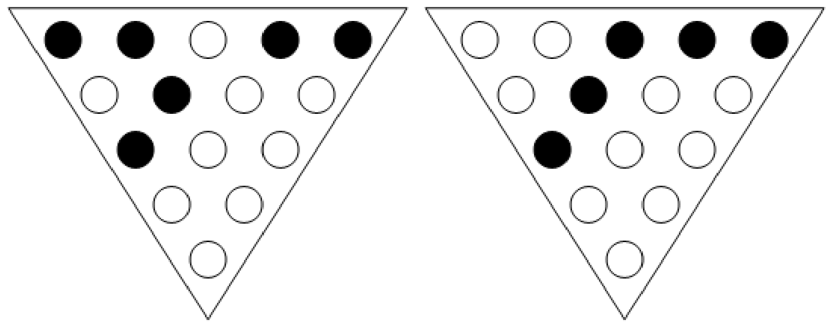


Figure 16

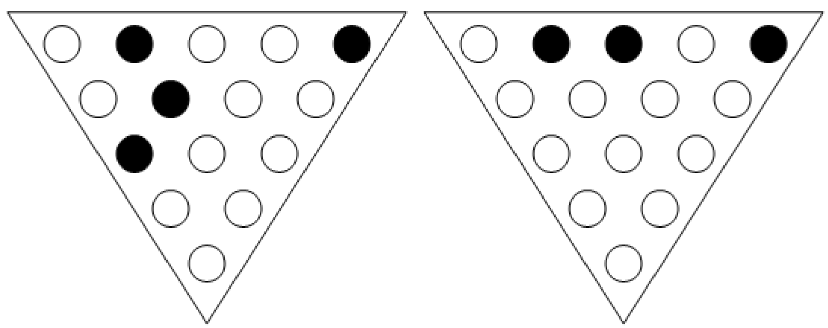


Figure 17

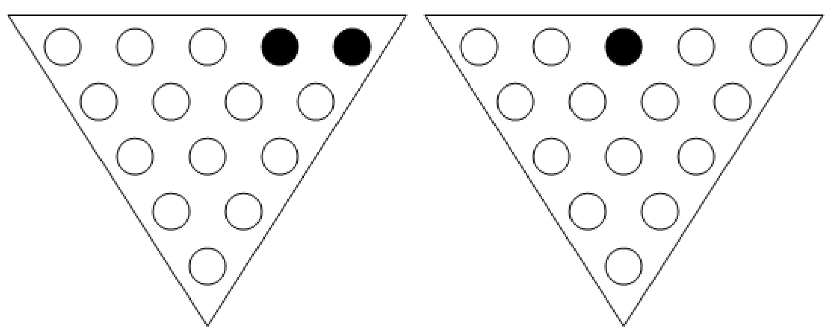


Figure 18

*Acknowledgements.* I would first of all like to acknowledge my advisor and professor, Dr. Sarah Cook for all of her help during the writing of this paper. Without her, I would not have had the background and encouragement to discover my findings. I would also like to thank the entire mathematics department of Washburn University through whose instruction and guidance I was able to reach this point in my mathematical endeavors. I would like to specifically acknowledge Dr. Gaspar Porta, the instructor of the class through which this research was done. Without his instruction, I would not have had the courage to explore mathematics in this way. Finally, I want to thank my husband and three daughters for all of their support and patience with me during my long hours of research and exploration.

## ***References***

[1] A. Bialostocki, An Application of Elementary Group Theory to Central Solitaire, *College Math. J.* **29** (1998).

[2] D. Burton, (1998). *Abstract Algebra*, Wm. C. Brown Publishers, 1988.

[3] S. Cook, Grandpa's Triangle, *J. of Recreational Math.* **31** (2003) 280-283.

[4] L. Grove, *Algebra*. Academic Press, Inc., 1983.

# *Phi Patterns in Nature and Beyond*

Leigh Johnson, Donna Marie Pirich, Heather O'Connor, & Theresa Sampson

NY Omicron

St. Joseph's College
Patchogue, NY 11772

Presented at the 2009 National Convention.

**Abstract**

In his paper, "The Distance of the Planets from the Sun and Their Atmospheric Composition," Charles William Johnson postulates the existence of a Phi pattern in planetary orbits. The conjecture hinges upon the inclusion of Ceres as a dwarf planet. The author claims this inclusion is necessary in order to properly represent the asteroid belt between Mars and Jupiter, but fails to give a valid mathematical proof. We, the authors of this paper, investigate the validity of Johnson's work, and offer a mathematical proof based on regression analysis. Furthermore, we apply the same analysis to the lunar orbits of Neptune, Uranus, and Saturn, as well as the rings of Uranus. We believe this data analysis technique can also be used to predict the location of undiscovered moons in our solar system, as well as planets beyond Pluto.

## 1. Introduction

The existence of Phi patterns in nature has been a topic of great interest for mathematicians, beginning with Leonardo of Pisa (1170 – 1250), who first popularized the idea in a problem that he posed involving the growth of a hypothetical rabbit population (Burton 289-294). Phi is related to the Fibonacci sequence, $\{F_n\}$, where $F_1 = 1$, $F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$

for $n \geq 1$, and is defined as $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$ (Bicknell-Johnson). Hence, the Fibonacci numbers are:

$$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 \ldots\},$$

and Phi is an irrational number, which can be aproximated by $1.61803$, correct to five decimal places. The figure below illustrates the rapid convergence of the sequence.

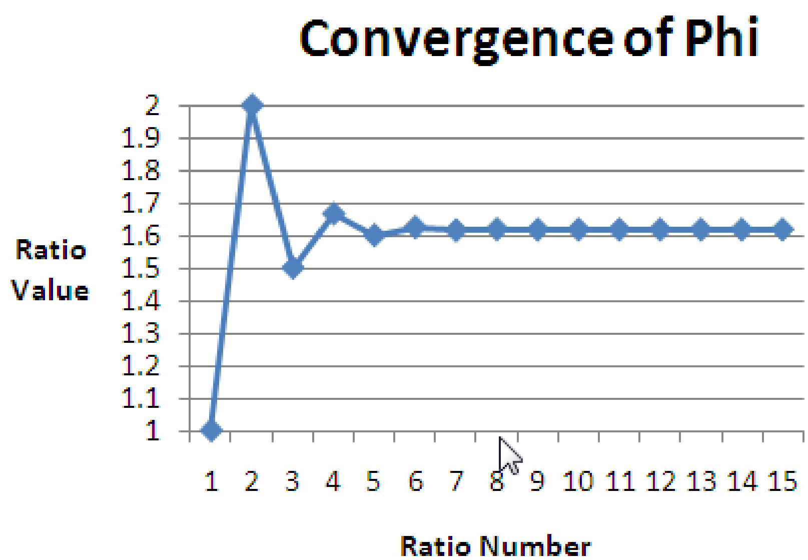



Figure 1

The Fibonacci sequence and the related Phi pattern have been observed throughout nature. The sprouting of new shoots during the growth process of various plants parallels the growth of Fibonacci's hypothetical rabbit population. This pattern can also be observed in the petals and seed heads of certain flowers, as well as in the spiral growth pattern of pinecones and nautilus shells (Knott).

For example, if one were to count the counterclockwise spirals created by the seeds of a sunflower, the number would be a Fibonacci number (Knott). Moreover, the number of clockwise spirals would be the previous Fibonacci number, and hence the ratio of these numbers is an approximation of Phi (Knott). As the sunflower grows, the approximation improves! The same phenomenon is also observed in pinecones (Knott).

## 2. Analysis of Planetary Data

The spiral pattern associated with Phi can also be observed in space. For example, the Milky Way is classified as a spiral galaxy (Morison and Penston 37). It appears to have a spiral pattern that resembles the growth pattern of pinecones, sunflowers, and nautilus shells (Harris). Therefore, the question arises as to whether a Phi pattern can be observed in our solar system.

Charles William Johnson postulates the existence of such a pattern in "The Distance of the Planets from the Sun and their Atmospheric Composition." The conjecture hinges upon the inclusion of Ceres as a dwarf planet. Johnson claims this inclusion is necessary in order to properly represent the asteroid belt between Mars and Jupiter, but fails to give a valid mathematical proof. Our research began with the development of such a proof, based on regression analysis. The result was a data analysis technique which we then applied to the lunar orbits of Neptune, Uranus, and Saturn, as well as the rings of Uranus.

We first analyzed the ratios of the distances from the sun of successive planets (normalized to Mercury), without including data on Ceres (see Table 1 and Figure 2). Using linear regression, with one-sigma error bars, we found Jupiter to be an outlier. (Note that since multiplication is commutative, dividing the distances in Column 2 of Table 1 by Mercury's distance from the sun, and then calculating successive ratios of distances, is equivalent to setting the first ratio in Column 3 to one, and calculating the remaining ratios directly from the planetary distances.)

| Planet | Distance from Sun (km) | Ratio |
|---|---|---|
| Mercury | 57,900,000 | 1 |
| Venus | 108,200,000 | 1.868739 |
| Earth | 149,600,000 | 1.382625 |
| Mars | 227,900,000 | 1.523396 |
| Jupiter | 778,600,000 | 3.416411 |
| Saturn | 1,433,500,000 | 1.841125 |
| Uranus | 2,872,000,000 | 2.003488 |
| Neptune | 4,485,100,000 | 1.561664 |
| Pluto | 5,870,000,000 | 1.308778 |
| | Mean | 1.767358 |
| | Std. Dev. | 0.653183 |

Table 1

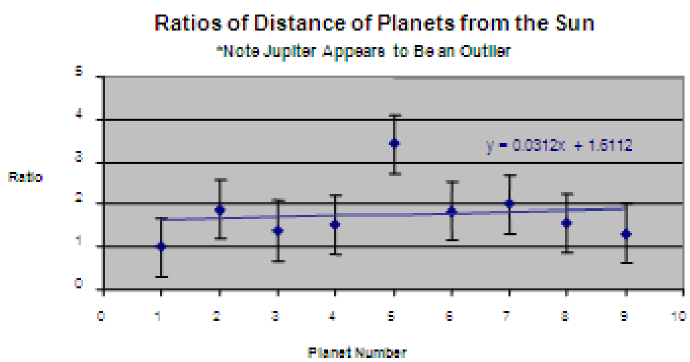



Figure 2

Using the linear regression equation established above,

$$y = .0312x + 1.6112,$$

we predict the location of a "missing planet" between Jupiter and Mars, recalculate the ratios of the distances of the planets from the Sun (normalized to Mercury), and finally establish a new regression line (see Table 2 and Figure 3).

| Planet | Distance from Sun (km) | Ratio |
|---|---|---|
| Mercury | $57,900,000$ | $1$ |
| Venus | $108,200,000$ | $1.868739$ |
| Earth | $149,600,000$ | $1.382625$ |
| Mars | $227,900,000$ | $1.523396$ |
| Estimate | $402,744,880$ | $1.7672$ |
| Jupiter | $778,600,000$ | $1.933234$ |
| Saturn | $1,433,500,000$ | $1.841125$ |
| Uranus | $2,872,000,000$ | $2.003488$ |
| Neptune | $4,485,100,000$ | $1.561664$ |
| Pluto | $5,870,000,000$ | $1.308778$ |
| | | |
| | Mean | $1.619025$ |
| | Std. Dev. | $0.3211$ |

Table 2

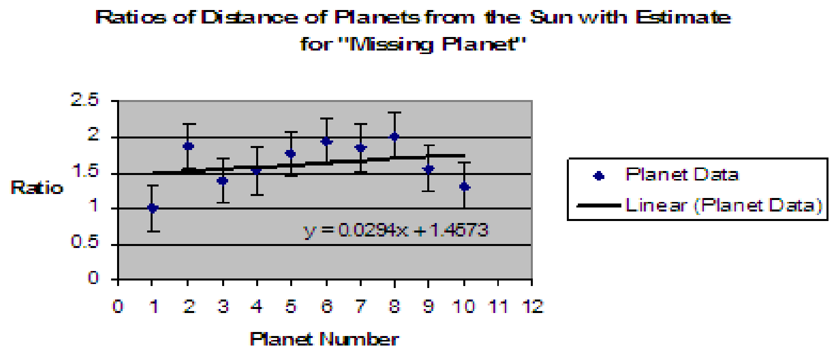



Figure 3

Inclusion of a "missing planet" resulted in a mean normalized planetary distance very close to Phi (1.619025). The location of the "missing planet" is within the vicinity of Ceres, thus justifying Johnson's inclusion of Ceres in his data analysis (see Figure 4).

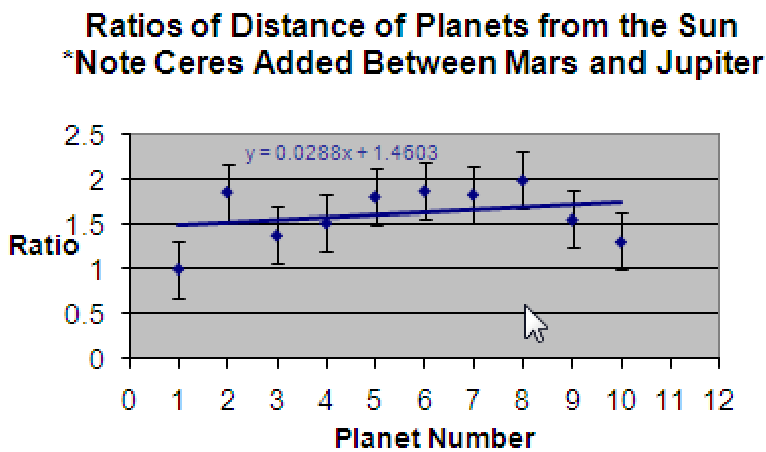



Figure 4

### 3. Analysis of Neptune

We began our study of lunar orbits with Neptune. The data pertaining to the thirteen known moons of Neptune (see Table 3) was collected by NASA (Williams, "Neptunian Satellite Fact Sheet"). Due to Neptune's great distance from Earth, and the limits of technology, discovery of these moons is relatively recent. Five of Neptune's moons were discovered in 2002 and 2003. As technology continues to improve, it is likely that still others will be found. In this section we will explore the possibility of the existence of a Phi pattern in the location of these moons. Our exploration was motivated by analogies that can be made between the Kuiper belt (which begins in the orbit of Neptune) and the asteroid belt between Mars and Jupiter discussed in the previous section.

Using the same approach as in the planetary data analysis, the distance between Neptune and its closest moon, Naiad, was taken to be the unit distance, and ratios of successive distances were calculated. The method of least squares was used to calculate the linear regression line determined by the moon numbers and corresponding distance ratios (normalized to Naiad). The results are plotted via Microsoft Excel, and one-sigma error bars are shown (see Figure 5). In the remainder of this section, we continue to iterate the technique until all outliers are eliminated, and predict the locations of possible undiscovered moons. Finally, the accuracy of our technique is analyzed by performing a regression analysis on the mean lunar distances resulting from the individual steps of the iterative technique.

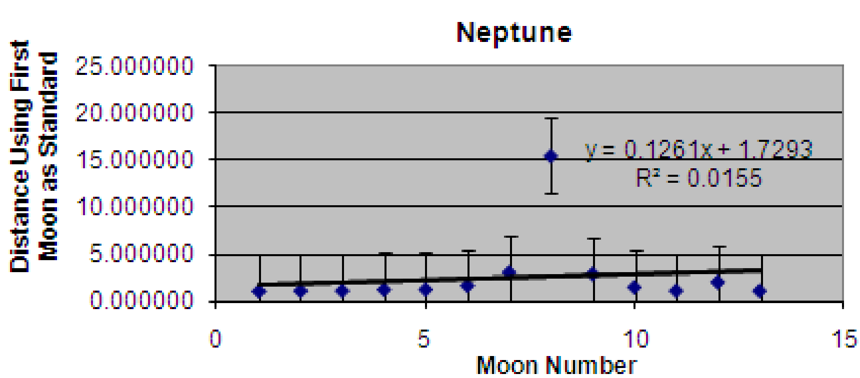



Figure 5

| Moon | Moon # | Distance (km) | Ratio |
|---|---|---|---|
| Naiad | 1 | 48,227 | 1.000000 |
| Thalassa | 2 | 50,075 | 1.038319 |
| Despina | 3 | 52,526 | 1.048947 |
| Galatea | 4 | 61,953 | 1.179473 |
| Larissa | 5 | 73,548 | 1.187158 |
| Proteus | 6 | 117,647 | 1.599595 |
| Triton | 7 | 354,760 | 3.015462 |
| Nereid | 8 | 5,513,400 | 15.541211 |
| Halimede | 9 | 15,730,000 | 2.853049 |
| Psamathe | 10 | 22,430,000 | 1.425938 |
| Sao | 11 | 46,700,000 | 1.050825 |
| Laomedeia | 12 | 46,700,000 | 1.981332 |
| Neso | 13 | 48,390,000 | 1.036188 |
| | | Mean | 2.612115 |
| | | Std. Dev. | 3.9447828 |

Table 3

Linear regression revealed that Nereid was an outlier. Using the regression equation

$$y = 0.1261x + 1.7293,$$

we postulate the existence of an undiscovered moon between Triton and Nereid (referred to as "Moon 1," in Table 4 below). We then recalculate the linear regression line and analyze the graph (see Figure 6).

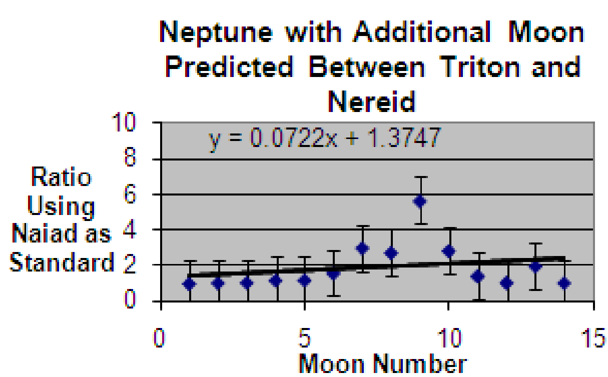



Figure 6

| Moon | Moon # | Distance (km) | Ratio |
|---|---|---|---|
| Naiad | 1 | 48,227 | 1.000000 |
| Thalassa | 2 | 50,075 | 1.038319 |
| Despina | 3 | 52,526 | 1.048947 |
| Galatea | 4 | 61,953 | 1.179473 |
| Larissa | 5 | 73,548 | 1.187158 |
| Proteus | 6 | 117,647 | 1.599595 |
| Triton | 7 | 354,760 | 3.015462 |
| Moon 1 | 8 | 971,368 | 2.738100 |
| Nereid | 9 | 5,513,400 | 5.675911 |
| Halimede | 10 | 15,730,000 | 2.853049 |
| Psamathe | 11 | 22,430,000 | 1.425938 |
| Sao | 12 | 46,700,000 | 1.050825 |
| Laomedeia | 13 | 46,700,000 | 1.981332 |
| Neso | 14 | 48,390,000 | 1.036188 |
| | | Mean | 2.612115 |
| | | Std. Dev. | 3.9447828 |

Table 4

Nereid continues to be an outlier. Using the new regression equation, $y = 0.0722x + 1.3747$, we postulate the existence of a second undiscovered moon, "Moon 2," between "Moon 1" and Nereid (see Table 5). We then recalculate the linear regression line and analyze the graph (see Fig. 7).

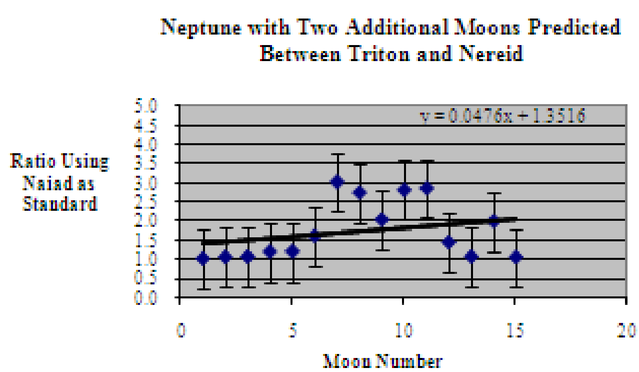



Figure 7

| Moon | Moon # | Distance (km) | Ratio |
|---|---|---|---|
| Naiad | 1 | 48,227 | 1.000000 |
| Thalassa | 2 | 50,075 | 1.038319 |
| Despina | 3 | 52,526 | 1.048947 |
| Galatea | 4 | 61,953 | 1.179473 |
| Larissa | 5 | 73,548 | 1.187158 |
| Proteus | 6 | 117,647 | 1.599595 |
| Triton | 7 | 354,760 | 3.015462 |
| Moon 1 | 8 | 971,368 | 2.738100 |
| Moon 2 | 9 | 1,966,535 | 2.024500 |
| Nereid | 10 | 5,513,400 | 2.803611 |
| Halimede | 11 | 15,730,000 | 2.853049 |
| Psamathe | 12 | 22,430,000 | 1.425938 |
| Sao | 13 | 46,700,000 | 1.050825 |
| Laomedeia | 14 | 46,700,000 | 1.981332 |
| Neso | 15 | 48,390,000 | 1.036188 |
| | | Mean | 1.732166 |
| | | Std. Dev. | 0.77263053 |

Table 5

After “Moon 2” is added, Triton, “Moon 1,” Nereid, and Halimede appear to be outliers. Using the new regression equation, $y = 0.0476x + 1.3516$, we postulate the location and associated distance ratios of four more undiscovered moons (see Table 6). At this point, there appear to be no more outliers (see Figure 8). Hence, we end our predictions here, and evaluate the accuracy of the technique by performing a regression analysis on the iterative mean lunar distance data.

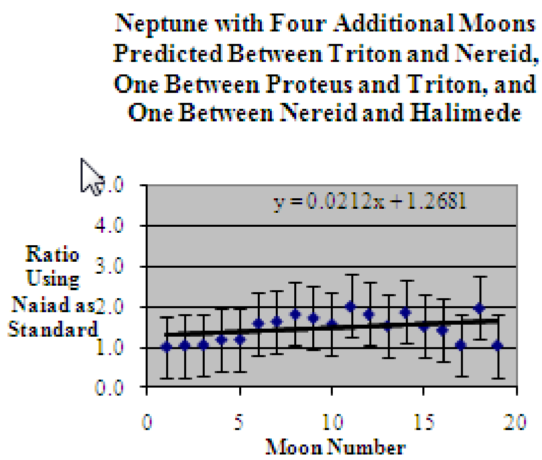



Figure 8

| Moon | Moon # | Distance (km) | Ratio |
|---|---|---|---|
| Naiad | 1 | 48,227 | 1.000000 |
| Thalassa | 2 | 50,075 | 1.038319 |
| Despina | 3 | 52,526 | 1.048947 |
| Galatea | 4 | 61,953 | 1.179473 |
| Larissa | 5 | 73,548 | 1.187158 |
| Proteus | 6 | 117,647 | 1.599595 |
| Moon 3 | 7 | 193,976 | 1.648800 |
| Triton | 8 | 354,760 | 1.828883 |
| Moon 4 | 9 | 614,586 | 1.732400 |
| Moon 1 | 10 | 971,368 | 1.580524 |
| Moon 2 | 11 | 1,966,535 | 2.024500 |
| Moon 5 | 12 | 3,594,040 | 1.827600 |
| Nereid | 13 | 5,513,400 | 1.534040 |
| Moon 6 | 14 | 10,338,728 | 1.8752 |
| Halimede | 15 | 15,730,000 | 1.521464 |
| Psamathe | 16 | 22,430,000 | 1.425938 |
| Sao | 17 | 46,700,000 | 1.050825 |
| Laomedeia | 18 | 46,700,000 | 1.981332 |
| Neso | 19 | 48,390,000 | 1.036188 |
| | | Mean | 1.564620 |
| | | Std. Dev. | 0.31763458 |

Table 6

Notice that Tables 3 through 6 show that with each iteration of our data analysis technique, the mean distance ratio for the moons of Neptune (normalized to Naiad) appears to be approaching Phi. Our results can be fit by a power regression curve, with correlation of 0.9931, which is quite accurate (see Table 7 and Figure 9).

| Fibonacci # | Fibonacci Ratio | Mean of Mon Distance Ratio |
|---|---|---|
| 1 | 1 | 2.612115 |
| 1 | 1 | 1.91645 |
| 2 | 2 | 1.732166 |
| 3 | 1.5 | 1.5642 |
| 5 | 1.666667 | |
| 8 | 1.6 | |
| 13 | 1.625 | |
| 21 | 1.615385 | |
| 34 | 1.619048 | |
| 55 | 1.617647 | |
| 89 | 1.618182 | |

Table 7

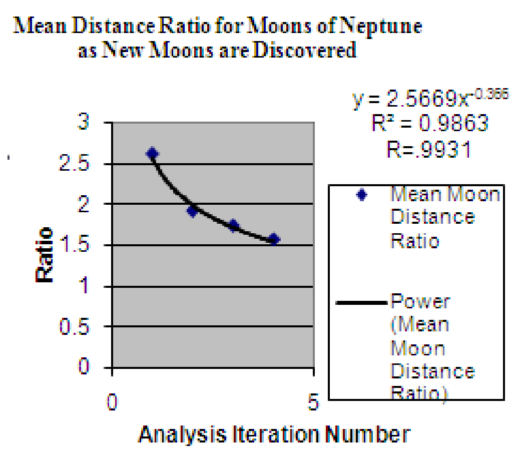



Figure 9

Note that Columns 1 and 2 of Table 7 illustrate the convergence of successive ratios of the Fibonacci numbers. Column 3 illustrates the mean lunar distance trend resulting from the four iterations of our regression analysis which were necessary to eliminate all lunar outliers. The ratios in Column 3 appear to be converging to a number which is close to Phi.

### 4. Analysis of Uranus

The next planet that was researched was Uranus, which has 5 major satellites and 22 minor satellites. The data pertaining to the moons of Uranus (see Table 8 at www.kappamuepsilon.org, the Kappa Mu Epsilon website) was collected by NASA (Williams, "Uranian Satellite Fact Sheet"). As in the planetary and Neptune data analyses, distances of satellites from Uranus were normalized to a unit distance equal to the distance between Uranus and its closest moon, Cordelia. The method of least squares was used to calculate the linear regression line determined by the moon numbers and corresponding distance ratios (normalized to Cordelia). The results are plotted via Microsoft Excel, and one-sigma error bars are shown (see Figure 10).

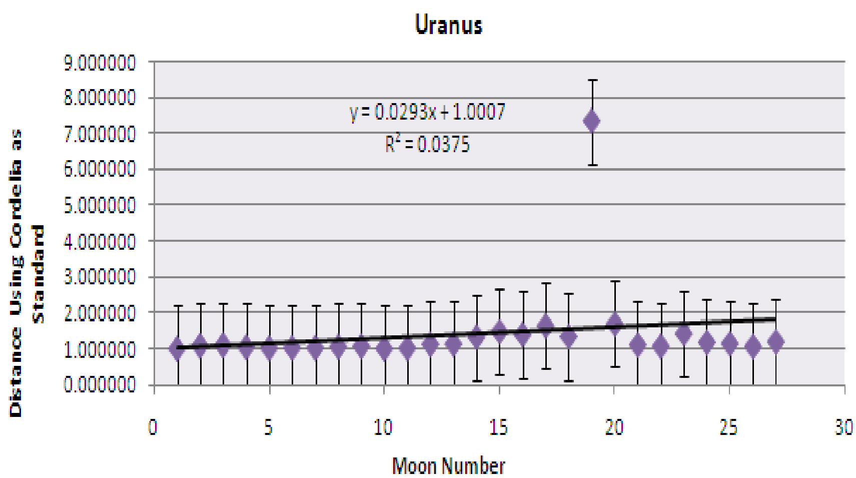



Figure 10

Calculation of the mean distance between Uranus and its moons did not reveal an observable connection to Phi. However, linear regression of the data shows a relationship between the distances, except for Francisco, which is an outlier. Adding possible undiscovered satellites, as was done in the case of Neptune (as well as the planetary data), would not be useful here due to the large number of minor moons, as well as the magnitude of the distance between these moons and the outlier, Francisco.

The moons of Uranus are classified as major or minor. The major moons are Miranda, Ariel, Umbriel, Titania, and Oberon. They are considered to be major moons because their radii are significantly larger than the radii of the minor moons (Williams, "Uranus Fact Sheet"). Due to their size, there is much more data available for the major moons. We decided to reanalyze our data using only the major moons to see if there was a Phi pattern, but we were unable to find one. In fact, without the minor moons, the mean is further from Phi.

We also analyzed the rings of Uranus, normalizing the radii of the rings to the radius of the equator of Uranus (see Table 9). Linear regression of the data reveals a strong correlation between ring number and normalized radius, and Phi lies within one standard deviation of the mean (see Figure 11).

| Rings of Uranus | Distance (km) | Radius/Equator Radius |
|---|---|---|
| Equator of Uranus | 25559 | 1 |
| 6 | 41837 | 1.636879377 |
| 5 | 42234 | 1.652412066 |
| 4 | 42571 | 1.665597246 |
| Alpha | 44718 | 1.749598967 |
| Beta | 45661 | 1.786493994 |
| Eta | 47176 | 1.845768614 |
| Gamma | 47627 | 1.863414062 |
| Delta | 48300 | 1.889745295 |
| Lambda | 50024 | 1.957197073 |
| Epsilon | 51149 | 2.00121288 |
| | | |
| | Mean | 1.732021099 |
| | Std. Dev. | 0.271740742 |

Table 9

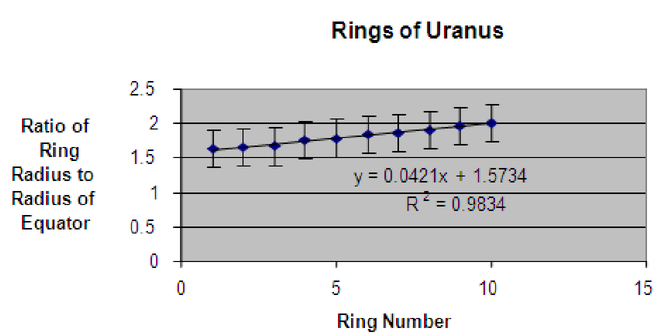



Figure 11

Further research uncovered many articles which discussed a relationship between the rings and moons of Uranus. The rings of Uranus are formed by dust particles released by surrounding moons (Goudarzi). These dust particles are formed when meteoric collisions occur with the moons. The dust particles become trapped in the lunar orbit by surrounding forces ("New Moons and Rings Found at Uranus"). In a 2007 MSNBC news release, "Planet Uranus Has a Rare Blue Ring," Goudarzi discusses the discovery of a rare Blue Ring about Uranus, and its relationship to the minor moon, Mab, which is believed to be the ring's "companion moon." According to the article, dust particles formed by meteoric collisions were released by Mab, and sent into the atmosphere to form this faint Blue Ring. The Blue Ring follows the orbit of Mab, and its blue color is due to the small size of the particles. On the other hand, the rings about Uranus which are predominantly red were formed by larger particles that reflect red light (Goudarzi). Further indications of a relationship between the moons and rings of Uranus were also noted in a 2005 press release, "New Moons and Rings Found at Uranus." A pair of rings and two new moons were discovered, due to the fact that one of the moons shared its orbit with a ring. This set of rings was so far from the rest, that they are considered to be their own system of rings.

The existence of a relationship between the rings and moons of Uranus led us to search for a Phi pattern based on this connection. Noticing that the mean of the ring data was an overestimate for Phi, while the mean of the moon data was an underestimate, we examined the average of these two estimates and found a much more accurate estimate for Phi:

$$\text{Mean Moon Distance from Uranus (normalized to Cordelia)} = 1.410173$$

$$\text{Mean Radius of Rings (normalized to the equator of Uranus)} = 1.7320210099$$

$$\text{Mean of Moon and Ring Data} = 1.57109705$$

The discovery of a Phi pattern which links the moons and rings of Uranus is not surprising. As discussed earlier, the rings are composed of particles from the moons, as in the small particles of the Blue Ring which are attributed to Mab (Goudarzi). Scientists have also found that the particles which comprise the rings of Uranus are being acted upon by surrounding forces which are influenced by the mass and orbit of the planet's satellites.

## 5. Analysis of Saturn

Finally, we investigated the moons of Saturn to determine whether or not they revealed a Phi pattern. Data pertaining to the moons of Saturn and their distances from Saturn was gathered from a NASA web site (Williams, "Saturnian Satellite Fact"). As of July 2007, sixty moons of Saturn have been identified. However, some of these discoveries are so recent that they are still unnamed. Using Johnson's study as a model, we set the distance of Pan (Saturn's nearest moon) from Saturn as the unit distance. We then calculated successive ratios of distances, as in the Fibonacci sequence. The data has been tabulated in Table 10, which can be found on the Kappa Mu Epsilon website, www.kappamuepsilon.org.

As in Johnson's work, we checked for the existence of a Phi pattern within these ratios, by computing the mean and standard deviation. The mean was calculated as 1.1246447, and the standard deviation was found to be 0.3698122. Figure 12 shows the results of a regression analysis of the raw data, along with error bars determined by the standard deviation.

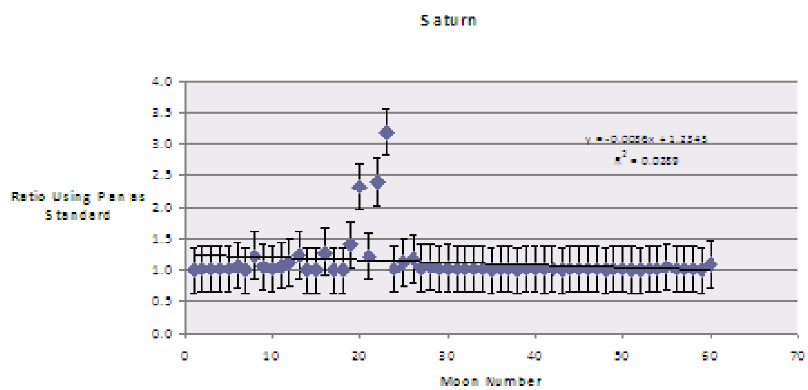



Figure 12

Titan, Iapetus, and Kiviuq appear to be outliers, and the data does not exhibit a readily identifiable Phi pattern. The average of the ratios is considerably less than Phi. In addition, two of the major moons of Saturn each have two Trojan moons which share the same orbit (Schombert). This forces the ratio unnaturally to one in the corresponding sequence of ratios. It was immediately apparent that the Trojan moons Calypso, Talesto, Helena, and Polydeuces, needed to be removed from the analysis. The list of data can be further reduced by considering only the most significant moons. Since the data was standardized to Pan, only those moons having mean density greater than or equal to that of Pan, 560km/m$^3$ , were considered (Williams, "Saturnian Satellite Fact"). This is justifiable since many of Saturn's moons are actually large chunks that broke away from other moons (Schombert). For example, Hyperion is the largest irregular shaped moon observed, and is highly "pock-marked" (Schombert). This indicates that pieces of Hyperion broke away and entered alternate orbits. Another moon that shows signs of contributing to the formation of smaller moons is Mimas. This moon has a large crater that indicates it was struck by an asteroid or other cosmic object (Schombert). Therefore, the orbits of low density moons really depend on the original moons at the time of impact. Restricting the data in this way gives rise to the data in Table 11, below, as well as the regression analysis in Figure 13. One-sigma error bars enable us to identify outliers.

| Moon | Moon # | Distance (km) | Ratio of Distance |
|---|---|---|---|
| Pan | 1 | 133,583 | 1.000000000 |
| Epimetheus | 2 | 151,422 | 1.133542442 |
| Janus | 3 | 151,472 | 1.000330203 |
| Mimas | 4 | 185,520 | 1.224780818 |
| Enceladus | 5 | 238,020 | 1.282988357 |
| Tethys | 6 | 294,660 | 1.237963196 |
| Dione | 7 | 377,400 | 1.280798208 |
| Rhea | 8 | 527,040 | 1.396502385 |
| Titan | 9 | 1,221,830 | 2.318287037 |
| Hyperion | 10 | 1,481,100 | 1.212198096 |
| Iapetus | 11 | 3,561,300 | 2.404496658 |
| Phoebe | 12 | 12,944,000 | 3.634627804 |
| | | Mean Ratio | 1.5938763 |
| | | Std. Dev. | 0.7923295 |

Table 11

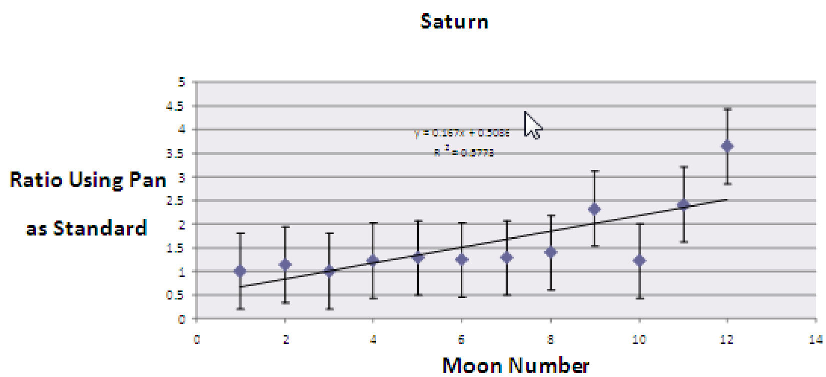



Figure 13

Saturn's moons suggest a strong correlation to Phi, similar to the pattern found in the planetary data. There were many factors to consider when analyzing the moons of Saturn, including the Trojan moons and the mean density of moons that were smaller than Pan.

## 6. Further Research

While working on any project one often wonders how the research can be extended in the future. One question that was raised by our research was whether or not a Phi pattern can be found in the moons of Jupiter. Considering the fact that Jupiter also has rings, if a Phi pattern was discovered, would it be similar to the pattern found in the moons of Saturn, or would it be more similar to the pattern exhibited by Uranus?

Another way our work can be extended is to draw a connection with Johannes Kepler's Laws of Planetary Motion. Kepler's Laws arose frequently in our research, as well as in discussions with mathematicians and scientists at various conferences. It would be interesting to see if our work is similar to Kepler's. For example, Kepler's Third Law states that $(\text{Period})^2 = (\text{Distance})^3$ (Morison and Penston 16), where the period is how long a planet takes to revolve around the Sun, and the distance is measured between the planet and the Sun. Using this information, Kepler knew where to look in the night sky for a particular planet. This is similar to our work, in that we found where a planet or moon should be using linear regression. Hopefully this project will lead others to discover new information about our universe.

Another avenue that this research could follow is an in-depth study of the asteroid belt between Mars and Jupiter. Could that have been a planet at one time? If so, what caused the planet's destruction? There has also been an asteroid belt discovered beyond Pluto, as well as a Plutiod. We would be very interested to know if that data would support the Phi pattern in our galaxy.

Finally, while researching the Milky Way, we learned about the classification of galaxies. For example, the Milky Way is a spiral class galaxy which exhibits a Phi pattern. Do other spiral class galaxies also exhibit a Phi pattern? Do the other two classifications, bar-spiral and elliptical, reveal a different pattern altogether, or none at all? We are sure that the quest for answers to these questions will lead to interesting discoveries in the future.

## 7. Conclusion

Further study of planetary data led to a data analysis technique based on linear regression which proved Johnson's postulated existence of a Phi pattern in the distance of the planets to the Sun (normalized to Mercury). This technique was applied to data collected on three planets in our solar system: Neptune, Uranus, and Saturn. It revealed a Phi pattern in all three cases.

An analysis of Neptune's satellite data led to the discovery of "missing" moons that fit a Phi pattern. This was similar to the pattern found by Johnson when he included Ceres (the largest asteroid in the asteroid belt between Mars and Jupiter) in his calculations. The mean distance between Neptune and its satellites (normalized to Naiad) was found to be close to Phi when "missing" moons were included in the analysis.

The search for a Phi pattern in the moon and ring data of Uranus proved to be tricky. Uranus has many moons, and our initial analysis produced a mean satellite distance that seemed too low for a Phi ratio. Therefore, linear regression would not correct this. Instead it would make the ratio even smaller. The ring data was taken into consideration, and at first it appeared that this data would not help our research, since it produced an estimate of Phi that was too high. However, information gained from the Voyager Mission revealed that the rings of Uranus did not form at the same time the moons did. The rings appear to be remnants of moons created prior to the rings, either broken up by a high-velocity impact or torn apart by gravitational effects. Therefore, a relationship exists between the satellite and ring data of Uranus. The average of the mean distance between the satellites and Uranus (normalized to Cordelia) and the mean of the radii of the rings of Uranus (normalized to its equator) resulted in an estimate of Phi.

Saturn's satellite data suggested that the mean ratio of the distance between Saturn and its moons (normalized to Pan) was an underestimate of Phi. There were many factors to consider when analyzing the moons of Saturn, including the Trojan moons and the mean density of moons that were smaller than Pan. Once these moons were removed from our analysis, a Phi pattern was revealed.

The results of our research uncovered further examples of Phi patterns in nature and beyond. These patterns are linked to the evolution of our solar system. We believe that similar patterns arise in other systems, and encourage future research in this area.

## *References*

[1] M. Bicknell-Johnson, A Generalized Magic Trick from Fibonacci: Designer Decimals, *College Math. J.* **35** (2004) 125-126.

[2] R. Britt, "New Uranian Moon Found in Months-long Effort," http://www.space.com/scienceastronomy/uranus_moon_021002.html.

[3] D. Burton, "The Fibonacci Sequence.", *The History of Mathematics: An Introduction*, 6th ed., McGraw-Hill, 2005, 289-94.

[4] S. Gourdazi, "Planet Uranus Has a Rare Blue Ring," http://www.msnbc.msn.com/id/12188326/.

[5] C. Hamilton, "The Voyager Planetary Mission," http://www.solarviews.com/eng/vgrfs.htm.

[6] J. Harris, "The Fibonacci Series, Phi-Series and Synodic Mean," http://www.spirasolaris.ca/sbb4d.html.

[7] C. Johnson, "The Distance of the Planets from the Sun and Their Atmospheric Composition," http://www.earthmatrix.com/extract62/mercury.html.

[8] R. Johnson, *Miller & Freund's Probability and Statistics for Engineers*, 7th ed., Prentice Hall, 2005.

[9] R. Knott, "Fibonacci Numbers and Nature," University of Surrey, Department of Engineering & Physical Sciences, http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html.

[10] M. Livio, "Painters and Poets Have Equal License,", *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*, Broadway Books, 2002, 159-200.

[11] I. Morison and M. Penston. *Pocket Guide to Stars and Planets*, New

Holland Publishers, 2005.

[12] "New Moons and Rings Found at Uranus," http://www.space.com/scienceastronomy/051222_uranus.html.

[13] J. Schombert, Astronomy 121: *The Formation and Evolution of the Solar System*: *Lecture 17: Moons*, Dept. of Physics, University of Oregon, http://abyss.uoregon.edu/~js/ast121/.

[14] D. Williams, "Neptunian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/neptuniansatfact.html.

[15] D. Williams, "Planetary Fact Sheet," . , http://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html.

[16] D. Williams, "Saturnian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/saturniansatfact.html.

[17] D. Williams, "Uranian Satellite Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uraniansatfact.html.

[18] D. Williams, "Uranus Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uranusfact.html.

[19] D. Williams, "Uranus Rings Fact Sheet," http://nssdc.gsfc.nasa.gov/planetary/factsheet/uranringfact.html.

# *Properties of Zero Divisor Graphs Associated to Commutative Semigroups*

Mark Pelfrey

Central Michigan University

## 1. Introduction

This article continues the study of the zero-divisor graph associated to a semigroup begun in [5] by F. R. DeMeyer, T. McKenzie, and K. Schneider entitled *The Zero-Divisor Graph of a Commutative Semigroup*. I. Beck introduced the zero-divisor graph $\Gamma(R)$ associated to a commutative ring $R$ [3]. This graph was the sole focus of a study by Anderson and Livingston in [2], and was studied further (see [4], [1], [6] and [7]). However, the first in-depth study of the subgraph in the context of a semigroup was performed by DeMeyer, McKenzie, and Schneider.

While *The Zero-Divisor Graph of a Commutative Semigroup* was the first paper to discuss zero-divisor graphs associated to semigroups, multiple other articles have focused on the classification of zero-divisor graphs and zero-divisor graph properties. This article will begin with a discussion of the algebraic and graph-theoretic concepts associated to the study of the zero-divisor graph, and will then expand upon the current literature associated with zero-divisor graphs.

## 2. Graph Theory

For all definitions in the glossary below, we loosely follow *Graph Theory* by Reinhard Diestel [8].

**Definition 1** *A graph $G$ is a set of vertices $V(G)$ and a set of edges $E(G)$ consisting of unordered pairs of vertices. Two vertices, $a$ and $b$, belonging to the set $V(G)$ are said to be adjacent to one another if the two vertices are connected by an edge $ab \in E(G)$. Further, a simple graph is a graph with no loops (edges from one vertex to itself) and with at most one edge between any two vertices.*

For the duration of this article, we will assume that all graphs are simple graphs.

**Definition 2** *A path is a graph G is a sequence of adjacent vertices*

$$V = \{x_0, x_1, ..., x_k\}$$

*connected by a sequence of edges*

$$E = \{x_0x_1, x_1x_2, ..., x_{k-1}x_k\}$$

*where all $x_i \in V$ are distinct. Moreover, a cycle is a path that begins and ends at the same vertex, and a graph G is said to be connected if every pair of vertices can be joined by a path.*

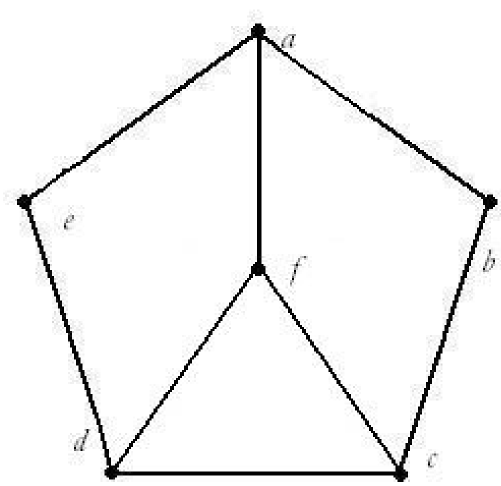



Figure 1

Consider, for example, Figure 1. The graph is connected since every vertex can be joined by a path; vertices $a$ and $c$ are joined by the path $a - f - c$, vertices $b$ and $d$ are joined by path $b - c - d$, and so one. Note that there can be multiple paths connecting distinct pairs of points. Also, observe that path $a - b - c - f - a$ is a cycle, as it begins and ends at the same vertex; however, there are also other cycles in Figure 1.

**Definition 3** *The distance $\rho$ between two vertices in a graph G is the length (number of edges) of the shortest path joining the two vertices. The diameter of a graph, given by* $\operatorname{diam}(G)$*, is the maximum distance between any pair of vertices in G.*

Again considering Figure 1, note that the distance $\rho$ between $d$ and $b$ is 2, as $d - c - b$ is the shortest path connecting the two vertices, and $\mathrm{diam}(G) = 2$, as the maximum distance between any two vertices in $G$ is length 2.

**Definition 4** *A vertex $v$ is a cut vertex if deleting the vertex and all incident edges increases the number of connected components in the graph.*

In Figure 2, the center vertex in the graph on the left is a cut vertex; deleting this vertex and its incident edges produces the graph on the right.

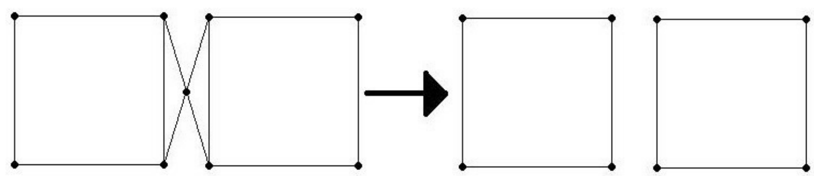


Figure 2. A Cut Vertex

**Definition 5** *A complete graph, denoted by $K_n$, is a graph with $n$ vertices and an edge between every pair of distinct vertices (see Fig. 3).*

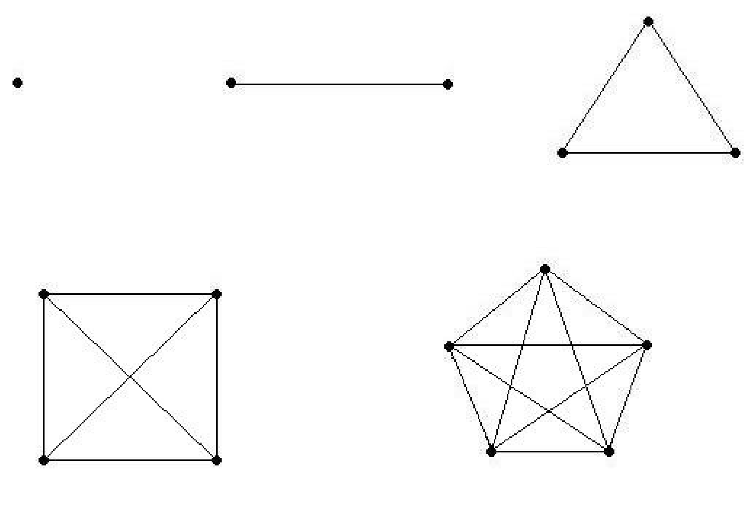


Figure 3. Complete Graphs $K_1 - K_5$

**Definition 6** *A complete bipartite graph, denoted by $K_{m,n}$, is a graph in which the vertices can be partitioned into two sets, $V_1$ and $V_2$, and the edges in the graph are the complete set of edges with one vertex in each part (see Fig. 4).*

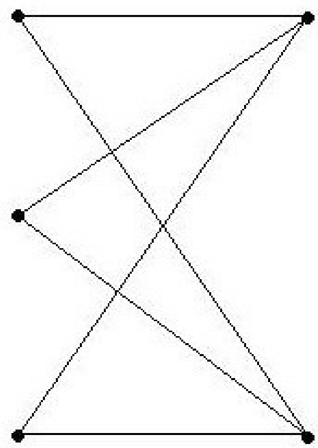


Figure 4. Complete Bipartite Graph $K_{3,2}$

**Definition 7** *A star graph is a special form of a complete bipartite graph of the form $K_{1,n}$ (see Fig. 5).*

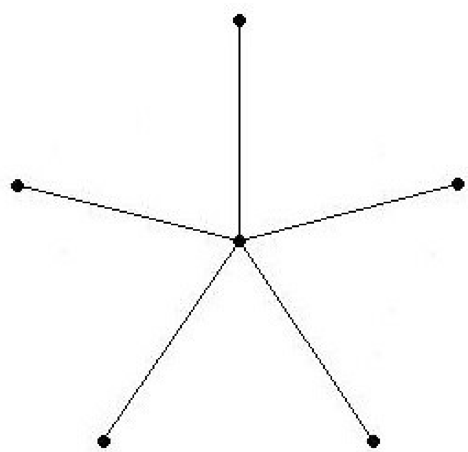


Figure 5. Star Graph $K_{1,5}$

**Definition 8** *The degree of a vertex is the number of edges incident to the vertex; a vertex of degree one is an end.*

**Definition 9** *The core of a graph is the largest subgraph contained within the graph such that every edge of the subgraph is the edge of a cycle (see Fig. 6).*

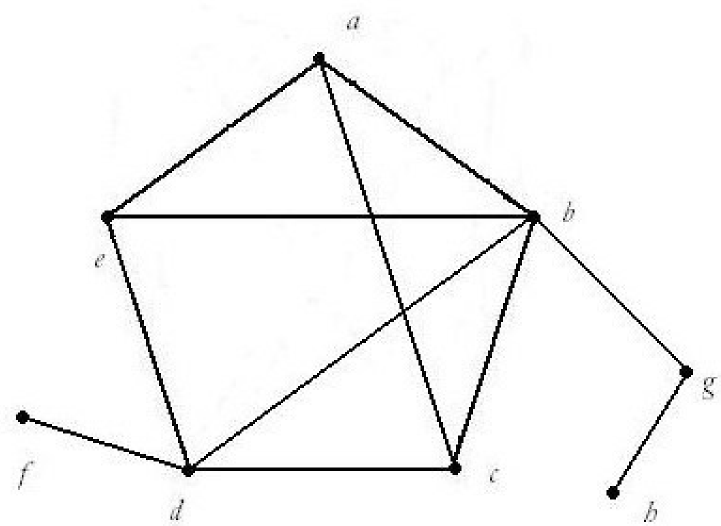



Figure 6. A graph induced by core graph $G = \{a, b, c, d, e\}$

**3. Algebra**

This section of the article will outline the algebraic objects, operations, and properties of zero-divisor graphs.

**Definition 10** *A closed binary operation on a set $A, *$ is a map $* : A \times A \longrightarrow A$. In other words, for $a, b \in A$, $a * b \in A$.*

**Definition 11** *A group is a nonempty set closed under a binary operation that satisfies three properties:*

1. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ *for all* $a, b, c \in S$ *(associativity);*
2. *There exists* $e \in G$ *such that* $e \cdot a = a \cdot e = a$ *for all* $a \in G$ *(identity);*
3. *For all* $a \in G$*, there exists* $b \in G$ *such that* $a \cdot b = b \cdot a = e$ *(inverses).*

For example, the set of all integers $\mathbb{Z}$ under addition is a group. Also, $\mathbb{R} - \{0\}$, the set of all real numbers not including $0$, is a group under multiplication.

**Definition 12** *A commutative semigroup with zero is a set $S$ with a binary operation $\cdot$ such that*

1. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ *for all $a, b, c \in S$ (associativity);*
2. $a \cdot b = b \cdot a$ *for all $a, b \in A$ (commutativity);*
3. *There exists $0 \in S$ such that $0 \cdot x = x \cdot 0 = 0$ for all $x \in S$ (zero).*

It is important to note that a semigroup is not necessarily a subset of a group. Consider the set of all integers under multiplication, $(Z, \times)$. By observing that $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ and that $a \cdot b = b \cdot a$, we can see that the set is indeed a semigroup. However, if we attempt to prove that $(Z, \times)$ is a group, we find that it is associative as shown above and that it has an inverse $(a \cdot 1 = 1 \cdot a = a)$, but cannot prove that there is a unique inverse for all elements in the set.

**Definition 13** *An ideal is a subset $I$ of a semigroup $S$ such that for all $x \in S$ and for all $a \in I$, $ax \in I$.*

Consider the set $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$. The set $\{0, 2, 4, 6\}$ is an ideal $I$ of $Z_8$. Because multiplying any element in $Z_8$ by any element in the proposed ideal of $Z_8$ carries the element back to the ideal, the set $\{0, 2, 4, 6\}$ is in fact an ideal of $Z_8$.

**Definition 14** *A non-zero element $a \in S$ is a zero divisor if there exists a non-zero element $b \in S$ such that $ab = 0$.*

**Definition 15** *A zero divisor semigroup is a semigroup in which every non-zero element is a zero divisor.*

In order to see an example of both zero divisors and a zero divisor semigroup, consider $Z_6 = \{0, 1, 2, 3, 4, 5\}$. The zero divisors of $Z_6$ with $0$ form the zero divisor semigroup $S = \{0, 2, 3, 4\}$, as $2 \cdot 3 = 0$ mod 6, and $3 \cdot 4 = 0$ mod 6.

## 4. The Zero-Divisor Graph

We will begin with a basic definition of a zero-divisor graph associated to a commutative semigroup, and then we will discuss some relevant results that have been found relating to the graph.

**Definition 16** *Let $S$ be a commutative semigroup. Associate a zero-divisor graph $G = \Gamma(S)$ by assigning a vertex to each zero divisor and connecting two distinct vertices $a$ and $b$ by an edge if and only if $ab = 0$.*

We give three examples of zero divisor graphs (see Figures 7, 8, and 9). In each, notice that vertices are represented by the zero divisors of the commutative semigroups associated to the graphs, and that vertices are connected by an edge if their product is 0.

Further, there are important things to note about the examples that we give here. First notice that the graph structure of the graph associated to $S_1$ is identical to the structure of the graph associated to $S_2$. However, we see that while zero divisor graphs can have identical structures, it is not necessarily true that their associated semigroups are identical.

Also, in Figure 9, note that $S_3$ is defined to include specific products that are not known by the structure of the graph, namely the products $ac$, $bd$, and the squares of all of the individual vertices. In order for a semigroup to be commutative and for it to be associated to a zero divisor graph, all triple products of elements of the semigroup must be associative, and these definitions allow us to show that the semigroup is indeed associative.

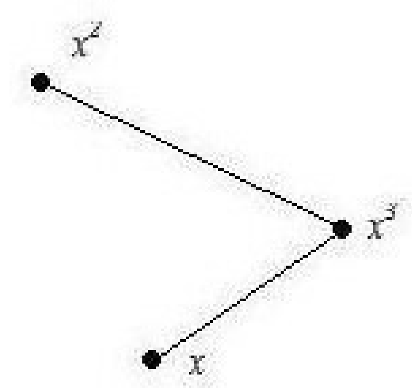



Figure 7. $S_1 = \{x, x^2, x^3 | x^4 = 0\}$

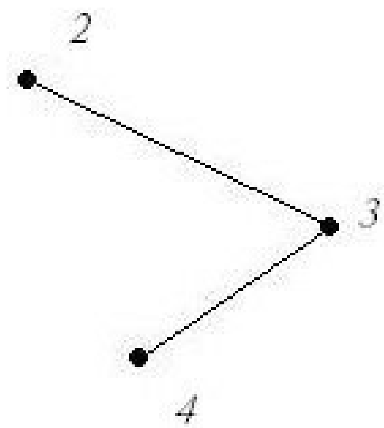



Figure 8. Zero Divisors of $Z_6 : S_2 = \{2, 3, 4\}$

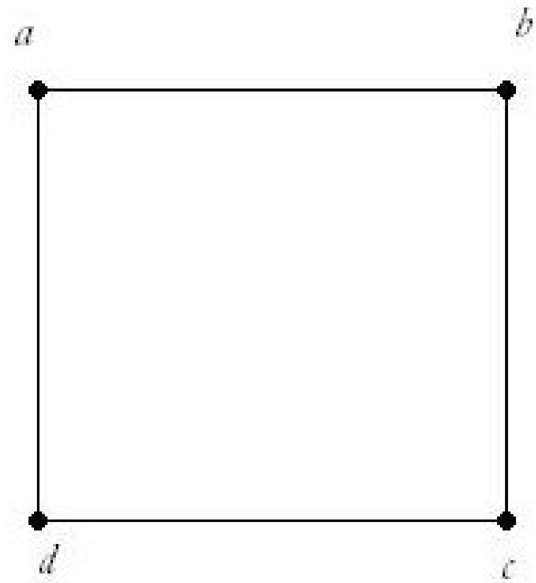



Figure 9. $S_3 = \{a, b, c, d | ac = a, bd = b, a^2 = a, b^2 = b, c^2 = c, d^2 = d\}$

Finally, we define a *neighbor* of a vertex in a graph to be a vertex connected to the given vertex by an edge. With this definition, we are now ready to dissect a fundamental theorem concerning zero divisor graphs. For reference, we will refer to this theorem as Theorem 1 for the duration of this article.

**Theorem 1** *If $G$ is a zero divisor graph, then $G$ satisfies all of the following conditions:*

1. *$G$ is connected.*
2. *Any two vertices belonging to $V(G)$ are connected by a path with at most three edges.*
3. *If $G$ contains a cycle, then the core of $G$ is a union of quadrilaterals and triangles, and any vertex not in the core of $G$ is an end.*
4. *For each pair $x, y$ of non-adjacent vertices of $G$, there is a vertex $z$ with $N(x) \cup N(y) \subset N(z) \cup z$.*

Property (1) states that any pair of vertices of a zero divisor graph must be connected by some path. Property (2) states that all vertices of the graph must be connected by a path of 3 edges or fewer, or, in other words, that the diameter of the graph must be less than or equal to 3.

Property (3) states that if a graph contains a cycle, then for the graph to be a zero divisor graph, the core of that graph must be a union of quadrilaterals and triangles, and any vertex not in the core of that graph must be an end, or a vertex of degree one. In the example below, the graph on the left is a zero divisor graph, while the graph on the right is not. Because vertex $c$ in the graph on the right has a cycle and a vertex of degree 2 that is not in the core of $G$, the graph fails part three of the theorem above.

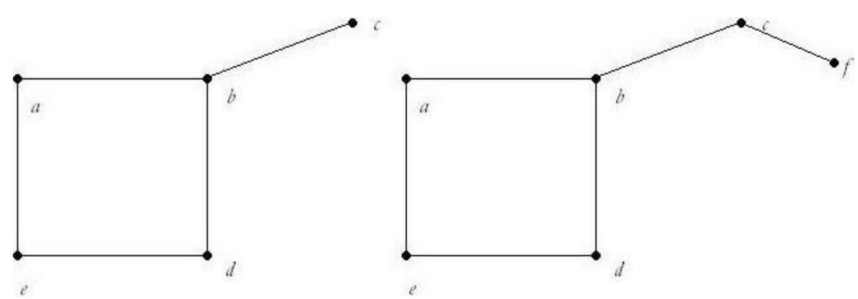



Figure 10. An illustration of part three of Theorem 1.

Property (4) of Theorem 1 arises because any product $a_i a_j$ within the semigroup associated to the zero divisor graph must be a vertex already in the semigroup that is adjacent to all of the vertices to which both $a_i$and $a_j$are adjacent. Consider the examples below. All of the products of non-connected vertices in the graph on the left can be defined within a semigroup $S$ associated to the graph; however, in the graph on the right, the product $cf$ would have to be a vertex existing in the graph adjacent to

$b, d, e$, and $a$. Because no such vertex exists, the graph on the right is not a zero divisor graph. In the latter example, we see that for vertices $c$and $f$, there does not exist any vertex $z$such that $N(x) \cup N(y) \subset N(z) \cup z$.

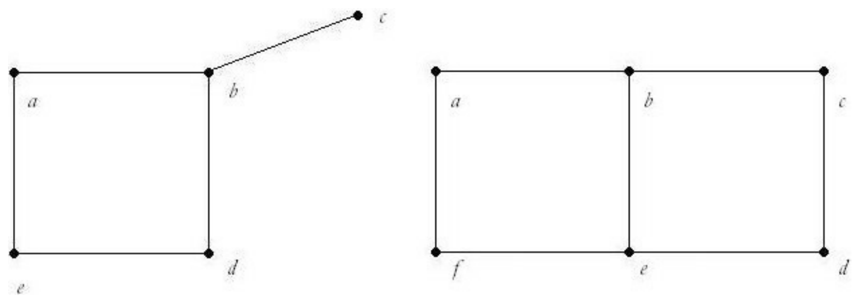



Figure 11. An illustration of part four of Theorem 1

For graphs of five vertices or fewer, the conditions in Theorem 1 are necessary and sufficient to classify a graph as a zero divisor graph [4]. However, for graphs of more than five vertices, the conditions are necessary to classify a graph as a zero divisor graph but are not sufficient to do so. The following graphs are all of the graphs with six vertices that adhere to the conditions in Theorem 1 but have been proven to not be associated to any zero-divisor semigroup. This list of graphs was compiled by the REU students at Central Michigan University in the summer of 2008 [7].

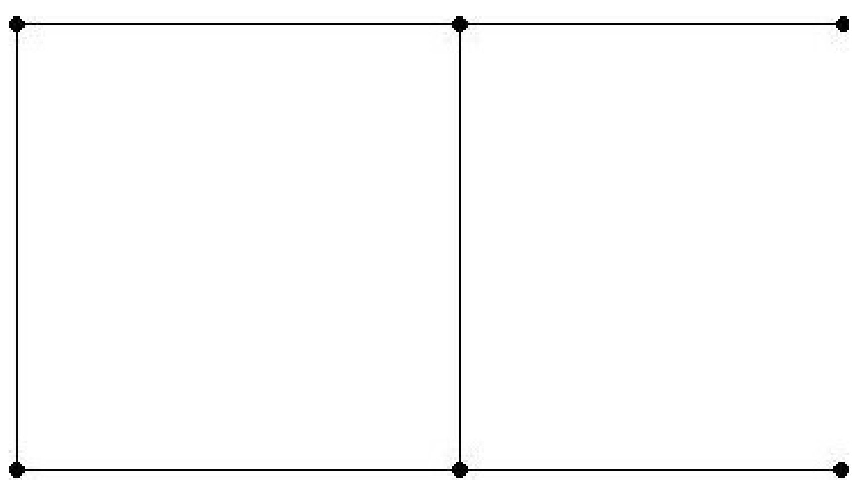


Figure 12. CMU REU Graph 1

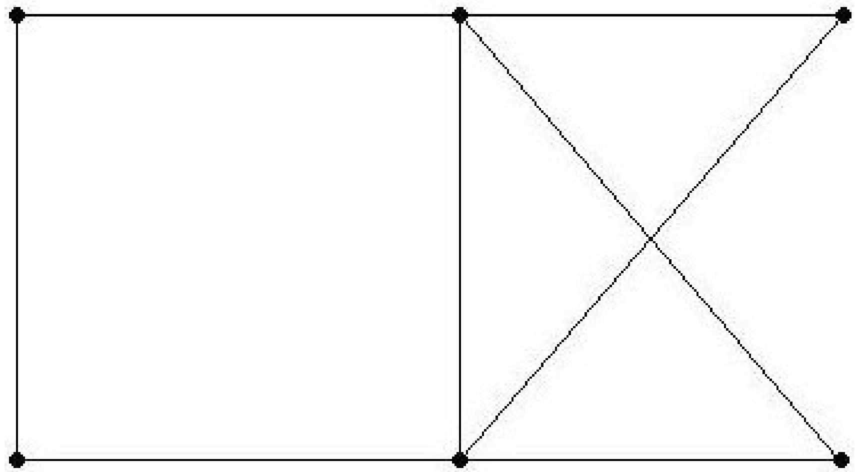


Figure 13. CMU REU Graph 2

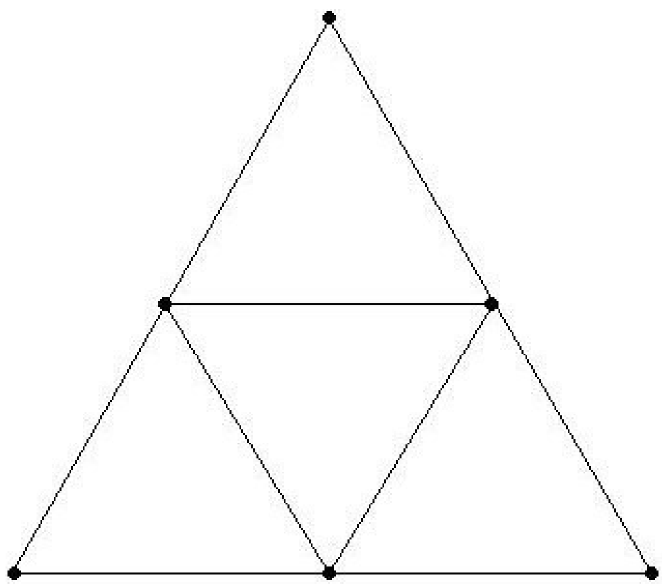


Figure 14. CMU REU Graph 3

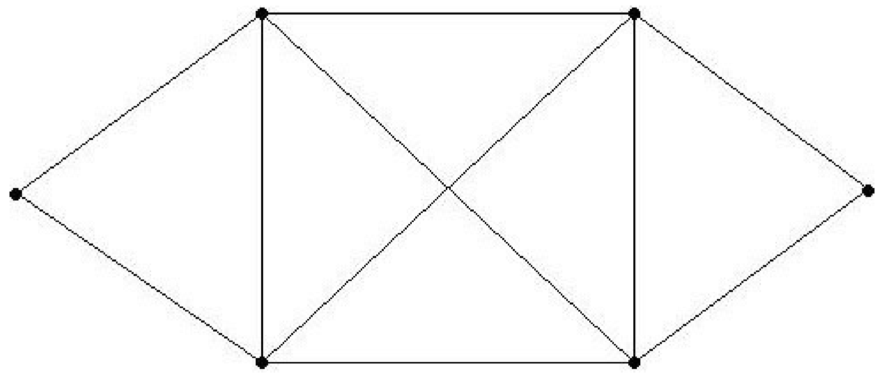


Figure 15. CMU REU Graph 4

Finally, before we move on to our findings, there is one more result that is helpful in studying zero divisor graphs.

**Theorem 2** *The following graphs are the graph of a semigroup.*

1. *A complete graph or a complete graph together with an end.*
2. *A complete bipartite graph or a complete bipartite graph together with an end.*
3. *A refinement of a star graph.*
4. *A graph which has at least one end and has diameter at most 2.*
5. *A graph which is the union of two star graphs whose centers are connected by a single edge.*

### 5. Results on Wedge Construction of Zero Divisor Graphs

In our exploration of the zero divisor graph, we studied new ways to classify graphs on more than 5 vertices. One way in which we studied these graphs was by wedging both zero-divisor and non zero-divisor graphs with multiple vertices.

Before we present our theorems, we must first describe a special type of graph construction used in our findings called wedging.

**Definition 17** *Let $K$ and $H$ be two graphs. We define the wedging of graphs $K$ and $H$ to be a new graph $G = K \vee H$ such that $V(G) = V(K) \cup V(H)$, $E(K), E(H) \subseteq E(G)$, and vertices $k$ and $h$ are joined by an edge for all $k \in V(K)$ and for all $h \in V(H)$.*

**Theorem 3** *A zero-divisor graph connected to 2 separate, distinct vertices (complete bipartite graphs of degree one) is also a zero divisor graph.*

**Proof.**

Suppose $G$ is a zero-divisor graph. Then $G \vee 2(K_1)$ is also a zero-divisor graph.Let $S$ be the semigroup associated to the graph $G$, and let $S^* = S \cup \{x, y\}$ with the multiplication inherited from $S$ and with $xy = x$, $x^2 = x$, $y^2 = y$, and $a_i x = a_i y = 0$ for all $a_i \in S$. Given that $G$ is a zero-divisor graph and $S$ is an commutative semigroup, to prove that $S^*$ is also an commutative semigroup we must prove that all triple products containing factors belonging to $\{x, y\}$ are associative.

- In the first case, with 0 terms belonging to $\{x, y\}$ involved a triple product, associativity holds.
- If there is one element belonging to $\{x, y\}$, then,since $a_i x = a_i y = 0$, the triple product will be associative because it will be equal to zero. Then, given any pair of elements $\{a_i, a_j\} \in S$, $(a_i a_j)\, y = (a_i a_j)\, x = 0$ because $a_i a_j \in S$, and both $x$ and $y$ multiply any element in $S$ to 0.
- If there are two elements belonging to $\{x, y\}$, the triple product will equal zero since $a_i xx = (a_i x)\, x = 0x = 0$. Also, $a_i xy = (a_i x)\, y = 0y = 0$. Similarly, $a_i yx = a_i yy = 0$.
- Finally, triple products involving three elements belonging to $\{x, y\}$ are also associative. We have $xxx = (xx)\, x = xx = x$; similarly, $yyy = y$. Also, $xxy = (xx)\, y = xy = x$, and $xxy = x\,(xy) = xx = x$. Finally, $xyy = (xy)\, y = xy = x$, and $xyy = x\,(yy) = xy = x$.

Observe that the graph associated to $S^*$ is $G \vee 2(K_1)$. ■

Our next theorem requires a bit of notational explanation. The symbol $\coprod$ represents a disjoint union, and for our purposes, we describe a disjoint union between two graphs as a graph including both graphs in their entirety with no edges connecting them. Equipped with that knowledge, we now describe a class of graphs that are always zero divisor graphs.

**Theorem 4** *Let $G$ be a disconnected graph such that $G = H \coprod nK_1$, where $H$ is a zero divisor graph, and $nK_1$ are $n$ isolated vertices. Then $G \vee 2K_1$ is a zero divisor graph.*

**Proof.**

Let $S$ be the semigroup associated to the graph $G$. Also, let $h, k, l \in H$, let $a_i, a_j, a_k \in nK_1$, and let $2K_1 = \{x, y\}$. Further, define the following products: $ha_i = h$ for all $a_i \in nK_1$; $a_i a_j = a_{\min\{a_i, a_j\}}$; $x^2 = x$; $y^2 = y$; $xy = x$.

In order to prove that $G$ is a zero divisor graph, we must prove that all triple products with factors belonging to $H$, $nK_1$, and $2K_1$ are associative. In order to complete this task systematically, we will deal with vertices from one subgraph at a time.

- First, consider $H$. Because $H$ is a zero divisor graph by definition, any triple product from $h, k, l \in H$ is associative. Given two elements from $H$, we have either $(hk)\,a_i = 0 \cdot a_i = 0 = hk = h\,(ka_i)$ or $(hk)\,x = 0 \cdot x = 0 = h \cdot 0 = h\,(kx)$ .Given one element from $H$, we have $(ha_i)\,a_j = ha_j = h = ha_i = h\,(a_i a_j)$, $(ha_i)\,x = hx = 0 = h \cdot 0 = h\,(a_i x)$, $(hx)\,x = 0 \cdot x = 0 = h \cdot 0 = hx = hx^2 = hxx$, and $(hx)\,y = 0 \cdot y = 0 = hx = h\,(xy)$ .
- Now, we will consider elements of $nK_1$. Given three elements of $nK_1$, we have either $(a_i a_j)\,a_k = a_i a_k = a_i = a_i a_j = a_i\,(a_j a_k)$ or $(a_i a_k)\,a_j = a_i a_j = a_i = a_i a_j = a_i\,(a_k a_j)$ .Given two elements of $nK_1$, we have $(a_i a_j)\,x = a_i x = a_i x = 0 = a_i \cdot 0 = a_i\,(a_j x)$ .And, given one element of $nK_1$, we have either $(a_i x)\,x = 0 \cdot x = 0 = a_i \cdot 0 = a_i x = a_i x^2 = a_i\,(xx)$or $(a_i x)\,y = 0 \cdot y = 0 = a_i x = a_i\,(xy)$.
- Finally, we must consider triple products with factors only from $\{x, y\}$; because $x^2$ and $y^2$ are defined in the semigroup definition, we only need to consider $xxy = (xx)\,y = xy = x = x^2 = xx = x\,(xy) = xxy$ and $yyx = (yy)\,x = yx = x = yx = y\,(yx) = yyx$.

This completes the proof. ■

**Theorem 5** *Let $G = H \coprod F$, where $H$ and $F$ are graphs that contain non-trivial connected components. Then $G \vee 2K_1$ is not a zero divisor graph.*

**Proof.** Consider the picture of $G$ below in Figure 16, where $a - b$ is an edge in $H$ and $c - d$ is an edge in $F$. The product $ac$ cannot be defined because of part 4 of Theorem 1; the product $ac$ must belong to either $\{a, b, c, d\}$ or $\{x, y\}$ and must be neighbors with $b$, $d$, $x$, and $y$ at the same time. Since no such vertex exists, $G \vee 2K_1$ is not a zero divisor graph. ■

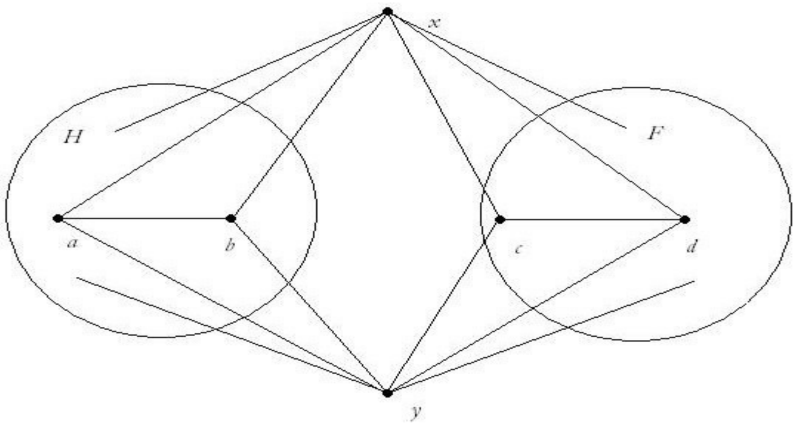



Figure 16. $G = H \coprod F$

**Theorem 6** *Let $G$ be a connected graph that fails to meet condition 4 of Theorem 1. Then $G \vee 2K_1$ is not a zero divisor graph.*

**Proof.** Let $S$ be the semigroup associated to the graph $G$. By hypothesis, there exist vertices $a_i$ and $a_j$ such that the product $a_i a_j$ cannot be defined because of the neighborhood condition of the DMS/DD Theorem. To illustrate this failure to meet the neighborhood condition, let there exist vertices $a_k$, $a_l$, and $a_m$ such that the product $a_i a_j$ must be adjacent to $a_k$, $a_l$, and $a_m$ but such that there is no such vertex in $G$. Then, wedge $2K_1$ = $\{x, y\}$ with $G$. Similar to the proof for Theorem 5, the product $a_i a_j$ must be adjacent to $a_k$, $a_l$, $a_m$, $x$, and $y$. Because no such vertex exists, we find that $G \vee 2K_1$ is in fact not a zero divisor graph, a contradiction. Therefore, the theorem is proved. ■

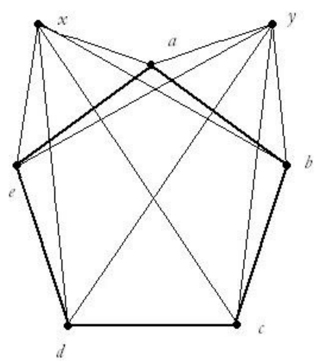



Figure 17. An illustration of Theorem 6

## 6. Conclusion and Open Questions

While the theorems contained in this paper help us to classify zero divisor graphs of a certain type, those created from the graph construction technique of wedging, there is much more to be discovered about zero divisor graphs and the commutative semigroups attached to them. Is there a way to determine if graphs of greater than 5 vertices are zero divisor graphs? Are there other graph construction techniques which we can use to determine that certain classes of graphs are zero divisor graphs?

Also, our paper focused on determining if given graphs were in fact zero divisor graphs. Another question to consider concerns the semigroup attached to a zero divisor graph. Is the semigroup for a given zero divisor graph unique? What are the algebraic properties concerning semigroups attached to zero divisor graphs?

### *References*

[1] D. D. Anderson and M. Naseer, Beck's Coloring of a Commutative Ring, *Journal of Algebra* **159** (1993), 500-514.

[2] D. F. Anderson and P. S. Livingston,. The Zero-Divisor Graph of a Commutative Ring, *Journal of Algebra* **217** (1999), 434-447

[3] I. Beck, Coloring of a Commutative Ring, *Journal of Algebra* **116** (1988), 208-226.

[4] F. DeMeyer and L. DeMeyer, Zero-divisor graphs of semigroups, *Journal of Algebra* **283** (2005), 190-198.

[5] F. DeMeyer, T. McKenzie, and K. Schneider, The Zero-divisor Graph of a Commutative Semigroup, *Semigroup Forum* **65** (2002), 206-214.

[6] L. DeMeyer, Zero-Divisor Graph Associated to a Semigroup, To appear, *Rocky Mountain Journal of Mathematics*.

[7] L. DeMeyer, A. S. Greve, and J. Wang, Zero Divisor graph associated to a semigroup, To appear, *Communications in Algebra*.

[8] Diestel, Richard. *Graph Theory*, Springer-Verlag: Heidelberg, 2005.

# *The Ballot Theorem*

Yawei Chu, *student*

St. Olaf College
Northfield, MN 55057

## 1. Introduction

For hundreds of years, mathematics scholars have studied complex math underlying ballot elections.

**The Ballot Problem.** *Suppose that Al gets A-votes in an election, Betty gets B-votes. Let $a$ be the number of A-votes, and $b$ be the number of B-votes, where $a > b$. Votes are tallied in a random order. Find the probability that Al always leads (ties are not allowed) during the counting of the votes.*

The answer, it turns out, is surprisingly simple.

**The Ballot Theorem.** *The probability that Al always leads is $(a - b) / (a + b)$, in other words, Al's margin of victory divided by the total number of votes.*

The Ballot problem and theorem are old. Mathematicians have made this problem popular by using different methods to solve the Ballot problem. Joseph Bertrand [2] proved the theorem by induction in 1887. Also, in [1], Rich Durrett used "backwards martingales" to solve the problem.

## 2. An Example

Here, we present another approach. Let's start with an easy example, where $a = 3$ and $b = 2$. Since there are five possible positions for the two B-votes, there are $\binom{5}{2} = 10$ possible outcomes. We can, of course, list all 10 outcomes. Instead, we will attempt to categorize the different possibilities. It is convenient to display a string of votes on a circle. For example, voting string of AAABB can be displayed as:

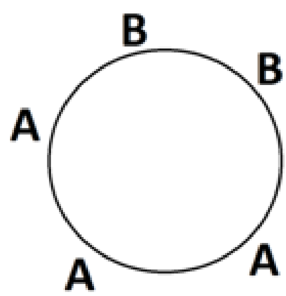



Figure 1

Notice that we do not need to list all circles from each individual string, since on any circle there are $a + b$ votes and each vote can start a string. Strings from any circle may be distinct from one another or some strings may be the same. We know that if any two circles are the same, then these two circles contain all the same strings. Therefore, the number of circles, which is sufficient to represent all possible strings, is less than the number of all possible strings. Thus, if we remove those repeat circles, then all remaining circles will be distinct from one another.

- Assume there is no gap between Betty's two votes as shown in Figure 1 above. Notice that Betty and Al must not be tied, and Al must be strictly ahead of Betty. If the string goes clockwise, then the only possible starting point is "one A," shown below:

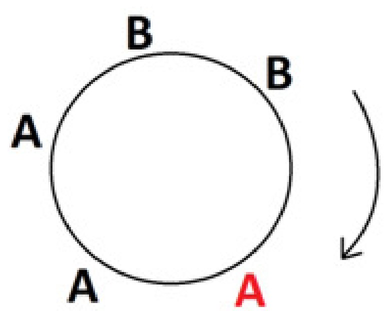



Figure 2

Since these categorized votes are around a circle, any vote on this circle can start a string. Since there are five votes, there are five strings.

- Assume there is one gap (one or two A's) between two B's (Betty's votes.)

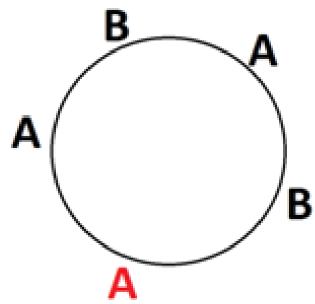



Figure 3

Then there is still one vote, on the circle, which starts a string that keeps Al wining at all stages. After listing all possible circles, we observe that in every circle, the probability that Al is ahead of Betty stays the same, which is 1/5. Therefore, the probability is 1/5 in this example. But, here is the question: is this just a coincidence that each circle has the same number of votes that can be counted as a starting vote for a string that keeps Al winning at any stage?

### 3. Proof of the Ballot Theorem

In Ballot problems, it is assumed that Betty has at least one vote; otherwise, the Ballot Theorem has no meaning. Let us revisit the question: what is the probability that Al always leads? If we want Al to always lead, then at any stage Al has to have more votes than Betty. Thus, as long as there is at least one vote left for Al after cancelling Betty's votes at any stage, Al leads all the time in such a string. Also, we know that the first and second votes of any successful string always have to be A-votes.

Suppose a string from a 'circle' meets the requirement that Al is always ahead of Betty. In such a string, whenever an A-vote meets a B-vote, we use that last A before a B-vote to cancel this vote of Betty. Once all Betty's votes are cancelled out, we will have some of Al's votes left which can be the starting votes for such a string in a circle. Why can each of these remaining A-votes be a starting vote of a string that satisfies the criteria? Only when we start with any of those remaining A-votes, are we guaranteed that some other A-votes will be available to be used to cancel B-votes. How do we find all possible leading A-votes? Note that if we know which successful string we start with to form a circle, then, we can work backwards on this circle to find the A-votes which are not used on the original string to cancel B's. For the leading A's to remain, it is necessary to use a non-leading A-vote to cancel a B-vote. If we use non-leading A-votes to cancel the B-votes on a string, then the remaining A's are all possible leading votes. Note that there are two cases for the location of the remaining A-votes on a string: one in which they are adjacent, and one in which at least one possible leading A-vote is separated from the other possible leading votes. Suppose we know one successful string that forms a circle.

**Case 1** *all possible leading votes adjacent on a string.*

Since we know that the first and second votes of any successful string always have to be A-votes, the remaining A's, which are all possible leading votes, start from the first vote on that string. Since the leading A's stay together on a string, there are no leading A-votes between or after non-leading A-votes on this string. Thus, a non-leading A-vote is the closest vote to some B on a string also on a circle (clockwise.) It follows that to find the leading A-votes backwards on a circle, we can use a B-vote to cancel the previous A-vote, which must be a non-leading vote. Since such strings are arranged around a circle, there is always an A before a B so there always exists a string in such a circle that satisfies the criteria which keeps Al winning all the time.

**Case 2** *at least one leading A-vote that is separated from the other leading A-votes*

Suppose we know all successful strings on such a circle, so we also know the leading A-votes. Pick any string from these successful strings on the circle, and then keep the votes from the first leading A-vote to the last B-vote before the A-vote which is the first leading vote separated from the other leading votes. If we string these votes around a circle, then this circle is exactly an example in Case 1. Back to the example of Case 2, since a B-vote cancels the previous A, the leading A-votes that come after the partial string will not be canceled by any B-vote from the previous partial string. Because the partial string that we pick is from any string of all successful strings on the circle, we can find all possible leading A's which remain on any circle after using a B to cancel the previous A-vote also in Case 2.

Since all strings are around circles, and on any circle $a > b$, there is always a closest previous A for a B to be cancelled. It follows that there exists a successful string in any circle in which Al always leads. Since one B is canceling the previous A-vote, there is a quantity $b$ of A-votes that would be cancelled. Then, there would be a quantity $a - b$ of A-votes left which can be leading votes for keeping the condition Al always leads on any circle. There are $a + b$ possible leading votes around any circle, which can lead a string whether Al always leads, or not. Therefore, the probability in this case is $\frac{a-b}{a+b}$, with $a > b$.

What if there are two or more strings on a circle that represent the same ordered string? For example:

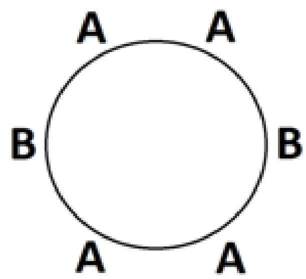



Figure 4

Is the probability, $\frac{a-b}{a+b}$, still accurate in such a circle? The answer is 'yes'. Assume there is a string which is repeated n times in a circle. Let X denote the leading vote of any of those repeated strings, and let Y denote the next leading vote after X of the repeated string. Since strings started with X and Y have the same ordered votes, votes between X and Y appear again after Y and before the next coming string repeating the same ordered votes as strings started with X and Y on the circle. Therefore, votes from X to the vote right before Y can be the leading votes for keeping Al leads all the time appear again after the last vote before Y. Thus, the number of votes satisfying the condition that Al leads all the time stays the same in any intervals between X and the last vote before Y inclusively. If we string these "intervals" into 'small circles,' then these circles will be exactly the same. Thus, the probability we get, $\frac{a-b}{a+b}$, is also the probability for each of the small circles that have no repeated strings on them. In conclusion, having repeated strings on any circles will not be a problem for counting the probability.

## 4. The Possibility of Ties

Suppose that we now modify the problem, asking for the probability that Al always leads or Al is tied with Betty? Unfortunately, the above method does not apply for finding the number of "ties." Since we now know which A's can be the leading votes of a string that has the property that Al always leads, obviously, the next A after those leading A's would lead to a string in which Al and Betty would be tied in at least one stage. When the A's are together, there is only one A available as a leading vote to have a tie string. In separated cases, there would be one A for leading a string having Al and Beth tied after each separated group of leading A-votes which can start strings that ensure that Al wins all the time. Some A's for keeping Al winning at any stage stay together, some A's are separated, and these A's could appear on any circles, so it is impossible to predict which circles have separated leading A-votes. Thus, we cannot find the probability that Al always leads or Al is tied with Betty by using circles to solve such a problem.

## *References*

[1] J. Bertrand, Solution d'un Problème, *Comptes Rendus de l'Académie des Sciences*, **105** (1887) 369.

[2] R. Durrett, *Essentials of Stochastic Processes*, Springer, 1999, p. 120.

# *The Problem Corner*

Edited by Pat Costello

*The Problem Corner* invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before February 1, 2012. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2012 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)-622-3051).

## NEW PROBLEMS 679-688

**Problem 679**. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Suppose that $f(x)$ is continuous and bounded on $(0,\infty)$ and the sequence $\{f(n)\}_{n=1}^{\infty}$ doesn't converge. Show that for any positive constant $M$, there exists an $x_0 > M$ such that $f(x_0+1) > f(x_0)$.

**Problem 680**. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Let

$$f_n(x) = \sum_{i=1}^{n}\sum_{j=1}^{n}\cos^i x\sin^j x - \sum_{i=1}^{n}\cos^i x - \sum_{j=1}^{n}\sin^j x + 1.$$

Show the following two things:

1. The function $f_n(x)$ has exactly two zeros in the interval $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ for $n = 2, 3, \ldots$.
2. If we denote the smaller zero and larger zero as $a_n$ and $b_n$, respectively, then
$$\lim_{n\to\infty} a_n = \frac{\pi}{6} \text{ and } \lim_{n\to\infty} b_n = \frac{\pi}{3}.$$

**Problem 681**. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $a$, $b$, $c$ be the lengths of the sides of an acute triangle $ABC$. Prove that

$$\sum_{\text{cyclic}} \left(\cos^a B \cos^b A\right)^{1/(a+b)} < 2.$$

**Problem 682**. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $a$, $b$, $c$ be three positive numbers such that $a^2 + b^2 + c^2 = 1$. Prove that

$$\left[\frac{1}{a^3 (b+c)^5} + \frac{1}{b^3 (c+a)^5} + \frac{1}{c^3 (a+b)^5}\right]^{1/5} \geq \frac{3}{2}.$$

**Problem 683**. *Proposed by Pedro H.O. Pantoja (student), University of Natal, Brazil.*

Let$F_n = 2^{2^n} + 1$, the $n$th Fermat number. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{\pi (F_1) + \pi (F_2) + \cdots + \pi (F_n)} \leq 2,$$

where $\pi (x)$ denotes the number of primes less than or equal to $x$.

**Problem 684**. *Proposed by Ovidiu Furdui, Campia Turzii, 405100, Cluj, Romania.*

Calculate $\int_0^1 x \ln\left(\sqrt{1+x} - \sqrt{1-x}\right)\, dx$.

**Problem 685**. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, MO.*

Assume that $f(x)$ is continuous and the integral

$$I = \int_c^d \frac{f(a-x)}{f(a-x) + f(x-b)}\, dx$$

exists. Evaluate $I$.

**Problem 686**. *Proposed by Panagiote Ligouras, Leonardo da Vinci High School, Noci, Italy.*

The lengths of the sides of the hexagon $ABCDEF$ satisfy $3AB = BC$, $3CD = DE$, $3EF = FA$. Prove that

$$\frac{AF}{CF} + \frac{CB}{EB} + \frac{ED}{AD} \geq \frac{9}{4}.$$

**Problem 687**. *Proposed by the editor*.

On a calculus test, one student wrote that the derivative of the product of three functions $f(x)$, $g(x)$, $h(x)$ was equal to $f'(x)\,g'(x)\,h(x) + f'(x)\,g(x)\,h'(x) + f(x)\,g'(x)\,h'(x)$. While this is not the correct formula, it does work sometimes. Do the following two things:

1. Prove that if the functions are all linear functions and this formula holds, either the functions are all constant or one is the zero function.
2. Find an infinite collection of sets of three non-constant functions
$$\{f(x), g(x), h(x)\}$$
where this formula gives the correct derivative of the product of three functions.

**Problem 688**. *Proposed by the editor.*

Find a 6-digit prime integer $x$ where all of its digits are prime and every pair of consecutive digits is a prime. Find a 12-digit prime integer $y$ with the same property.

# SOLUTIONS 659-668

**Problem 659**. *Proposed by Andrew Cusumano, Great Neck, NY.*

Find the value of the infinite series

$$\sum_{n=1}^{\infty} \frac{2n}{n^4+n^2+1}.$$

**Solution** *by Rustyn VanDeventer (student), OK Alpha, Northeastern State University, Tahlequah, OK.*

The series is a telescoping series. The denominator factors as

$$\left(n^2-n+1\right)\left(n^2+n+1\right).$$

When we set

$$\frac{A}{n^2-n+1}+\frac{B}{n^2+n+1}=\frac{2n}{n^4+n^2+1},$$

we get $A=1$ and $B=-1$. So

$$\frac{2n}{n^4+n^2+1}=\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}.$$

The partial sum is

$$\begin{aligned} S_k &= \sum_{n=1}^{k} \frac{2n}{n^4+n^2+1}=\sum_{n=1}^{k}\left(\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right) \\ &= \left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{13}\right)+\cdots \\ &\quad +\left(\frac{1}{k^2-k+1}-\frac{1}{k^2+k+1}\right) \\ &= 1-\frac{1}{k^2+k+1} \text{ since } (n+1)^2-(n+1)+1=n^2+n+1. \end{aligned}$$

Thus

$$\sum_{n=1}^{\infty} \frac{2n}{n^4+n^2+1}=\lim_{k\to\infty} S_k=\lim_{k\to\infty}\left(1-\frac{1}{k^2+k+1}\right)=1.$$

*Also solved by PA Kappa Problem Solving Group, Holy Family University, Philadelphia, PA; Amanda Goodrick (student), PA Pi, Slippery Rock University, Slippery Rock, PA; Carly Campbell and Brieann Pedro (students), Cal State University - Fresno, Fresno, CA; Jessie Deering (student), TN Beta, East Tennessee State University, Johnson City, TN; Pedro H. O. Pantoja, UFRN, Natal - RN, Brazil; and the proposer.*

**Problem 660**. *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Let $n$ be a positive integer greater than 1. Let $f^{(0)}(x) = f(x) = (x-1)(x-2)\cdots(x-n)$. Let $f^{(i)}$ be the $i^{th}$ derivative of $f(x)$. Let $S_i$ denote the sum of all zeros of $f^{(i)}(x)$. Show that

$$\sum_{i=0}^{n-1} S_i = \frac{1}{n}\sum_{i=1}^{n} i^3.$$

**Solution** *by Robert Gardner, TN Beta, East Tennessee State University, Johnson City, TN.*

First we prove a lemma.

**Lemma**: *The average of the zeros of a polynomial is the same as the average of the zeros of its derivative.*

Proof: Let the zeros of a polynomial $p$ of degree $n$ be $z_1$, $z_2$, ..., $z_n$, so that

$$p(z) = \sum_{k=0}^{n} a_k z^k = a_n \prod_{k=1}^{n} (z - z_k).$$

Equating coefficients of $z^{n-1}$ gives

$$a_{n-1} = -a_n (z_1 + z_2 + \cdots z_n),$$

so that the average of the zeros of $p(z)$ is

$$\frac{z_1 + z_2 + \cdots z_n}{n} = -\frac{a_{n-1}}{na_n}.$$

Let the zeros of $p'$ be $w_1$, $w_2$, ..., $w_{n-1}$. Then

$$p'(z) = \sum_{k=1}^{n} k a_k z^{k-1} = a_n \prod_{k=1}^{n-1} (z - w_k).$$

As above, the average of the zeros of $p'$ is

$$\frac{w_1 + w_2 + \cdots + w_{n-1}}{n-1} = \frac{1}{n-1}\left[\frac{-(n-1)a_{n-1}}{na_n}\right] = -\frac{a_{n-1}}{na_n}.$$

Therefore, the averages of the zeros of $p$ and $p'$ are the same.

Now we can prove the claim. For

$$f^{(0)}(x) = f(x) = (x-1)(x-2)\cdots(x-n),$$

and we see that the average of the zeros is $\dfrac{1+2+\cdots+n}{n} = \dfrac{n+1}{2}$.

Since there are $n$ zeros of $f^{(0)}(x)$, the sum of the zeros is $S_0 = \dfrac{n(n+1)}{2}$. Next, $f^{(i)}(x)$ is an $n-i$ degree polynomial which, by repeated application of Rolle's Theorem, has $n-i$ distinct real zeros. The average of its zeros is $\dfrac{n+1}{2}$. Thus the sum of the zeros of $f^{(i)}$ is $S_i = \dfrac{(n-i)(n+1)}{2}$. Summing the $S_i$'s, we have

$$\begin{aligned}\sum_{i=0}^{n-1} S_i &= \sum_{i=0}^{n-1} \frac{(n-i)(n+1)}{2} = \frac{n+1}{2}\sum_{i=1}^{n} i = \frac{n+1}{2}\cdot\frac{n(n+1)}{2} \\ &= \frac{1}{n}\left[\frac{n(n+1)}{2}\right]^2 = \frac{1}{n}\sum_{i=1}^{n} i^3.\end{aligned}$$

*Also solved by Gerhardt Hinkle (student), Central High School, Springfield, MO; and the proposer.*

**Problem 661.** *Proposed by Hongbiao Zeng, Fort Hays State University, Hays, KS.*

Evaluate the double sum

$$\sum_{k=0}^{\infty}\sum_{i=1}^{\infty}(-1)^k \int_0^1 \frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}\,dx.$$

**Solution** *by Gerhardt Hinkle (student), Central High School, Springfield, MO.*

We note that

$$\begin{aligned}&\frac{d}{dx}\frac{1}{i^2(2k+1)}\left[1+\frac{-1+x^{i^2(2k+1)}}{\left(1+x^{2k+1}\right)^{i^2}}\right] \\ =\ & \frac{1}{i^2(2k+1)}\frac{d}{dx}\left[1+\left(-1+x^{i^2(2k+1)}\right)\left(1+x^{2k+1}\right)^{-i^2}\right] \\ =\ & \frac{1}{i^2(2k+1)}(A-B),\end{aligned}$$

where

$$A = i^2(2k+1)\,x^{i^2(2k+1)-1}\left(1+x^{2k+1}\right)^{-i^2},$$

and

$$B = i^2(2k+1)\,x^{2k}\left(1+x^{2k+1}\right)^{-i^2-1}\left(-1+x^{i^2(2k+1)}\right).$$

Thus,

$$\begin{aligned}
&\frac{d}{dx}\frac{1}{i^2\left(2k+1\right)}\left[1+\frac{-1+x^{(2k+1)(i^2)}}{\left(1+x^{2k+1}\right)^{i^2}}\right]\\
=\ & x^{2k}\left[\frac{x^{(i^2-1)(2k+1)}}{\left(1+x^{2k+1}\right)^{i^2}}-\frac{-1+x^{i^2(2k+1)}}{\left(1+x^{2k+1}\right)^{i^2+1}}\right]\\
=\ & \frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}.
\end{aligned}$$

Thus,

$$\int\frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}\,dx=\frac{1}{i^2\left(2k+1\right)}\left[1+\frac{-1+x^{(2k+1)(i^2)}}{\left(1+x^{2k+1}\right)^{i^2}}\right],$$

and

$$\int_0^1\frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}\,dx=\frac{1}{i^2\left(2k+1\right)}\left(1-0\right)=\frac{1}{i^2\left(2k+1\right)}.$$

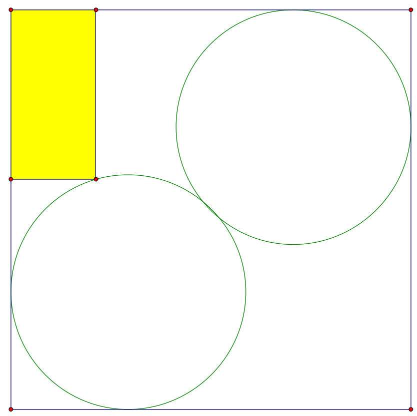
Plugging this into the double sum gives

$$\begin{aligned}
\sum_{k=0}^{\infty}\sum_{i=1}^{\infty}(-1)^k\int_0^1\frac{x^{2k}\left[1+x^{(2k+1)(i^2-1)}\right]}{\left(1+x^{2k+1}\right)^{i^2+1}}\,dx &= \sum_{k=0}^{\infty}\sum_{i=1}^{\infty}\frac{(-1)^k}{i^2\left(2k+1\right)}\\
&= \sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\sum_{i=1}^{\infty}\frac{1}{i^2}\\
&= \frac{\pi^2}{6}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}\\
&= \frac{\pi^2}{6}\cdot\frac{\pi}{4}\\
&= \frac{\pi^3}{24}.
\end{aligned}$$

*Also solved by the proposer.*

**Problem 662.** *Proposed by Ken Dutch, Eastern Kentucky University, Richmond, KY.*

In the following diagram, the shaded rectangle measures 2 cm by 4 cm. What is the radius of the circles in centimeters?



**Solution** *by the proposer.*

Let $r$ be the radius of the circles. Notice that the SW-NE diagonal of the square is made up of four segments with lengths (from left to right): $r\sqrt{2}$, $r$, $r$, $r\sqrt{2}$. Thus by the Pythagorean theorem, the length, $s$, of the side of the square must satisfy $s^2 + s^2 = \left(2r + 2r\sqrt{2}\right)^2$. Hence

$$s = \frac{\left(2 + 2\sqrt{2}\right) r}{\sqrt{2}} = \left(\sqrt{2} + 2\right) r.$$

In the lower circle, draw a radius to the point at which the rectangle touches the circle, and drop a vertical segment to the horizontal diameter. By the Pythagorean theorem, we have

$$(r-2)^2 + (s-r-4)^2 = r^2.$$

Combining these two equations gives a quadratic which can be solved by the quadratic formula. The only solution that works is

$$r = \frac{6 + 4\sqrt{2} + 2\sqrt{2 + 2\sqrt{2}}}{3 + 2\sqrt{2}} \approx 2.754.$$

*Also solved by Jacob Curley and Jon Janzen (students), OK Alpha, Northeastern State University, Tahlequah, OK; and Elias Alvarez (student), Cal State -Fresno, Fresno, CA.*

**Problem 663.** *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let $0 < a < b$ and $f : [a, b] \to \mathbb{R}$ be a continuous function. If $A(x)$ is a polynomial with real coefficients for which $A(a) < (b-a)^2 < A(b)$, show that there exist $\alpha, \beta \in (a, b)$ such that

$$\int_a^b f(x)\, dx = f(\alpha)\sqrt{A(\beta)}.$$

**Solution** *by the proposer.*

Let $0 < a < b$, and let $f : [a, b] \to \mathbb{R}$ be a continuous function. Applying the Mean Value Theorem for Integrals, we have that there exists $\alpha \in (a, b)$ such that $\int_a^b f(x)\, dx = (b-a) f(\alpha)$. Let $B(x) = A(x) - (a-b)^2$. Then from $A(a) < (b-a)^2 < A(b)$, we get

$$B(a) B(b) = \left[A(a) - (a-b)^2\right]\left[A(b) - (a-b)^2\right] < 0.$$

Applying Bolzano's Theorem, we have that there exists $\beta \in (a, b)$ such that $B(\beta) = 0$, from which we obtain $A(\beta) = (b-a)^2$. Taking square roots in both terms of the preceding expression yields $\sqrt{A(\beta)} = b - a$, and we are done.

**Problem 664.** *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all triplets $(a, b, c)$ of real numbers that satisfy the equations

$$a^6 = 5b^2 - 2, b^6 = 5c^2 - 2, c^6 = 5a^2 - 2.$$

**Solution** *by Amanda Goodrick, PA Pi, Slippery Rock University, Slippery Rock, PA.*

Suppose $a^2 \geq b^2$. Then $c^6 = 5a^2 - 2 \geq 5b^2 - 2 = a^6$. This means that $c^6 - a^6 \geq 0$, so that $\left(c^2 - a^2\right)\left(c^4 + c^2a^2 + a^4\right) \geq 0$. Thus $c^2 \geq a^2$ so that $b^6 \geq c^6$, and $b^2 \geq c^2$. We have shown that $c^2 \geq a^2 \geq b^2 \geq c^2$ and so $a^2 = b^2 = c^2$. A similar argument leads to the same result when $b^2 \geq a^2$. Therefore the given hypotheses imply that $a^2 = b^2 = c^2$. Thus $a^6 - 5a^2 + 2 = 0$, and so $\left(a^2 - 2\right)\left(a^4 + 2a^2 - 1\right) = 0$. Therefore $a^2 = 2$ or $a^2 = -1 + \sqrt{2}$, implying that $a = \pm\sqrt{2}$ or $a = \pm\sqrt{-1 + \sqrt{2}}$. There are eight solutions corresponding to $a^2 = 2$ and eight corresponding to $a^2 = -1 + \sqrt{2}$, for a total of 16 solutions.

*Also solved by Robert Gardner, TN Beta, East Tennessee State University, Johnson City, TN; and the proposer.*

**Problem 665.** *Proposed by Jason Gibson, Eastern Kentucky University, Richmond, KY.*

Let $T$ be the set of integers greater than 1 whose prime divisors live in the set $\{2, 3, 5\}$. What is the sum of the reciprocals of the integers in the set $T$?

**Solution** *by Cade Herron (student), TN Beta, East Tennessee State University, Johnson City, TN.*

The particular solution is $\frac{11}{4}$. We prove a more general case. Let $T$ be the set of integers greater than 1 whose prime divisors live in the set $\{p_1, p_2, p_3\}$, where these are distinct primes. Let $z \in T$. Then $z = p_1^i p_2^j p_3^k$, where $i, j, k$ are nonnegative integers. Thus $z^{-1} = \frac{1}{p_1^i p_2^j p_3^k}$. Since $T$ does not include 1, in order to find the sum of the reciprocals of all the integers in $T$, we compute

$$\begin{aligned}
\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{p_1^i p_2^j p_3^k} - 1 &= \sum_{i=0}^{\infty}\frac{1}{p_1^i}\sum_{j=0}^{\infty}\frac{1}{p_2^j}\sum_{k=0}^{\infty}\frac{1}{p_3^k} - 1 \\
&= \frac{1}{1-\frac{1}{p_1}} \cdot \frac{1}{1-\frac{1}{p_2}} \cdot \frac{1}{1-\frac{1}{p_3}} - 1 \\
&= \frac{p_1}{p_1 - 1} \cdot \frac{p_2}{p_2 - 1} \cdot \frac{p_3}{p_3 - 1} - 1 \\
&= \frac{p_1 p_2 p_3}{(p_1 - 1)(p_2 - 1)(p_3 - 1)} - 1.
\end{aligned}$$

For $(p_1, p_2, p_3) = (2, 3, 5)$, we get 11/4.

*Also solved by Rex Edmonds (student), PA Pi, Slippery Rock University, Slippery Rock, PA; OK Alpha chapter, Northeastern State University, Tahlequah, OK; Jeff Hanson and Conlan Simons, Cal State - Fresno, Fresno, CA; and the proposer.*

**Problem 666.** *Proposed by Ovidiu Furdui, Campia Turzii, 405100, Cluj, Romania.*

Let $a$ be a positive real number. Find the value of

$$\lim_{n\to\infty} \sqrt[n]{\int_0^1 (1+ax^n)^n \, dx}.$$

**Solution** *by the proposer.*

The limit equals $1+a$. Let $x_n = \sqrt[n]{\int_0^1 (1+ax^n)^n \, dx}$. Integrating by parts, we have

$$\begin{aligned}
& \int_0^1 (1+ax^n)^n \, dx \\
= \quad & x(1+ax^n)^n\big|_0^1 - n^2 a \int_0^1 x^n (1+ax^n)^{n-1} \, dx \\
= \quad & (1+a)^n - n^2 \left[ \int_0^1 (1+ax^n)^n \, dx - \int_0^1 (1+ax^n)^{n-1} \, dx \right] \\
= \quad & (1+a)^n - n^2 x_n^n + n^2 \int_0^1 (1+ax^n)^{n-1} \, dx \\
\geq \quad & (1+a)^n - n^2 x_n^n.
\end{aligned}$$

It follows that $x_n \cdot \sqrt[n]{1+n^2} \geq 1+a$. We find, since $x_n \leq 1+a$, that

$$\frac{1+a}{\sqrt[n]{1+n^2}} \leq x_n \leq 1+a,$$

and the limit is $1+a$.

*Also solved by Luis Belman and Cheng Siong (students), Cal State University - Fresno, Fresno, CA.*

**Problem 667.** *Proposed by Russell Euler, Northwest Missouri State University, Maryville, MO.*

For $n \geq 1$, find all Pythagorean triples $(T_n, S_n, P_n)$ where $T_n$, $S_n$, and $P_n$ are the $n^{th}$ triangular, square, and pentagonal numbers, respectively.

**Solution** *by Rex Edmonds (student), PA Pi, Slippery Rock University, Slippery Rock, PA.*

Below are the formulas for the $n$th triangular, square, and pentagonal numbers.

| Number | Formula |
|---|---|
| Triangular | $\frac{n(n+1)}{2}$ |
| Square | $n^2$ |
| Pentagonal | $\frac{n(3n-1)}{2}$ |

Placing these formulas into the Pythagorean equation and solving for $n$, we have

$$\begin{aligned} \left[\frac{n(n+1)}{2}\right]^2 + \left(n^2\right)^2 &= \left[\frac{n(3n-1)}{2}\right]^2 \\ n^4 + 2n^3 + n^2 + 4n^4 &= 9n^4 - 6n^3 + n^2 \\ 4n^4 &= 8n^3 \\ n &= 2. \end{aligned}$$

Putting 2 into the above formulas produces only the Pythagorean triple $(3, 4, 5)$.

*Also solved by Catawba College Math Club, NC Zeta, Catawba College, Salisbury, NC; Jessie Dering (student), TN Beta, East Tennessee State University, Johnson City, TN; William Jamieson (student), TN Beta, East Tennessee State University, Johnson City, TN; Ed Wilson, Eastern Kentucky University, Richmond, KY; and the proposer.*

**Problem 668.** *Proposed by the editor.*

Prove that the sum of the cubes of three consecutive positive integers can never equal the sum of the squares of two integers which are relatively prime.

**Solution** *by Patrick James and Brian Tucker (students), Cal State - Fresno, Fresno, CA.*

Preliminary assumption and facts:

1. Fact: Let $a > 0$. Then the three consecutive integers $at, a+1$, $a+2$ have the property $a^3 + (a+1)^3 + (a+2)^3 = 3a^3 + 9a^2 + 15a + 9 = 3\left(a^3 + 3a^2 + 5a + 3\right) \equiv 0 \bmod 3$.
2. Fact: Any integer squared is congruent to either 0 or 1 $\bmod 3$.
3. Assumption: The integers $b$ and $c$ are relatively prime.

We proceed by contradiction. Assume that $a$, $b$, and $c$ are integers with $a > 0$ and $(b, c) = 1$, and suppose that

$$a^3 + (a+1)^3 + (a+2)^3 = b^2 + c^2.$$

By fact 1, $0 \equiv b^2 + c^2 \bmod 3$, so that $b^2 \equiv -c^2 \bmod 3$. By fact 2, we can consider two cases.

- Suppose $b^2 \equiv 1 \bmod 3$. This implies that $c^2 \equiv -1 \equiv 2 \bmod 3$, contradicting fact 2.
- Suppose $b^2 \equiv 0 \bmod 3$. This implies that $c^2 \equiv 0 \bmod 3$. Then $3|b^2$ and $3|c^2$ so that $3|b$ and $3|c$. This implies that $(b, c)$ is a multiple of 3, contradicting $(b, c) = 1$.

*Also solved by Rho Middleton (student), OK Alpha, Northeastern State University, Tahlequah, OK; and the proposer.*

# *Kappa Mu Epsilon News*

Edited by Peter Skoner, Historian

**Updated information as of April 2011**

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive
313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

## Installation Report

Georgia Epsilon
Wesleyan College

The Georgia Epsilon Chapter of Kappa Mu Epsilon was installed at 11:15 a.m. on Tuesday, March 30, 2010, at a ceremony in Munroe Science Center on the campus of Wesleyan College, located in Macon, Georgia. The meeting was conducted by Jennifer Aust. KME President Ron Wasserstein served as the Installing Officer. The charter members, Supriya Shrestha, Bhumika Thapa, Mona Shrestha, Ankit Pokhrel, Shreejaya Shrestha, Sudha Regmi, Xiaochen Dong, Yiwei Han, Feiya Zhao, Dahlia Wright, Azea Mustafa, Sadikshya Adhikary, and Sadichha Sitaula were initiated into the chapter. The first officers of Georgia Epsilon, President Azea Mustafa, Vice President Shreejaya Shrestha, Recording Secretary Xiaochen Dong, Treasurer Bhumika Thapa, and Corresponding Secretary/Faculty Sponsor Joe Iskra.

About twenty people were in attendance. After the formal ceremonies, Ron Wasserstein presented a talk entitled "What Probability and Forrest Gump Teach Us About the Georgia Lottery."

Massachusetts Beta Chapter
Stonehill College

The installation of the Massachusetts Beta Chapter of Kappa Mu Epsilon was held in the Merkert-Tracy Administrative Building, on the campus of Stonehill College in North Easton on Friday, April 8, 2011, at 4:30 p.m.

In attendance were the seven charter faculty members who were initiated, including Professors Ralph Bravaco, Fr. Rudy Carchidi, Carlos Curley, Norah Esty, Eugene Quinn, Hsin-hao Su, and Timothy Woodcock; nine of the 12 charter students members including Laura Bercume, Sarah Chiodi, Meghan Galiardi, Cortney Logan, Stephanie Martino, Kristen Mattson, Daniel Perry, and Jamie Long; installing officer National Historian Peter Skoner; and 22 family members of initiates for a total of 39 people in attendance. Charter student members Lauren Balla, Alyssa Harel, Katherine McCue, and Kathleen Zarnitz did not attend.

The afternoon celebration began with a buffet dinner including salad, shrimp, ravioli, and very popular chocolate cake. Professor Woodcock welcomed the initiates and guests to the ceremony. Installing officer Peter Skoner followed with a welcome from the Kappa Mu Epsilon national council, and an introduction to the aims, activities, and history of the organization.

For the installation ceremony, Laura Bercume and Cortney Logan had speaking parts in the installation ritual. After initiates accepted their membership pledges, each initiate was invited to sign the Massachusetts Beta Chapter Roll, and accepted a membership card, a KME brochure, a program announcing the charter initiates, their KME certificate, and a KME jewelry pin. Following the description of the crest, Cortney accepted the framed charter for the newly installed Massachusetts Beta Chapter of Kappa Mu Epsilon. Then the charter chapter officers were installed including: President Cortney Logan, Vice President Laura Bercume, Secretary Kathleen Zarnitz (with member Jamie Long accepting), Treasurer Alyssa Harel (with member Dan Perry accepting), Faculty Sponsor Professor Ralph Bravaco, and Corresponding Secretary Professor Timothy Woodcock. Each officer was charged with the responsibilities of the office, and each chose to accept those responsibilities. Several large rounds of applause followed each significant part of the ceremony.

Following the ceremony, charter member Meghan Galiardi presented "Facial Recognition using Conformal Geometry," a summary of her summer 2010 REU research at Central Michigan University. The evening concluded with many camera flashes, congratulations, fellowship, pleasant conversation, and the partitioning of the remaining chocolate cake.

# Chapter News

**AL Alpha – Athens State University**

*Chapter President – Melisa Dutton; 240 Current and 21 New Members*
*Other Fall 2010 Officers: Carl Kuby, Vice President; Shannon Harwell, Secretary; and Patricia Glaze, Corresponding Sec. and Faculty Sponsor*

We participated with the Math and Computer Science (MACS) Club for two events: During the SGA-sponsored "Welcome Back Week" we helped promote membership in MACS. We also assisted MACS during The Old Time Fiddler's Convention by selling smoked BBQ ribs to raise money for charities.

**AL Epsilon – Huntingdon College**

*Dr. Sally Clark, Corresponding Secretary*

New Initiates – Ashleigh Karis Anderson, Tin May Aye, Johnathan Brett Barnett, and Hannah Elizabeth Correia.

**AL Zeta – Birmingham Southern College**

*Chapter President – Rebecca Terry; 17 Current Members*
*Other Fall 2010 Officers: Amy Schumacher, Vice President; Stephanie Gosset, Secretary; Bernadette Mullins, Corresponding Secretary and Faculty Sponsor*

Alabama Zeta hosted Dr. John Mayer of the University of Alabama at Birmingham in November who spoke on Mathematical Models for Fairness; he presented an interactive colloquium in which the audience was responsible for the fair division of property and power. They wrestled with issues such as that posed by the following problem: Andy, Bert, and Connie are farmers. Their neighbor who is also a farmer is retiring next month and wishes to sell her 12 pigs for $480. Andy, Bert, and Connie can only afford to purchase the pigs if they pool their money. Andy can contribute $97, Bert can contribute $210, and Connie can contribute $173. How many pigs each should Andy, Bert, and Connie get? Explain why your distribution is a fair division of the pigs. (Note: No pigs may be harmed or shared in your solution.)

**AL Theta – Jacksonville State University**

*Dr. David W. Dempsey, Corresponding Secretary*

New Initiates - William Justin Beam, Wesley Stone Campbell, Allison Leighanne Clark, Felisha Nelson Cleland, Stephen Coggins, Courtney Marie Crosby, Rachel Lynn Howell-Farley, April Dianne Franklin, Jordan Comelus Fuller, Elizabeth Mae Garnett, Tiffany D. Hill, Megan Michelle Lightsey, Evan Michael Mince, Tara Modesa Naugher. Noel Overton, Jr., Sharon Padgett, Amanda Camille Pitts, Russell Price, Sarah Elizabeth Pugh, Matthew Allen Sosebee, Christian Hope Whitfield, Ashlin Donielle Young, and Kara Marie Young.

**CO Delta – Mesa State College**

*Erik Packard, Corresponding Secretary*

New Initiates – Melissa Asay, Krystal Arnett, Robert Atkins, Katee Denham, Gordon Gibson, Jonathan Lusk, David Miller, Ethan Stanley, and Brittelle Thorpe.

**FL Beta – Florida Southern College**

*Allen Wuertz, Corresponding Secretary*

New Initiates – Melissa J. Adams, Brian R. Covello, Robert Wesley Crues, Kelly A. Madden, Joshua Ryan Newell, Alex Paradis, Spencer D. Parry, Lindsay C. Snyder, Christopher G. Stahl, and Sarah Ilyta Studebaker.

**GA Beta – Georgia College and State University**

*Laurie Huffman, Corresponding Secretary*

New Initiates – Kelsey Davis, Trey Gay, Thomas Pangia, Lauren Tripi, Kendyl Wade, and Scott Wofford.

**IA Alpha – University of Northern Iowa**

*Chapter President – Jaime Zeigler; 30 Current and 5 New Members*

*Other Fall 2010 Officers: Tristam Nebelsick, Vice President; Kelsey Staudacher, Secretary; David Rygh, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor*

Our first meeting was held on September 27, at Professor Mark Ecker's house where student member Samantha Jaeger presented her paper entitled "Factors of Household Income." Student member Khang Ng presented his paper entitled "Goal Scorers for Manchester United Soccer Players" at our second meeting on November 3 at Professor Russ Campbell's home. Student member Kelsey Staudacher addressed the initiation banquet with "Analysis of State Graduation Rates." Our banquet was held at Pepper's Grill and Sports Pub in Cedar Falls on December 8, where five new members were initiated.

New Initiates – Hannah Andrews, Allison Meier, Elizabeth Mastalio, David Ta, and Nicole Weis.

**IL Beta – Eastern Illinois University**

*Nancy Van Cleave, Corresponding Secretary*

New Initiates - Sylvia Carlisle, Marlon Chatman, Christopher DeSanto, Daniel Dulaney, Renee Fietsam, Julie Huber, Ethan Ingram, Catherine Kruger, Kelly Price, Jessica Ringler, James Romack, and Will Zukowski.

**IL Zeta – Dominican University**

*Chapter President – Kim Plesnicar*

*Other Fall 2010 Officers: Daniel Dziarkowski, Vice President; Eva Mehta, Secretary; Lisa Gullo, Treasurer; and Aliza Steurer, Corresponding Secretary and Faculty Sponsor*

This semester we focused our efforts on fundraising. We held a popcorn sale and an origami ornament sale. For the letter, Professor Paul Coe gave a workshop on how to make a few simple origami platonic solids. We spent a couple of afternoons making several copies of these ornaments in different colors and sold them at the end of the semester. It was a great bonding experience and we earned a decent amount of money!

**IL Theta – Benedictine University**

*Chapter President – Michael Whitley; 253 Current Members*

*Other Fall 2010 Officers: Michael Mutersbaugh, Vice President; Victoria Blumen, Secretary; Jared Gustafson, Treasurer; Dr. Thomas Wangler, Corresponding Secretary; and Dr. Jeremy Nadolski, Faculty Sponsor*

**IN Beta – Butler University**

*Chapter President – Sarah Prusinski; 20 Current and 13 New Members*

*Other Fall 2010 Officers: Ashley Drees, Vice President; Kristen Allen, Secretary; Eric Buenger, Treasurer; and Dr. Amos Carpenter, Corresponding Secretary and Faculty Sponsor*

In addition to our monthly meetings, we had two invited speakers.

New Initiates - Roshni Agarwal, Kristen Allen, Eric Buenger, Xi Chen, Rachel Colby, Ben Craw, Ashley Drees, Ashley Hanson, Alaina Kenney, Zachary Lovall, Sarah Prusinski, Casey Szulc, and Katherine Wainwright.

**IN Gamma – Anderson University**

*Dr. Stanley L. Stephens, Corresponding Secretary*

New Initiates – Matthew J. Danskey, Dorothy G. Clements, Mathew S. Preston, Joseph A. Davidson, Mengjiao Tan, Carrie M. Steinke, Nabin Timsina, and Amy L. Wuestefeld.

**KS Alpha – Pittsburg State University**

*Chapter President – Vanessa Peach*

*Other Fall 2010 Officers: Jordan Jameson, Vice President; Aisha Ford, Secretary; Wes Brook, Treasurer; Dr. Tim Flood, Corresponding Secretary; and Dr. Cynthia Woodburn, Faculty Advisor*

New Initiates – Bilal Abdullah, Matthew Haffner, Steven Huskey, and Lissa Mentzer.

**KS Beta - Emporia State University**

*Chapter President - Yuchen Chen; 35 Current and 11 New Members*

*Other Fall 2010 Officers: Jennifer Long, Vice President; Yuying Cao, Secretary; Whitney Turley, Treasurer; and Dr. Connie Schrock, Corresponding Secretary and Faculty Sponsor*

We had a very busy semester with mathematics and social events monthly. Events included bowling, movies and mathematical games. At the end of the semester we took a trip to Kansas City to visit the rare mathematics book collections at the Linda Hall Library. After the visit we went to the Plaza for dinner. One of our continuing service projects is to help College Algebra and other students learn how to use graphing calculators.

New Initiates - Jessica Anderson, Yuying Cao, Daria Cuznetova, Heather Czechowski, Jared Dyche, Keely Grossnickle, Yuan Guo, Yangrong Jia, Ye Kang, Lezley Lawson, Xiongya Li, Fan Liu, Jennifer Long, Chase McIver, Andrew Rees, Sheila Sarrafi, Alexandra Schmaderer, Emily Schmar, Yusuke Suita, Jungshuang Sun, Chris Teeter, Cade Witte, and Tiayue Zhao.

**KS Delta – Washburn University**

*Chapter President – Stephen Littleton; 26 Current Members*

*Other Fall 2010 Officers: Sean Van Dyke, Vice President; Anna Lischke, Secretary and Treasurer; Dr. Mike Mosier, Corresponding Secretary; and Dr. Kevin Charlwood, Faculty Sponsor*

**KY Alpha – Eastern Kentucky University**

*Chapter President – Jennifer Fischesser; 11 Current Members*

*Other Fall 2010 Officers: Kristin Eppinghoff, Vice President; Michael Mazzotta, Secretary; Ryan Whaley, Treasurer; and Pat Costello, Corresponding Secretary and Faculty Sponsor*

The Fall semester included a meeting to elect officers and discuss plans for the new year. In December, we had a meeting where we did a White Elephant gift exchange.

**KY Beta – University of the Cumberlands**

*Chapter President – Amy Roberts; 29 Current Members*

*Other Fall 2010 Officers: Megan Barrowman Brown, Vice President; Jerrid Neeley, Secretary; Clint Creekmore, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary; and Dr. John Hymo, Faculty Sponsor*

Along with the Mathematics and Physics Club and Sigma Pi Sigma, the chapter had a picnic at Briar Creek Park on October 7. On December 10, the entire department, including the Kentucky Beta chapter, had a Christmas party with about 41 people in attendance.

New Initiates – Megan Barrowman, Aaron Bruce, Nathan Centers, Marie Dennison, Delilah Devore, Alissa Ellis, Lindsey Embry, Erin Engel, Whitney Horn, Candace Mack, Natalia McClellan, Jerrid Neeley, Olivia Neeley, Amy Roberts, Melodye Smith, and Michelle Weber.

**LA Gamma – Northwestern State University**

*Chapter President – Jessica Bass; 14 Current Members*

*Other Fall 2010 Officers: Baylen Johnson, Vice President; Carrie Faulk, Secretary; Phillip Adams, Treasurer; Leigh Ann Myers, Corresponding Secretary; and Lisa Galminas, Faculty Sponsor*

The Louisiana Gamma chapter of Kappa Mu Epsilon collected toys for children in the LSU Health Sciences Center Hospital in Shreveport, LA.

New Initiates – Carrie Falke, Baylen Johnson, and Jessica Ricks.

**MA Beta – Stonehill College**

*Timothy Woodcock, Corresponding Secretary*

New Initiates – Lauren Balla, Laura Bercume, Ralph Bravaco, Rudy Carchidi, CSC, Sarah Chiodi, Carlos Curley, Norah Esty, Meghan Galiardi, Alyssa Harel, Cortney Logan, Jamie Long, Stephanie Martino, Katherine McCue, Kristen Mattson, Daniel Perry, Eugene Quinn, Hsin-hao Su, Timothy Woodcock, and Kathleen Zarnitz.

**MD Delta – Frostburg State University**

*Chapter President – Joshua Wilson; 21 Current Members*

*Other Fall 2010 Officers: Rachel Skipper, Vice President; Jesse Otto, Secretary; Kevin Loftus, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor*

The Maryland Delta Chapter's fall activities commenced with a meeting in September featuring a lecture by our Vice President Rachel Skipper concerning her summer research experience with the McNair Program, giving an interesting presentation on Zipf's Law and the GDP's of emerging economies. Also in September was the chapter's participation in the Mathematics Department's annual Meet and Greet event for Mathematics, Physics and Engineering students and faculty. During October, the chapter represented the Mathematics Department at the annual Major's Fair on campus. Our October meeting featured a presentation on the fascinating Farey Sequence given by Professor Emeritus Edward White, a former faculty sponsor of the Maryland Delta Chapter. Our November meeting involved a workshop on Mathematica software given by Dr. Barnet, one of our current faculty sponsors.

New Initiates – Marcus Carter, Justin Good, Aaron Littlejohn, Jacob Pickwoad, Adam Rexroad, Luke Valenta, and Meghan Voelkel.

**MD Epsilon – Stevenson University**

*Chapter President – Rebecca Hollins; 35 Current and 19 New Members*

*Other Fall 2010 Officers: Megan Staudenmaier, Vice President; Diane Swale, Secretary; Rachel Buchanan, Treasurer; and Dr. Christopher E. Barat, Corresponding Secretary and Faculty Sponsor*

On September 21, the Chapter initiated 19 new members in a ceremony on Stevenson's Greenspring campus; the guest speaker was Dr. Bonita Saunders of the National Institute of Standards and Technology (NIST), who gave a talk entitled "Creating Interactive 3D Graphs for a Digital Library: So How is This Math?" The participants also watched a video tribute to Dr. Susan P. Slattery, Chair of the Department of Mathematics, who was killed in a car accident in August prior to the beginning of Fall classes. The Chapter's share of the proceeds from the Fall 50-50 Raffle, near $300, was donated to the newly established Dr. Susan P. Slattery Scholarship Fund, to provide financial support to female students in science or mathematics. KME members also assisted in the Stevenson School of the Sciences Reef Ball Project, which created concrete balls to be used in an artificial reef in Chesapeake Bay.

New Initiates - Amanda Boodhoo, Rachel Buchanan, Maria Carrera, Jennifer DeHoff, Tyler Demasky, Aline Dzaringa, Kellie Forsyth, Thomas Fuller, Grace Guerrier, Marie Guerrier, Amanda Hieatzman, Rebecca Hollins, Staci Hoover, Amanda King, Daniela Poss, Megan Staudenmaier, Monalee Swale, Joshua Vogel, and Lindsay Ward.

**MI Beta – Central Michigan University**

*Chapter President - David Creech*

*Other Fall 2010 Officers: Katerina Tiles, Vice President; Marie Ermete, Secretary; Nick Stephenson, Treasurer; Abram Demski, Public Relations; and Dr. Sivaram K. Narayan, Faculty Sponsor*

During the academic year KME met once every two weeks. Ten new members were initiated in the spring 2010. KME members raised money through a book sale held jointly with other student organizations in the department. KME members designed and sold t-shirts for Pi day (March 14) and "Never Drink and Derive" t-shirt during October. The money raised was used for buying pizza on meeting days and for conducting initiation ceremonies. Additional funding was sought through the College of Science and Technology. Dr. Tim Pennings from Hope College spoke on April 20, 2010, “Do Dogs Know Calculus?,” that was attended by over 150 students. Dr. Narayan gave a 5-10 minute talk at every meeting on different topics in mathematics and its applications, and spoke on the research opportunities for undergraduates here at CMU with both the REU and LURE programs. Two members presented their research from summer projects on September 15th, and one gave a talk at the Michigan Undergraduate Mathematics Conference, held at Grand Valley State University on October 16th; other members also attended the conference. On October 23rd, members spent an evening at Uncle Jon’s Cider Mill (the haunted barn was awesome) followed by the Terror on 27. Six members formed two teams and took part in the 16th Annual Michigan Autumn Take Home (MATH) Challenge on November 6th; one team placed 6th and the other placed 18th out of the 67 teams. Dr. Xiaomeng Zheng spoke on application of mathematics in cancer research on November 10th. On December 4th six members took part in the William Lowell Putnam Mathematical Competition.

New Initiates - Joan Barry, Alyssa Benetti, Joseph Bibi, Michael Black, Michael Came, David Creech, Earle Crosswait, Robert Cundy, Brittany DeGroot, Angela Enck, Zachary Gillette, Veronica Lach, Mark Pelfrey, Mickey Redmond, Katherine Revenaugh, Nick Stephenson, Katerina Tiles, Shawn Witte, and Philip Zerull.

**MI Delta – Hillsdale College**

*Chapter President – Kerry Frost; 46 Current and 14 New Members*

*Other Fall 2010 Officers: Juliana O'Neill, Vice President; Jonathan Gregg, Secretary; Meredith Longlois, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor*

To announce the new officers elected, we held a Kick-Off Picnic on September 10. At that event, our newly elected Secretary, Jonathan Gregg, was also awarded the Second Place Trophy for last spring's Honorama, an annual bowling tournament for honoraries of Hillsdale College. The KME team came in fifth in the overall competition and we are looking to do even better this year. On October 1, KME and the Math Department co-sponsored a Student Mathematics Symposium, where KME students Jonathan Gregg, Ian Markwood and Hannah Yee presented the results of their summer research while Heidi Schweizer talked about her Budapest Semester in Mathematics. Three more students (two doing research and one doing an actuarial internship) were unavailable to speak. Three students attended the Michigan Undergraduate Mathematics Conference held at Grand Valley State University, and Hannah Yee presented her summer REU research. We hosted our second annual Euchre Night on October 27, and recognized our 14 new members. On December 15, we sponsored a Final Exam Study Break.

New Initiates - Aubrey Childs Annis, Patricia Bassett, Jaclyn A. Beattey, Brigitta Estelle Burguess, Casey Gresenz, Casey Haggerty, David S. Montgomery, Miriam L. Poole, Jamin M. Rager, Daniel Rhodes, Ethan Thomas Smith, Edward Leo Sutherland, Jennifer Waller, and Roxanna C. West.

**MI Epsilon – Kettering University**

*Chapter President – Jessi Harden (A Section) and Matthew Sornig (B Section); 198 Current Members*

*Other Fall 2010 Officers: Brian Curbin (A Section) and Starla Walters (B Section), Vice Presidents; Keishawna Baker (A Section) and Shahnoor Amin (B Section), Secretaries; Derek Hazard, Kasey Simons, and Michael Steinert, Officers; Boyan N. Dimitrov, Corresponding Secretary; and Ruben Hayrapetyan (Section A – Winter and Summer terms), and Ada Cheng (Section B – Spring and Fall terms), Faculty Sponsors*

At Kettering University, we the traditional Pizza/Movie Parties with the movie "Infinite Secrets" about Archimedes lost book on August 11 and 25. For 10th consecutive year, we hosted the KU High School Mathematics Olympiad, organized by the Mathematics group of enthusiastic faculty, (http://paws.kettering.edu/~acheng/Olympiad/new-winners.html). The competition is designed to identify and encourage students with interests and abilities in mathematics, and our goal is to develop the Olympiad into one of the most prestigious mathematical competitions in the region. The examination is designed for students in grades 9 through 12, consists of six challenging problems and has a time limit of four hours. The problems range from "mind-benders" that require little mathematical skills to problems that require the knowledge of geometry, trigonometry and beginning calculus. The winners were: First Place: Joseph Renzi,10th grade at University Liggett School; Second Place: Mason Liang, 12th grade at Troy High School. Third Place: Dalton Allan, 12th grade at Saginaw Arts and Science Academy and SVSU. Fourth - Seven Place: Matthew Bauerle, 11th grade and homeschooled; Magda Lee Hlavacek, 10th grade in Saginaw Arts and Science Academy and SVSU; Alex Kitchin, 12th grade at Flushing Senior High School; and Mayank Patke, 10th grade at Okemos High School.

**MO Alpha – Missouri State University**

*Chapter President – Christina Tharp; 36 Current and 7 New Members*
*Other Fall 2010 Officers: Brett Foster, Vice President; Ashley Lewis, Secretary; Lee Hicks, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor*

Seminars were held on the following dates with the following speakers: 09/29/10–KME Annual Picnic; 09/22/10–KME Seminar with speaker John Havel (Biology), MSU; 10/28/10–KME Seminar and Math Power Hour with Math games and contests; and11/30/10–KME Seminar with speakers Jeff Chapman and Ashley Lewis (Mathematics), MSU.

New Initiates - Brian Barnhouse, Ashley Bartkoski, Miles Collins, Josh Hartman, Peng Hou, Sarah Kramer, and Kelsey Ryan.

**MO Beta – University of Central Missouri**

*Rhonda McKee, Corresponding Secretary*

New Initiates - Amy Billups, John Crooker, Codey Davis, Jennifer Granicke, Zachary Foster, Sara Kennedy, David Lewis, Kevin Loeffler, Annie Lowe, Christopher Purcell, Emilee Rice, Hannah Williams, and Alexandra Wolf.

**MO Theta – Evangel University**

*Chapter President – Rosemary Sherwood; 11 Current Members*
*Other Fall 2010 Officers: Lindsay Paur, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor*

Meetings were held monthly. In September, we had our first meeting at the home of Don Tosh. In November, most members were able to attend the math conference held at Missouri State University.

New Initiates - Elizabeth Baumeister, Rebecca E. Dalstein, Jonathan Ryan Faggart, Richard Grauberger, Nathaniel McGinnis, Danika C. Lindsey, and Katie Strand.

**MO Iota - Missouri Southern State University**

*Chip Curtis, Corresponding Secretary; Grant Lathrom and Rich Laird, Faculty Sponsors*

New Initiates - Nicole Green, Adebayo Orunpekun, Jared Smith, and Peter Thompson.

**MO Mu - Harris-Stowe State College**

*Dr. Ann Podleski, Faculty Sponsor*

KME activities are combined with an open Math Club. We sponsor review sessions for the mathematics certification exams. We also have a series of hands-on math activities that are open to the entire university community. In December we had a session entitled "Coloring Pascal's Triangle." We began planning for the KME National Convention, which Harris-Stowe is hosting in April 2011.

**MO Nu – Columbia College**

*Tomas Horvath, Chapter President; 10 Current Members*

*Other Fall 2010 Officers: Chris Hawkins, Vice President; Kyle Christian, Secretary; Austin Miller, Treasurer; and Dr. Kenny Felts, Corresponding Secretary and Faculty Sponsor*

New Initiates - Serena Jenkins, Ran Kim, Giang Le, Rahel Lemma, Olim Negmatov, Carolyn Summers, AnniLauri Villeme, and Tabitha Williams.

**MS Alpha – Mississippi University for Women**

*Chapter President – Kerri Dewitt; 11 Current Members*

*Other Fall 2010 Officers: Matthew Toncrey, Vice President; Tyler Greer, Secretary; Jami Henry, Treasurer; Dr. Shaochen Yang, Corresponding Secretary; and Dr. Joshua Hanes, Faculty Sponsor*

On September 29th, we discussed future projects for KME, interesting topics in mathematics, and had some tasty snacks. And on November 17th, we assembled four boxes for "Operation Christmas Child" of Samaritan's Purse, and had refreshments.

**NC Epsilon – North Carolina Wesleyan College**

*Bill Yankosky, Corresponding Secretary*

New Initiates – Holly Lauren Deaver, Trevour Andrew Huber, Jenalee Michele McFadden, Linh Su Nguyen, Brittany Nichols, Tyler Kevin Olkowski, and Deanna Petersen.

**NC Zeta – Catawba College**

*Chapter President – Cynthia Cook; 16 Current Members*

*Other Fall 2010 Officers: Spencer Ashley, Vice President; Zachary Owen, Secretary; Bridgett Hendersen, Treasurer; and Doug Brown, Corresponding Secretary and Faculty Sponsor*

KME and Math Club members ran a weekly help session, open to any students with questions about any of their mathematics courses, and planned and hosted events such as a movie night (A Beautiful Mind), fundraising activities and activities for Pi Day next spring. Five new members were initiated on February 3.

New Initiates – Alan Burgess, Jacob Hill, Mark Ketterer, Joseph Manser, and Jonathon McNeill.

**NC Eta – Johnson C. Smith University**

*Chapter President, Niketa Jones; 14 Current Members*

*Other Fall 2010 Officers: Maurice Scott, Vice President; Shimeca Bowman, Secretary; Quadashia Walker-Moss, Treasurer; Dr. Lakeshia Legette, Corresponding Secretary; and Dr. Brian Hunt, Faculty Sponsor*

New Initiates – Gerald Agbegha, Jerran Banks, Merischia Griffin, Dr. Nailong Guo, Dr. Dawn McNair, Ashley Moore, Brigette Pitts, Amber Shoecraft, Mikkita Stevens, Sasha Thornhill, and Dr. Hampton Wright.

**NE Alpha – Wayne State College**

*Chapter President – Hannah Lee; 3 Current and 10 New Members*

*Other Fall 2010 Officers: Baili Klein, Vice President; Katie Svec, Secretary; Kyle Martin, Treasurer; and Dr. Jennifer Langdon, Corresponding Secretary and Faculty Sponsor*

This semester, we initiated 10 members. It had been over four years since we've had initiates, so this was a banner year! We also won third prize in the homecoming banner competition, decorated a math-themed Christmas tree as part of a fundraiser for underprivileged children, and painted/decorated the math department's computer lab rescuing it from its former basement-drab condition.

New Initiates - Deena Bignell, Amy Doerr, Emily Gardner, April Groteluschen, Jennifer Haselhorst, Jake Hirz, Jennifer Langdon, Amy Maika, Eric Snitily, and Christy Wilson.

**NE Beta – University of Nebraska Kearney**

*Chapter President – Valerie Sis; 13 Current and 5 New Members*

*Other Fall 2010 Officers: Kandi Young, Vice President; Brian Flannery, Secretary; Kali Anderson, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor*

This fall, five KME members participated in the Homecoming parade. Three marched, each with a sign with a letter, spelling out KME; the other two gave out candy. We sent thank you letters, including photos, to the five charter members who came to our 50th Anniversary Celebration last spring. We were asked for information for a university press release on KME, and our president designed a brochure about KME and our chapter, copies of which were given out at a student function. We are planning a Math Fun Day at a local school, to be held in April.

New Initiates - Claire Aylward, Josh Brummer, Koichi Sato, Laura Slaymaker, and Brent Wheaton.

**NE Delta – Nebraska Wesleyan University**

*Chapter President – Brent McKain; 13 Current Members*

*Other Fall 2010 Officers: Macklin Warrington, Vice President; Abigail Raasch, Secretary; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor*

In the autumn we had an event where participants in various research experiences for undergraduates spoke. At another event a rubix cube was solved slowly with explanation, and then a rubix cube race ensued. To close the term we had a holiday party with the Physics Club where chili was eaten, gifts were exchanged, and mathematical carols were sung.

New Initiates – Dana Anderson, Amanda Ardito, Linda Arthur, Laura Booton, Mary Canarsky, Michelle Koke, and Joseph Menousek.

**NH Alpha – Keene State College**

*Vincent J. Ferlini, Corresponding Secretary*

New Initiates - Abigail Ball, Joshua Binder-Brantley, Jessica Boland, Caitlin Bowen, Heather Burbine, Matthew Caputo, Megan Ferm, Alexandra Petrilli, Eric Sansone, and Katlyn Santosuosso.

**NJ Delta – Centenary College**

*Chapter President – Kim Kupper; 18 Current Members*

*Other Fall 2010 Officers: Ashley Burger, Vice President; Carissa Utter, Secretary; Brandon Iuzzolin, Treasurer; and Kathy Turrisi, Corresponding Secretary and Faculty Sponsor*

**NJ Epsilon – New Jersey City University**

*Chapter President – Peter Morin; 22 Current Members*

*Other Fall 2010 Officers: Phil Carrillo, Vice President; Tracy Goycochia, Secretary; Cody Ching, Treasurer; Dr. Beimnet Teclezghi, Corresponding Secretary; and Dr. Yi Ding, Faculty Sponsor*

**NY Mu – St. Thomas Aquinas College**

*Dr. Marie Postner, Corresponding Secretary*

New Initiates - Amy Aquilina, Renee C. Bluszcz, Stephen M. De Paul, Heather Lynn Edsall, James C. Joy, Jeannine M. Mulder, and Leanne Nicole Urbancik.

**NY Nu – Hartwick College**

*Chapter President – Amanda Cappelli; 18 Current Members and 1 New Member*

*Other Fall 2010 Officers: Dechhin Lama, Vice President; Julie Kessler, Secretary; Rebecca Lounsbury, Treasurer; and Ron Brzenk, Corresponding Secretary*

**NY Omicron – St. Joseph's College**

*Chapter President – Melissa A. Bernstein; 40 Current Members*
*Other Fall 2010 Officers: Charles C. Essig, Vice President; Maggie Kumpas, Secretary; Gabriela Rodrigues, Treasurer; Elana Reiser, Corresponding Secretaries; and Dr. Donna Marie Pirich, Faculty Sponsor*

KME members spent the Fall semester volunteering to tutor in the Mathematics Clinic, where local high school students can get free tutoring. We have also held bake sales and raffles to raise money to send representatives to the national conference.

New Initiates - Salvatore J. Alfredson, Lauren Beaudoin, Steven Brucato, Christina Calvarese, Thiessen Charles, Alessandro da Luz, Alexander De Ridder, James Ehrhardt, Megan E. Fensterer, Edward M. Gocinski, Jillian Kearney, Philip Lombardo, Alison Nunziata, Melissa O'Connell, Samantha R. O'Connor, Kerry Ojakian, Alyssa Quagliata, Brittany Michele Silver, Shannon M. Stark, Alison E. Stephens, Jennifer Turturro, Maria C. Werner, Michael Wheaton, and Robert J. Woods.

**NY Pi – Mount Saint Mary College**

Lee Fothergill, Corresponding Secretary

New Initiates – Bridget M. Costello, Theresa Dabroski, Gregory J. Dowling, Matthew Brandt Fowler, Jessica Giordano, Amy Goldstein, Allison E. Hasse, Christine D. Lauber, Sara Ann Soll, and Jennifer Weber.

**NY Rho - Molloy College**

*Chapter President – Kimberly Thompson; 51 Current and 31 New Members*
*Other Fall 2010 Officers: Jennifer Zontini, Vice President; Marissa Cusa, Secretary; Amin Hashimi, Treasurer; Manyiu Tse, Corresponding Secretary; and Deborah Upton, Corresponding Secretary and Faculty Sponsor*

Our chapter piloted "Calculus Corner," a walk-in for those that need help in Calculus (as well as other math courses). All the tutors volunteered their time to make it happen.

New Initiates – Lisa Marie Amabile, Genevieve Brzezinski, Megan Butterworth, Erika Capogna, Sanna Cheema, Brian Ciampo, Brigid Damm, Jillian Dutra, JoBeth Dutra, Vanessa Estevez, Marissa Felice, Daniel Flanick, Taylor Flinn, Alfeen Hasmani, Amanda Kovacs, Patricia Lyons, Stefanie Macaluso, Joanna Mantone, Christina Marra, Annmarie Pagano, Gillian Plaia, Nicole Reverberi, Meghan Schmidt, Claire Troiano, and Andrea Turrisi.

**OH Gamma – Baldwin-Wallace College**

*David Calvis, Corresponding Secretary*

New Initiates – Adam J. Bianchi, Michelle A. Blevins, Matthew J. Ciha, Christopher L. Cramer, Julia A. Donajkowski, Gina Mingo, Adam E. Pengal, Hannah V. Shoemaker, Anthony M. Testa, Alexander J. Trzeciak, and Sarah L. Widener.

**OK Alpha – Northeastern State University**

*Chapter President – Toni Slagle; 55 Current and 12 New Members*

*Other Fall 2010 Officers: Seth Vansell, Vice President; Jacob Curley, Secretary; Jonathan Moyer, Treasurer; and Dr. Joan E. Bell, Corresponding Secretary and Faculty Sponsor*

Our fall initiation brought 12 new members into our chapter. At our September meeting, Dr. Giovanni Petris from the University of Arkansas spoke on "Bayesian statistics, or how to combine historical information with data," and also spoke with students about their graduate program in mathematics. Dr. Bell showed the "classroom edition" DVD of the Disney movie Donald in Mathmagic Land, which features scene selection and clips correlated for three grade bands (3-5, 6-8, and 9-12). We spent one evening calling alumni of the College of Science and Health Professions and asking for their support. We ended the semester with a Christmas party for KME members, math majors and faculty. After eating pizza and Christmas treats, we played the logic game Mafia.

New initiates – Summer L. Bingham, Roderick L. Bledsoe, Shelbi N. Bowin, Kalin M. Bradshaw, Blane H. Burge, Xue Dang, Molly A. Erwin, Tatsuya Eto, Rebecca C. Folsom, Erik J. Friend, Jonathan H. Garcia, Leah L. Imboden, Ashley K. Keys, Randee J. McBride, Abraham Middleton, Gregory S. Palma, Tanisha N. Payne, Taylor M. Pride, Joshua L. Qualls, Tandy R. Roberts, Wen Shao, Jordan D. Smith, Brent A. Spencer, and Amanda L. Willinger.

**OK Delta – Oral Roberts University**

*Chapter President – Lori Fielding; 203 Current Members; 12 New Members (Fall 2010), 8 (Spring 2010)*

*Other Officers: Daniel Holman (Fall 2010) and Grant Shaida (Spring 2010), Vice Presidents; Jessica Shearer (Fall 2010) and Jesse Patsolic (Spring 2010), Secretary/Treasurers; and Dr. Vincent Dimiceli, Corresponding Secretary and Faculty Sponsor*

**OK Epsilon – Oklahoma Christian University**

*Chapter President – Jacob Clark; 31 Current Members and 11 New Members*

*Other Fall 2010 Officers: Cady Block, Vice President; Jordan Courtemanche, Secretary and Treasurer; Dr. Ray Hamlett, Corresponding Secretary and Faculty Sponsor; and Craig Johnson, Faculty Sponsor*

Oklahoma Epsilon projects for the current year include facilitating our third annual High School Mathematics contest in March and tutoring at-risk inner-city children in South Oklahoma City.

New initiates – Lexi Brown, Anna Hyldahl, Jonathan McCallum, Matthew Miller, Shaylee Patzer, Nathaniel Spencer, Teaven Taylor, and Ivan Yeah.

**PA Beta - LaSalle University**

*Chapter President – Veronica Ventura*

*Other Fall 2010 Officers: Stephen Kernytsky, Vice President; Rose Venuto, Secretary; Ryan Cunningham, Treasurer; Luke Giordano, Events Coordinator, and Stephen Andrilli, Corresponding Secretary and Faculty Sponsor*

We hosted the first joint meeting of the EPaDel (Eastern PA and Delaware) and NJ (New Jersey) sections of the MAA on November 6; there were three major talks and two workshops, as well as undergraduate and graduate paper sessions. Ten members of our Math Club (aka KME) served as student-volunteers during the meeting handling registration and book sales, and moderating student paper sessions, etc. This was a wonderful opportunity for our students to attend interesting talks and meet faculty and students from other local colleges and universities.

**PA Iota – Shippensburg University**

*Chapter President – Laura Henzy; 734 Current and 4 New Members*

*Other Fall 2010 Officers: Chad Nunemaker, Vice President; Lauren Robinson, Secretary; Drew Snyder, Treasurer; Dr. Paul Taylor, Corresponding Secretary and Faculty Sponsor.*

**PA Kappa – Holy Family University**

*Chapter President – Michael Browning; 9 Current Members*

*Other Fall 2010 Officers: Jacqueline Gallelli, Vice President; Alyssia Overline, Secretary; Michelle Kustra and Katie Blumenstock, Treasurers; and Sister Marcella Louise Wallowicz, CSFN, Corresponding Secretary and Faculty Sponsor*

On October 30, our KME members and the Math Club hosted its 4th annual Evening of Mathematical Suspense, a Halloween-themed event in the form of a Math Murder Mystery/Dinner Theatre in which participants solve math problems in order to obtain the clues to solve the murder mystery. Approximately 35 students participated, enjoying pizza, refreshments, and university logo items as prizes. In December, KME members performed during the university's annual Christmas celebration, singing both 'Twas the Night Before Finals and Rudolph the Tangent Function. We established an after school mathletes program at a South Philadelphia elementary school. Mike Browning, Jackie Gallelli and Alyssia Overlive coached approximately 25 elementary students who participated in the program during the Fall. The 4 candidates for Spring 2011 initiation began a peer math tutoring program at the University. Each candidate tutored for 10 hours during the Fall semester. Members and candidates held several bake sales to raise money to support planned Spring activities.

New Initiates – Emily Anick, Angela Hand, and Gidget Montelibano.

**PA Lambda – Bloomsburg University**

*Elizabeth Mauch, Corresponding Secretary*

New Initiates – Rodrigo Cano, Lauren Rumberger, Ray Steffen, and Jarid Yanos.

**PA Mu – Saint Francis University**

*Chapter President – Michelle Wetzel; 53 Current and 19 New Members*

*Other Fall 2010 Officers: Katie Dacanay, Vice President; Colin Trout, Secretary; Laura Stibich, Treasurer; Peter Skoner, Corresponding Secretary; and Katherine Remillard, Faculty Sponsor*

On August 26, several members presented their research from the summer at the Fifth Annual Undergraduate Research Poster Symposium. On September 28th, an audio conference sponsored by the Association of American Colleges & Universities was presented entitled "More Options for Women in Science." On October 6, several KME members participated in the Commissioning Service in the University Chapel for students who perform community service. At the 17th Annual Science Day held November 23, KME members served as session moderators for faculty making presentations, and moderators, judges, scorekeepers, and timers for the Science Bowl; a total of 435 high school students from 26 area high schools attended.

New Initiates - Jenna Bailey, Marissa Basile, Quy Cao, Dane-Marie Greaves, Addison Fox, Courtney Francis, Theodore Jagielski, Maura Jones, Sean Kane, Ryan Knee, Dr. Ying Li, Adam Mengel, Lucas Mignogna, Brittany Miller, Julie Moore, Amber Shaikh, Jessica Ulishney, Matt Warfel, and Mara Weinzierl.

**RI Alpha – Roger Williams University**

*Chapter President – Raveena Siegel; 18 Current Members*

*Other Fall 2010 Officers: Erin Gilliam, Vice President; Adrianna Johnson, Secretary; Sarah Jeanfavre, Treasurer; and Annela Kelly, Corresponding Secretary and Faculty Sponsor*

Our chapter held several meetings and a bake sale fundraiser before the holidays. A student team attended a mathematics team competition at MAA NES meeting in Providence, RI. We are making plans to hold an initiation in the spring semester.

**SC Epsilon – Francis Marion University**

*Damon Scott, Corresponding Secretary*

New Initiates - Curtis M. Jones, Charles J. Nettles, Kristen Dione Shaw, Daniel Stone, Abbey E. Sullivan, Derek J. Turner, and Zachary Wilson.

**SC Gamma – Winthrop University**

*Dr. Trent Kull, Corresponding Secretary*

New Initiates – Matthew Harrison Neal, Ryan Patrick Nikin-Beers, Heather Marie Schneck, Kimberly Ann Schneck, and Whitney Anne Taylor.

**TN Alpha – Tennessee Technological University**

*Andrew J. Hetzel, Corresponding Secretary*

New Initiates - Stephanie Amato, Bridgette Buchanan, Samuel Carruthers-Thorne, John Carter, Kevin Casler, Erin Chambers, Raven Cross, Evan Dirube, Micah Eller, Rebecca Escue, Jackson Ewton, Sarah Flanigan, Douglas Ford, Leah Frauendienst, Sarah Frizzell, Cathleen Fry, Robert Griffin, Elizabeth Hess, Eric James, Seth Latture, James Leverette, Jeremy Miller, Leslie Moore, Eric Morgan, Brittany Murphy, Paige Nash, Annie Powers, Cassie Putman, Nicole Reese, Arturo Santa Ruiz, Katlyn Smegelsky, Mark Straussberger, Jie Tang, Matthew Thompson, David Velez, and Jake Wilson.

**TN Beta – East Tennessee State University**

*Chapter President – Jeffrey Bonnell; 11 New Members*

*Other Fall 2010 Officers: Jeremy Brooks, Vice President; Elizabeth Harris, Secretary; Andrew Herron, Treasurer; Robert Gardner, Corresponding Secretary and Faculty Sponsor*

Our fall semester started with a meeting to discuss our budget, department "logo" contest, possible solutions to the problems presented in the Pentagon, and a presentation by our KME coadvisor Dr. Gardner of "Permutation Groups: Cycles, Transpositions, and Futurama." We had a "problem day" that resulted in a submission of Pentagon problem solutions by three of our chapter members: Deering, Jamieson, and Herron. We have also made a few purchases this year, including "The Story of Math" on DVD. Visit http://faculty.etsu.edu/gardnerr/KME/KME.html for our home page.

**TN Gamma – Union University**

*Chapter President – Rebecca Eaton; 22 Current Members*

*Other Fall 2010 Officers: Emilie Huffman, Vice President; Kim Lukens, Secretary/Treasurer; Seth Kincaid, Historian/Webmaster; Michelle Nielsen, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor*

On September 13, the Tennessee Gamma Chapter held the annual back-to-school cookout at the Union University campus. Several Union mathematics students and faculty, along with their family members, were in attendance.

**TX Gamma – Texas Woman's University**

*Dr. Mark Hamner, Corresponding Secretary*

New Initiates – Cammy Boaz, Melanie Cannon, Meg Chetwood, Lilian Chu, Yolanda Flores, Greg Gengo, Loree Johnson, Violeta Rodriguez, Crystal Smith, Brittany Watson, Emma Zemler, and Preeti Paliwal.

**TX Iota – McMurry University**

*Dr. Kelly McCoun, Corresponding Secretary*

New Initiates – Michael Herriage, Tiffany Keasler, Mike Luval, Tylar Murray, Aaron Ward, Kamron Ward, Austin Wegner, and Robert Wheeler.

**TX Mu – Schreiner University**

*Chapter President – Audra Burnap; 18 Current Members*

*Other Fall 2010 Officers: Denise Begley, Vice President; Caitlin Gayle, Secretary; Antonio Rameriz, Treasurer; William M. Sliva, Corresponding Secretary*

This fall, Matthew Moreno and Antonio Ramirez presented their ongoing research each at a separate noon meeting. They are hoping to attend and present at the national conference.

New Initiates – Brittany Elise Cardwell, Rebecca Mary Chiaro, Danielle Jean DeBacker, Molly K. Hutcherson, William Geoffrey Keaton, Austin F. Loza, Amanda Noel Ludwig, Marcus Paul Myhaver, and Madison Catherine Nelson.

**VA Delta – Marymount University**

*Chapter President – Hannah Korbach; 31 Current and 6 New Members*

*Other Fall 2010 Officers: Matthew Villemarette and Eric Kamta jointly hold the positions of Vice President, Secretary, and Treasurer; William Heuett, Corresponding Secretary; and Elsa Schaefer, Faculty Sponsor*

We had one meeting on December 5, at Dr. Elsa Schaefer's residence to initiate new members and to enjoy an evening together with games and food. Fifteen people, including students, faculty, family and friends, members and non-members, were in attendance.

New Initiates - Amanda Billy, Mike Bokosha, Atanaska Dobreva, Eric Kamta, Hannah Korbach, and Matthew Villemarette.

**WI Gamma – University of Wisconsin-Eau Claire**

*Chapter President – Mark Bauer; 80 Current and 26New Members*

*Other Fall 2010 Officers: Joshua Frinak, Vice President; Lindsay Brunshidle, Secretary; Hong Yang, Treasurer; and Dr. Simei Tong, Corresponding Secretary and Faculty Sponsor*

UWEC students Josh Frinak and Austen Ott received a poster award at undergraduate poster section at the annual Joint Mathematics Meetings for their project "Constructing Moduli Spaces of Low Dimensional $A\infty$-Algebras by Extensions" under the direction of Dr. Michael Penkava. This was the fourth consecutive year that his students received an award at the conference. Others from UWEC who presented posters included:

- Shawn Peters and Becky Sippert, faculty advisor Dr. Simei Tong, *Classifying Complemented Subspaces of $L_p$, $2 < p < \infty$, with Alspach Norm*;

- Chelsey Drohman, Ying Yang, and Alice Oswalt, faculty advisers Dr. Kate Masarik and Dr. Tong, *An International Study of Mathematics in the Middle Grades: China, Russia, and the United States*;
- Bret Meier and Austen Ott, faculty adviser Dr. Colleen Duffy, *Polynomial Equations over Matrices*;
- Tristan Williams with research team members from the University of St. Thomas, Worcester State University and the University of Indianapolis, *An Exploration of Ideal-Divisor Graphs*.

New Initiates - Lindsey Alger, Joseph Anderson, Patrick Bagan, Julia Baranek, Brittany Bauer, Travis Bischel, Kristina Bleess, Jake Bohlmann, Sam Brueggen, Wai Shan Chan, Cole Cook, Tim Deckers, Brian Fastner, Kimberly Finco, Kurt Flesch, Mitch D. Gardner, Kaisey Garrigan, Adam Gewiss, Lindsey Gohr, Ashley Grunau, Kevin Thomas Hankes, Eileen Heughins, Jeremy Kieser, Jacob Korinek, Wendell Tan Vooi Ley, Alyssa Markuson, Bret Meier, Hannah Miller, Aaron Moe, Michael North, Deana Petersen, Kyle Riesen, Stephanie Anne Ringsred, Corey Schulz, Jessica Spurr, Trevor Thompson, Anton Tillmann, Chun Yang Tang, Krystal Urness, Reba Van Beusekom, and Ying Yang.

# *Active Chapters of Kappa Mu Epsilon*

*Listed by date of installation*

| Chapter | Location | Installation Date |
|---|---|---|
| OK Alpha | Northeastern State University, Tahlequah | 18 April 1931 |
| IA Alpha | University of Northern Iowa, Cedar Falls | 27 May 1931 |
| KS Alpha | Pittsburg State University, Pittsburg | 30 Jan 1932 |
| MO Alpha | Missouri State University, Springfield | 20 May 1932 |
| MS Alpha | Mississippi University for Women, Columbus | 30 May 1932 |
| MS Beta | Mississippi State University, Mississippi State | 14 Dec 1932 |
| NE Alpha | Wayne State College, Wayne | 17 Jan 1933 |
| KS Beta | Emporia State University, Emporia | 12 May 1934 |
| AL Alpha | Athens State University, Athens | 5 March 1935 |
| NM Alpha | University of New Mexico, Albuquerque | 28 March 1935 |
| IL Beta | Eastern Illinois University, Charleston | 11 April 1935 |
| AL Beta | University of North Alabama, Florence | 20 May 1935 |
| AL Gamma | University of Montevallo, Montevallo | 24 April 1937 |
| OH Alpha | Bowling Green State University, Bowling Green | 24 April 1937 |
| MI Alpha | Albion College, Albion | 29 May 1937 |
| MO Beta | University of Central Missouri, Warrensburg | 10 June 1938 |
| TX Alpha | Texas Tech University, Lubbock | 10 May 1940 |
| KS Gamma | Benedictine College, Atchison | 26 May 1940 |
| IA Beta | Drake University, Des Moines | 27 May 1940 |
| TN Alpha | Tennessee Technological University, Cookeville | 5 June 1941 |
| MI Beta | Central Michigan University, Mount Pleasant | 25 April 1942 |
| NJ Beta | Montclair State University, Upper Montclair | 21 April 1944 |
| IL Delta | University of St. Francis, Joliet | 21 May 1945 |
| KS Delta | Washburn University, Topeka | 29 March 1947 |
| MO Gamma | William Jewell College, Liberty | 7 May 1947 |
| TX Gamma | Texas Woman's University, Denton | 7 May 1947 |
| WI Alpha | Mount Mary College, Milwaukee | 11 May 1947 |
| OH Gamma | Baldwin-Wallace College, Berea | 6 June 1947 |
| CO Alpha | Colorado State University, Fort Collins | 16 May 1948 |
| MO Epsilon | Central Methodist College, Fayette | 18 May 1949 |
| MS Gamma | University of Southern Mississippi, Hattiesburg | 21 May 1949 |
| IN Alpha | Manchester College, North Manchester | 16 May 1950 |
| PA Alpha | Westminster College, New Wilmington | 17 May 1950 |
| IN Beta | Butler University, Indianapolis | 16 May 1952 |
| KS Epsilon | Fort Hays State University, Hays | 6 Dec 1952 |
| PA Beta | LaSalle University, Philadelphia | 19 May 1953 |
| VA Alpha | Virginia State University, Petersburg | 29 Jan 1955 |
| IN Gamma | Anderson University, Anderson | 5 April 1957 |
| CA Gamma | California Polytechnic State University, San Luis Obispo | 23 May 1958 |
| TN Beta | East Tennessee State University, Johnson City | 22 May 1959 |
| PA Gamma | Waynesburg College, Waynesburg | 23 May 1959 |
| VA Beta | Radford University, Radford | 12 Nov 1959 |
| NE Beta | University of Nebraska—Kearney, Kearney | 11 Dec 1959 |
| IN Delta | University of Evansville, Evansville | 27 May 1960 |

| | | |
|---|---|---|
| OH Epsilon | Marietta College, Marietta | 29 Oct 1960 |
| MO Zeta | University of Missouri—Rolla, Rolla | 19 May 1961 |
| NE Gamma | Chadron State College, Chadron | 19 May 1962 |
| MD Alpha | College of Notre Dame of Maryland, Baltimore | 22 May 1963 |
| CA Delta | California State Polytechnic University, Pomona | 5 Nov 1964 |
| PA Delta | Marywood University, Scranton | 8 Nov 1964 |
| PA Epsilon | Kutztown University of Pennsylvania, Kutztown | 3 April 1965 |
| AL Epsilon | Huntingdon College, Montgomery | 15 April 1965 |
| PA Zeta | Indiana University of Pennsylvania, Indiana | 6 May 1965 |
| AR Alpha | Arkansas State University, State University | 21 May 1965 |
| TN Gamma | Union University, Jackson | 24 May 1965 |
| WI Beta | University of Wisconsin—River Falls, River Falls | 25 May 1965 |
| IA Gamma | Morningside College, Sioux City | 25 May 1965 |
| MD Beta | McDaniel College, Westminster | 30 May 1965 |
| IL Zeta | Domincan University, River Forest | 26 Feb 1967 |
| SC Beta | South Carolina State College, Orangeburg | 6 May 1967 |
| PA Eta | Grove City College, Grove City | 13 May 1967 |
| NY Eta | Niagara University, Niagara University | 18 May 1968 |
| MA Alpha | Assumption College, Worcester | 19 Nov 1968 |
| MO Eta | Truman State University, Kirksville | 7 Dec 1968 |
| IL Eta | Western Illinois University, Macomb | 9 May 1969 |
| OH Zeta | Muskingum College, New Concord | 17 May 1969 |
| PA Theta | Susquehanna University, Selinsgrove | 26 May 1969 |
| PA Iota | Shippensburg University of Pennsylvania, Shippensburg | 1 Nov 1969 |
| MS Delta | William Carey College, Hattiesburg | 17 Dec 1970 |
| MO Theta | Evangel University, Springfield | 12 Jan 1971 |
| PA Kappa | Holy Family College, Philadelphia | 23 Jan 1971 |
| CO Beta | Colorado School of Mines, Golden | 4 March 1971 |
| KY Alpha | Eastern Kentucky University, Richmond | 27 March 1971 |
| TN Delta | Carson-Newman College, Jefferson City | 15 May 1971 |
| NY Iota | Wagner College, Staten Island | 19 May 1971 |
| SC Gamma | Winthrop University, Rock Hill | 3 Nov 1972 |
| IA Delta | Wartburg College, Waverly | 6 April 1973 |
| PA Lambda | Bloomsburg University of Pennsylvania, Bloomsburg | 17 Oct 1973 |
| OK Gamma | Southwestern Oklahoma State University, Weatherford | 1 May 1973 |
| NY Kappa | Pace University, New York | 24 April 1974 |
| TX Eta | Hardin-Simmons University, Abilene | 3 May 1975 |
| MO Iota | Missouri Southern State University, Joplin | 8 May 1975 |
| GA Alpha | State University of West Georgia, Carrollton | 21 May 1975 |
| WV Alpha | Bethany College, Bethany | 21 May 1975 |
| FL Beta | Florida Southern College, Lakeland | 31 Oct 1976 |
| WI Gamma | University of Wisconsin—Eau Claire, Eau Claire | 4 Feb 1978 |
| MD Delta | Frostburg State University, Frostburg | 17 Sept 1978 |
| IL Theta | Benedictine University, Lisle | 18 May 1979 |
| PA Mu | St. Francis University, Loretto | 14 Sept 1979 |
| AL Zeta | Birmingham-Southern College, Birmingham | 18 Feb 1981 |
| CT Beta | Eastern Connecticut State University, Willimantic | 2 May 1981 |
| NY Lambda | C.W. Post Campus of Long Island University, Brookville | 2 May 1983 |
| MO Kappa | Drury University, Springfield | 30 Nov 1984 |
| CO Gamma | Fort Lewis College, Durango | 29 March 1985 |

| | | |
|---|---|---|
| NE Delta | Nebraska Wesleyan University, Lincoln | 18 April 1986 |
| TX Iota | McMurry University, Abilene | 25 April 1987 |
| PA Nu | Ursinus College, Collegeville | 28 April 1987 |
| VA Gamma | Liberty University, Lynchburg | 30 April 1987 |
| NY Mu | St. Thomas Aquinas College, Sparkill | 14 May 1987 |
| OH Eta | Ohio Northern University, Ada | 15 Dec 1987 |
| OK Delta | Oral Roberts University, Tulsa | 10 April 1990 |
| CO Delta | Mesa State College, Grand Junction | 27 April 1990 |
| PA Xi | Cedar Crest College, Allentown | 30 Oct 1990 |
| MO Lambda | Missouri Western State College, St. Joseph | 10 Feb 1991 |
| TX Kappa | University of Mary Hardin-Baylor, Belton | 21 Feb 1991 |
| SC Delta | Erskine College, Due West | 28 April 1991 |
| SD Alpha | Northern State University, Aberdeen | 3 May 1992 |
| NY Nu | Hartwick College, Oneonta | 14 May 1992 |
| NH Alpha | Keene State College, Keene | 16 Feb 1993 |
| LA Gamma | Northwestern State University, Natchitoches | 24 March 1993 |
| KY Beta | Cumberland College, Williamsburg | 3 May 1993 |
| MS Epsilon | Delta State University, Cleveland | 19 Nov 1994 |
| PA Omicron | University of Pittsburgh at Johnstown, Johnstown | 10 April 1997 |
| MI Delta | Hillsdale College, Hillsdale | 30 April 1997 |
| MI Epsilon | Kettering University, Flint | 28 March 1998 |
| KS Zeta | Southwestern College, Winfield | 14 April 1998 |
| TN Epsilon | Bethel College, McKenzie | 16 April 1998 |
| MO Mu | Harris-Stowe College, St. Louis | 25 April 1998 |
| GA Beta | Georgia College and State University, Milledgeville | 25 April 1998 |
| AL Eta | University of West Alabama, Livingston | 4 May 1998 |
| NY Xi | Buffalo State College, Buffalo | 12 May 1998 |
| NC Delta | High Point University, High Point | 24 March 1999 |
| PA Pi | Slippery Rock University, Slippery Rock | 19 April 1999 |
| TX Lambda | Trinity University, San Antonio | 22 November 1999 |
| GA Gamma | Piedmont College, Demorest | 7 April 2000 |
| LA Delta | University of Louisiana, Monroe | 11 February 2001 |
| GA Delta | Berry College, Mount Berry | 21 April 2001 |
| TX Mu | Schreiner University, Kerrville | 28 April 2001 |
| NJ Gamma | Monmouth University | 21 April 2002 |
| CA Epsilon | California Baptist University, Riverside | 21 April 2003 |
| PA Rho | Thiel College, Greenville | 13 February 2004 |
| VA Delta | Marymount University, Arlington | 26 March 2004 |
| NY Omicron | St. Joseph's College, Patchogue | 1 May 2004 |
| IL Iota | Lewis University, Romeoville | 26 February 2005 |
| WV Beta | Wheeling Jesuit University, Wheeling | 11 March 2005 |
| SC Epsilon | Francis Marion University, Florence | 18 March 2005 |
| PA Sigma | Lycoming College, Williamsport | 1 April 2005 |
| MO Nu | Columbia College, Columbia | 29 April 2005 |
| MD Epsilon | Stevenson University, Stevenson | 3 December 2005 |
| NJ Delta | Centenary College, Hackettstown | 1 December 2006 |
| NY Pi | Mount Saint Mary College, Newburgh | 20 March 2007 |
| OK Epsilon | Oklahoma Christian University, Oklahoma City | 20 April 2007 |
| HA Alpha | Hawaii Pacific University, Waipahu | 22 October 2007 |
| NC Epsilon | North Carolina Wesleyan College, Rocky Mount | 24 March 2008 |

| | | |
|---|---|---|
| CA Zeta | Simpson University, Redding | 4 April 2009 |
| NY Rho | Molloy College, Rockville Center | 21 April, 2009 |
| NC Zeta | Catawba College, Salisbury | 17 September, 2009 |
| RI Alpha | Roger Williams University, Bristol | 13 November, 2009 |
| NJ Epsilon | New Jersey City University, Jersey City | 22 February, 2010 |
| NC Epsilon | Johnson C. Smith University, Charlotte | 18 March, 2010 |
| AL Theta | Jacksonville State University, Jacksonville | 29 March, 2010 |
| GA Epsilon | Wesleyan College, Macon | 30 March, 2010 |
| FL Gamma | Southeastern University, Lakeland | 31 March, 2010 |
| MA Beta | Stonehill College, Easton | 8 April, 2011 |